Malthus in State Space: Macro Economic-Demographic Relations in English History, 1540 to 1870

The history of preindustrial Europe provides an opportunity to examine the causes and consequences of population change at a macro level. However, serious statistical problems arise from the endogeneity of all observed variables in a Malthusian system (fertility, mortality, population size, and real wages), and from unobserved influences such as shifts in the demand for labor and variations in health. These problems have undermined both informal inference from the data and more complex econometric investigations. This paper takes a new statistical approach, finding the maximum likelihood estimate of a state space representation of the Malthusian system by repeated application of Kalman filter methods, using annual data from England, 1540 to 1870. The new estimates confirm some findings of the earlier literature and contradict others. Some variables are estimated for the first time. Implications are discussed for the interpretation of English economic-demographic history.

Keywords: Malthus, England, Kalman, history, population, demography

Ronald Lee
Demography and Economics
University of California
2232 Piedmont Ave
Berkeley, CA 94720
rlee@demog.berkeley.edu
Phone: 510-642-4535
Fax: 510-643-8558

Michael Anderson
Mountain View Research
2251 Grant Road, A
Los Altos, CA 94024
mike@mvr.org>
Phone: 650-937-1280
Fax: 650-937-1284

We are grateful for comments on an earlier draft by David Brillinger, Joel Cohen, Kenneth Wachter, Jose Antonio Ortega and the Berkeley Economic History Seminar. Research was supported in part by a grant from NICHD, RO1-HD24982.

1. INTRODUCTION

The history of preindustrial England provides an opportunity to examine the causes and consequences of population change at a macro level, drawing on data that span more than three centuries. During this period, population size fluctuated widely and slowly about its long-term trend, reflecting variations in fertility and mortality. These fluctuations in population size apparently caused major swings in the level of the real wage. The real wage variations may, in turn, have been driving fertility and mortality. Untangling the relations among these interrelated variables involves serious statistical problems. These arise from the endogeneity of all observed variables in the Malthusian system (fertility, mortality, population size, and real wages), and from unobserved influences such as shifts in the demand for labor, climate change, and variations in health, many of which change very slowly. These problems have undermined both informal inference from the data and more complex econometric investigations. In this paper we take a new statistical approach, finding the maximum likelihood estimate of a state space representation of the Malthusian system by repeated application of Kalman filter methods, using annual data from England, 1540 to 1870. This approach allows us to deal effectively with many of the econometric problems that have weakened previous studies.

Our goal is not to explain the rich detail of levels, changes, and class differentials in the fertility, mortality, population size and real wage levels in England of this period. If it were, we would take a very different approach, with much more attention to specific historical influence and change. Instead, our purpose is to consider whether a Malthusian system was operating in the background, perhaps quite weakly, to shape the evolution and interaction of a few fundamental variables. The effects of such a system would necessarily operate through changes in disease, war, exposure or starvation; or through celibacy, age at marriage, marital fertility, or extra-marital fertility. These changes in turn might reflect changes in laws, occupational distribution, or a host of other economic and social changes. We could easily miss a weak but persistent Malthusian influence if we looked in one or another of these detailed proximate boxes. We believe that the rather abstract and broad-brush approach we take in this paper is appropriate to our task.

1.1. Importance of Problem

Malthusian population theory has three essential elements. *First*, it asserts that for a given demand for labor, labor force growth due to growing population encounters diminishing returns in agriculture, leading to falling real wages throughout the economy. Empirical tests of this assertion are particularly important, because this evidence may be relevant for the primarily agricultural economies of the Third World today. *Second*, it asserts that through the "positive check," higher wages cause lower mortality due to better nutrition, clothing and housing. Through the "preventive check," higher wages cause higher fertility, largely by permitting earlier marriage, but perhaps also because biological constraints on fertility are loosened, leading to higher fertility, or because couples choose higher fertility or shorter birth intervals. *Third*, when these causes and consequences of population change are considered together, they form an equilibrating system which regulates population in relation to the demand for labor so as to bring the level of wages to the "natural wage"; its long-run equilibrium. This equilibrating tendency of the system implies the "iron law of wages." The central contributions of Malthusian Theory were that population and wages tend toward equilibrium levels which are determined by the demand for labor, reproductive practices, the level of mortality, and the comparative static analysis of these equilibrium levels.

From these ideas, it follows that in the long run, population size grows or declines in proportion to shifts in the demand for labor, and in such a way as to leave the standard of living unchanged. Technical progress, new land, and investment in capital would all raise the demand for labor, which would serve only to enlarge the population in the long run, leaving living standards unchanged.

If this were so, then most efforts to improve the lot of humanity would be doomed to failure, leading only to larger populations. A further implication is that although population might be temporarily shocked away from equilibrium, and thereby have some independent effect on social and economic life, in the long run it must respond passively to changes in the demand for labor, and therefore be incapable of exerting an independent influence in history. An opposing view is that population size and growth reflect the mere accumulation of historical accidents, so that population size is itself an historical accident, and consequently, is an exogenous factor capable of operating as a fundamental cause of other historical developments. However, there is a middle ground, which leaves some room for exogenous influences on fertility and mortality to cause long-term population and wage changes, even if the overarching system is Malthusian. Malthus himself viewed reproductive behavior as an important explanation for differences across cultures and over time in levels of living. Here “reproductive behavior” is to be understood as a shift in the relation between wage levels and fertility, as opposed to movements along the fertility-wage schedule.

1.2. Econometric Background

To examine the impact of population growth on real wages with these historical data, the usual approach (Lee, 1973) is simply to regress the log of real wages, $\ln(W)$, on the log of population, $\ln(Pop)$, including a polynomial in time to allow for shifts in the demand for labor, for example:

$$\ln(W) = \alpha - \beta \ln(Pop) + \rho t + \varepsilon$$

where the time subscripts have been omitted. The equation has a straightforward interpretation: In the short run, the wage is determined by the intersection of a short-run supply of labor schedule and a short-run demand for labor schedule. Over time, both of these shift, leading to changes in the wage. Labor supply is assumed to be inelastic to the wage in the short run, and to be proportional to the size of the population in the long run. The demand for labor is assumed to have an elasticity of $-\beta$ with respect to the wage,[1] and it is assumed to shift outwards at exponential rate ρ/β. This means that if the population were to grow at the rate ρ/β, the wage would remain constant, while faster population growth would depress the wage, and slower growth would cause the wage to rise. ρ / β is called the "rate of labor absorption." Alternatively, if population were to remain constant, then the wage would rise at rate ρ. The following expression is proportional to the deterministic component of the wage:

$$\left[\frac{e^{\left(\frac{\rho}{\beta}\right)t}}{Pop_t}\right]^{\beta}$$

Regression estimates of the equation for ln(W) typically yield a highly significant positive estimate for β, consistent with the Malthusian hypothesis.

There are, however, several difficulties with this approach. *First*, according to Malthusian theory, the growth rate of population itself depends on the level of W, so population is an endogenous variable (see Lee, 1973, 1985a), and estimates of β will be biased by the correlation of $\ln(Pop)$ with ε.

Second, the disturbance term ε is highly autocorrelated, due to the patterns of climatic variation over time and to other sources of fluctuation in the demand for labor that are imperfectly reflected in the linear trend term in the regression (trade cycles, investment booms, waves of innovation). Many procedures for reducing the autocorrelation, such as taking first or second differences of the time series, remove the longer-run variance in the series and leave the shorter-run variance. Unfortunately, this worsens the simultaneity bias referred to in the first point above, since this bias arises most strongly in the effects of short-run variations of real wages on those in fertility and mortality (see Lee, 1981).

Third, as Lindert (1983, 1985) has pointed out, the surge of European population growth in the 16th century coincided closely with the surge in the general nominal price level, as did the attenuation of growth in the seventeenth centuries. Consequently, population size and the general price level are highly positively correlated. If the price level is entered in the equation above, then the coefficient on population becomes insignificant (Lindert, 1983, 1985).

Fourth, Bailey and Chambers (1994) assert that population is integrated to a higher order than are real wages, and therefore that real wages cannot properly be analyzed in relation to population size.

Fifth, some theory and some empirical work suggest that population growth drove technological progress (Boserup, 1965, 1981; Tsoulouhas, 1992), and if this were so, estimation would of course be complicated.

Sixth, neither the demand for labor nor its underlying influences - capital and technology - are observed, so previous work has assumed polynomial time trends. This imposes a strong constraint on the trajectory of the demand for labor, a constraint that is almost certainly inconsistent with the data.

There are additional problems in estimating the influence of real wages on the vital rates. Simultaneity bias is again a problem, since the vital rates affect population size, which in turn affects real wage levels. There is also good reason to believe that the disturbance terms in the vital rate equations are highly autocorrelated, so much so that one might prefer to think in terms of long-run shifts in the intercepts. There were long, deep swings in life expectancy that were not driven by wages and for which the origin remains obscure (see Wrigley and Schofield, 1981 and Lee, 1973). It appears that real wages were a relatively minor influence on vital rates except in the short run, and that there were many more powerful forces at work. In previous studies, the historical intrusion of these omitted variables has made it very difficult to estimate relations between wages and vital rates, other than for short-run fluctuations. The possibility of a thirty-year lag in the relationship of fertility to wages was raised by Wrigley and Schofield (1981); a lag which Lee (1985a) interpreted as an artifact of the simultaneity problem. It is also possible that a component of adult mortality responds with a very long lag to nutritional conditions in childhood (see Fogel, 1994). An additional problem is the strong cross-equation correlations in the disturbances to the mortality and the fertility equations, presumably due to the influence on both of the unobserved variations in health. There are also strong cross-equation correlations in the disturbances to the fertility and nuptiality equations, as analyzed in Lee (1975).

The literature has approached this collection of problems in three general ways: by widening the data base to include more European countries and deepening it to include earlier time periods (Lee, 1987; Weir, 1991), by imposing more theoretical structure on the empirical analysis (Lee, 1985a, 1988), and by refining the econometric techniques (Bailey and Chambers, 1994). Many problems, such as the simultaneity issue, the unobserved variation of the demand for labor, and shifting vital rate intercepts, remain unresolved.

Given that all four observed variables are endogenous, one might wonder how it is possible to identify the system at all, and what might be the status of previous estimates in the literature. Identification of this system comes from theoretically based assumptions about the temporal structure of the relationships; about stocks versus flows. These assumptions then have strong implications for the contrast between short-run and long-run covariances. Consider a simpler system in which intercepts in the equations are constant, rather than random walks. In this case, each of the four variables in a given year can be expressed as its initial value in 1540 plus a weighted sum of all the errors from 1540 to that year in the equations for fertility, mortality, and wages, with linear functions of the coefficients providing the weights (see Lee, 1985a and 1993a for explicit expressions). All four variables are functions of exactly the same historical streams of shocks. It can be shown that the short-run covariance is dominated by the causal effect of wage variation on fertility and mortality, because population size averages over centuries worth of variations in the vital rates, attenuating the effects of any current or recent shocks. The long-run covariance mainly, but not entirely, reflects the influence of population size on wages, but the feedback from wage changes through vital rates to population size operates so slowly and weakly. This is why earlier work was able to make progress by studying short-run fluctuations in the vital rates in relation to wages, and by studying the effect of population size on wages using very long-term averages of these variables as in Lee (1973) and Weir (1991).

For the first time, the formulation and estimation of state space models has the potential to deal with some of these problems in a straightforward and rigorous way within a coherent and flexible context, as opposed to the piece-meal approach of treating the long-run and short-run covariance separately. We have not attempted to deal with all of these problems in this paper, however, since estimation of even the basic model puts heavy demands on the data. In particular, we have not attempted to address the problem raised by Lindert (coincidental population growth and inflation) or Boserup (population size or growth as a cause of technological progress), although both can, in principle, be resolved using the state space model.

1.3. Substantive Controversies

The history of the period, and the statistical studies that illuminate it, pose a number of problems and controversies which the present analysis may help to resolve. First, English history, and indeed Western European history, reveals long and powerful fluctuations in the real wage (as well as other economic variables -- see Lee, 1980). Economic historians have suggested that these might have been driven by population change (Habakkuk, 1965; Phelps-Brown and Hopkins, 1957). Statistical studies have estimated elasticities of wages with respect to population size that are large in absolute value, ranging from -1.0 to -2.7 (see for example, Lee, 1980; Weir, 1991). If the elasticity of substitution of labor for land were 1, as assumed in a Cobb-Douglas production function, then the wage-population elasticity would be the negative of the share of labor in output, or perhaps about -.5[2]. The very large negative elasticities that are found seem to suggest a much lower elasticity of substitution of labor for land, which does not seem plausible based on results found from more direct studies of agriculture in other settings.[3]

Another controversy surrounds the influence of wages on fertility and the dynamic behavior of the whole system. It appears from the data that all the variables exhibit waves about two or three centuries long, with different phase patterns. Wrigley and Schofield (1981) suggest that wages are driving fertility with a lag of 30 to 50 years. Lee (1985a) suggests instead that if the wave in fertility were independent, then wages would appear to lead fertility by about a quarter cycle (50 years) when in fact they were being driven by variations in population size due to fertility fluctuations. Similar issues arise with mortality. How can we determine what is driving the motion of the system, when every variable we observe is endogenous?

The trend in the demand for labor is also of historical interest. In earlier work, Lee and others found that up until around 1800, the rate of labor absorption was about .4 percent per year. After 1800, the demand for labor appears to have accelerated dramatically, and the rate of absorption to have risen correspondingly. Because of the need to use polynomials in time to proxy for the demand for labor in earlier studies, details of its trajectory have been obscured, however.

Other matters of substantive interest include the estimates of the positive and preventive check, although their lag patterns appear to have been quite satisfactorily estimated in past research (Lee, 1981, 1993b, 1997; Galloway, 1988; Weir, 1984a;), at least for short-run fluctuations. Because the state space estimates do not involve prior detrending of the data series to remove longer-run variations, state space estimates may differ in their implications for longer-run relations.

1.4. Levels of Integration of the Series

A recent econometric study of these same variables (Bailey and Chambers, 1994), using the same data, asserted: “For the time period studied in this paper, the relationship between ln (w) and ln(P) of equation (4) cannot be sustained by the data, for ln(w) appears to be integrated of order one while ln(P) appears to be integrated of a higher order. Reliable inferences could not be drawn from a linear relationship between two such series. For this reason, the estimation of the parameters of equation (4) is not pursued further in this paper."

Malthusian theory in fact predicts that ln(P) would be integrated of a higher order than ln(w), just as Bailey and Chambers claim. Over the long run, the wage should trendlessly fluctuate around its equilibrium level, which is constant, while population will have a rising trend, reflecting secular increase in the demand for labor. Trendless fluctuation would not be consistent with a process integrated of order 1, but in practice it is very difficult or impossible to distinguish between a process that equilibrates, but is very highly autocorrelated, and a process that has no equilibrium and tends to drift away from its initial value in the long run.[4]

However, Malthusian Theory also tells us that what matters for wages is variation in the size of the population *relative to the demand for labor*, and that these *relative* variations should themselves be statistically stationary, and integrated of the same order as wages. The key is that the intercept term in the regression of ln(W) on ln(Pop) is not a constant, but instead reflects the level of the demand for labor. In our analysis below, the demand for labor is modeled as a random walk with drift. Alternatively, the equation could contain a proxy for the demand for labor, such as a polynomial in time as in Lee (1973, 1980), the measure of urbanization, as in Stavins (1988), or a more direct measure of progress, as in Tsoulouhas (1992). With any of these approaches, the deviations of population size from the demand for labor should be integrated of the same order as wages, permitting sound estimation. Put differently, the equation linking the level of wages to population size and the demand for labor is a co-integrating equation.

2. DATA

The population and wage data we use are taken from the classic Wrigley and Schofield (1981) reconstruction of English population history, and are the same as those used by most other researchers.[5] Some questions have been raised about the real wage data, which involve interpolation. Other questions have been raised about the data on population, fertility and mortality, which were estimated using the method of back projection, a method which has been heavily criticized by Lee (1985b, 1993b). In part because of these criticisms, and in part in the interests of simplicity, we have not used any age-distributed data, but have stuck to population size, and crude birth and death rates. Estimates of net migration we use only to complete the identity, linking population growth to the vital rates. We do not treat migration as an endogenous variable because we do not believe it is adequately measured.[6] We have used the real wage data as presented in Wrigley and Schofield.

Figure 1 shows the birth rate in raw and smoothed versions. [7] Figure 2 shows the death rate, raw and smoothed. Both series, particularly the birth rate, show broad swings over hundreds of years, and short-term fluctuations are substantial, especially prior to 1750. The natural log of the real wage is plotted in Figure 3. The wage series was calculated by Wrigley and Schofield from the Phelps-Brown and Hopkins (1957) price and wage data. The series is essentially a craftsmen's nominal wage divided by the price of wheat.[8] The wage series also shows broad swings over time, but the general trend is upwards. The natural log of population size is shown in Figure 4, exhibiting a strong upward trend, but also a long fluctuation about the trend. As we would expect, it appears much smoother than the other series.

3. BASIC MODEL

Our basic model has three behavioral equations describing influences on the real wage, fertility and mortality. An identity linking the change in population to the crude birth and death rates plus net migration (treated as exogenous) closes the system. In addition to these four equations, there are others that describe the structure of the disturbance terms in each of the three behavioral equations. In each case these are specified to be second order autoregressive. One is accustomed to seeing such specifications of the error terms incorporated in the behavioral equations, but in state space form, they are described separately.

The novel aspect of the state space specification comes into play with additional equations which describe the evolution of the intercept terms in the equations for fertility, mortality and wages, as random walks. These intercepts themselves thereby become stochastic variables which are permitted to vary across the 330 years of the sample period. In the case of the wage equation, the intercept describes the level of the (log of the) demand for labor. We model this intercept as a random walk with drift, where the drift reflects the rate of change of the demand for labor. For convenience we will refer to this rate of change as the rate of technological progress. One further equation then represents the rate of technological progress as a random walk. In all, therefore, our system includes eleven simultaneous equations that we hope provide a reasonable description of the more complex, true relationships among wages, population size, and vital rates. Only three of these eleven equations describe behavioral relationships; the remaining eight describe the structure of shocks.

The wage equation posits that the natural log of the real wage at time *t*, denoted w_t, is a function of the natural log of the population level, p_t, an unobserved intercept a_t representing the level of the demand for labor, and an unobserved disturbance term s_t representing the impact of short-term shocks on wages such as the weather and harvest conditions:

(3.1) $$w_t = a_t - \beta p_t + s_t \, .$$

As mentioned earlier, this is a co-integrating equation. Here β is the elasticity of wages with respect to labor and the disturbance term is AR(2):

(3.2) $$s_t = \gamma_1 s_{t-1} + \gamma_2 s_{t-2} + \varepsilon_t \, .$$

ε_t is an i.i.d. random variable (referred to herein as noise) and γ_1, γ_2 are real-valued parameters such that s_t is stationary. However, the annual rate of *change* in the demand for labor, c_t, is expected to have a random-walk structure, or:

(3.3) $$a_t = a_{t-1} + c_t \, ,$$

(3.4) $$c_t = c_{t-1} + \nu_t$$

where ν_t is noise. We expect a_t to rise in a roughly linear manner throughout the period in question (implying exponential growth in the demand for labor in real terms) with relatively small disturbances. An acceleration in its rate of increase is expected as industrialization becomes important; this would emerge as an upward drift or shift in the value of c_t.

In an OLS estimate of equation 3.1 using these data, Lee (1980) tested for change in β, the elasticity of wages with respect to population size, and found it remarkably constant across the three subperiods examined. It will be assumed to be constant in our specification.

The crude death rate d_t at time t is linearly related to the natural log of the real wage, together with an intercept m_t and a disturbance term u_t:

$$d_t = m_t + \delta_1 w_t + \delta_2 w_{t-1} + \delta_3 w_{t-2} + \delta_4 w_{t-3} + \delta_5 w_{t-4} + u_t \, . \tag{3.5}$$

The real-valued parameters δ_1 through δ_5 describe the impact on the death rate of wage levels contemporaneous with the death rate, one year previous, and so on up to four years previous. We expect the sum total of these coefficients, the lag sum, to be negative, in accordance with the positive check. These lagged terms are included because the effects of good or bad nutrition may be delayed to an unknown degree, and because a period of heavy mortality may be followed by a period of light mortality, since the most sickly and vulnerable will have been removed from the population by the preceding crisis. The choice to truncate the lags at t-4 was based on previous studies that explored models with longer lags (Lee, 1981; Galloway, 1988). The unobserved disturbance term u_t represents the short term shocks to mortality caused by environmental fluctuations such as the weather and disease prevalence (to the extent that this is due to factors other than the wage); we expect this term to be correlated from year to year, hence its AR(2) structure:

(3.6) $$u_t = \lambda_1 u_{t-1} + \lambda_2 u_{t-2} + \rho_t \, .$$

ρ_t is noise and λ_1, λ_2 are real-valued parameters such that u_t is stationary.[9] The intercept term m_t is also unobserved, and reflects long-term exogenous influences on mortality, such as the international exchange of diseases following voyages of exploration, climatic change, alterations in building materials or dietary staples, and cultural practices affecting death rates and morbidity. This term is modeled as a random walk:

(3.7) $$m_t = m_{t-1} + \phi_t$$

where ϕ_t is noise.

The equations describing the crude birth rate b_t are identical in structure to those for the death rate:

(3.8) $$b_t = n_t + \mu_1 w_t + \mu_2 w_{t-1} + \mu_3 w_{t-2} + \mu_4 w_{t-3} + \mu_5 w_{t-4} + r_t$$

where μ_1 through μ_5 express a mix of the internal dynamics of the birth interval and the lagged direct effects of real wages on the birth rate. We expect the sum of the μ's to be positive, consistent with the Malthusian preventive check. r_t is the disturbance term, modeled as AR(2), and n_t is the intercept term, modeled as a random walk:

(3.9) $$r_t = \kappa_1 r_{t-1} + \kappa_2 r_{t-2} + \psi_t,$$

(3.10) $$n_t = n_{t-1} + \sigma_t$$

where ψ_t and σ_t are noise, and κ_1, κ_2 are parameters such that r_t is stationary.

The distributed lag coefficients describe the effects of contemporaneous and earlier variations in the wage, up to four years previous. In particular, these allow for a nine month lag between conception and delivery, and they allow for well-known rebound effects of fertility arising from the length of the average closed birth interval, which was typically two and a half to three years. If an unusually large proportion of women gave birth in one year, then an unusually small number of women would be at risk of birth in the following year or two, after which there would be an unusually large number of women all ready to bear their next child again. This dynamic of the reproductive cycle tends to induce a two to three-year cycle in randomly perturbed birth series, as first noted and explained by Yule in 1906.

The intercept term, n_t, varies under the influence of the changing occupational structure of the population, variations in the "moving standard" against which people measured their relative well being, changes in rules and customs regulating marriage, changes in the Poor Laws, and so on. The contemporaneous disturbance, r_t, represents more transient influences such as variations in the weather (Lee, 1981), exogenous variations in health and morbidity, the separation of spouses, the effects of wars and political upheavals on the popular view of whether the times were good for bearing children, and many other influences. We wish to stress that the wage was only one of many influences on fertility, and its variations account for only a fraction of the variance in fertility. Note also that we do not include lagged marriages in the specification of the birth rate equation. We believe that nuptiality and marital fertility were both strongly influenced by unobserved forces which largely account for their strong empirical association, so that it is preferable to exclude them (see Lee, 1975 and 1981).[10]

Previous research has shown that within the 330-year period, the sums of the estimated coefficients differ across subperiods in OLS type specifications otherwise similar to those given above for the vital rates (Lee, 1981; Galloway, 1988). However, the irregular nature of these changes does not suggest a systematic trend. For example, the cumulative elasticity of mortality with respect to wages is -.107 for 1546-1674; -.380 for 1675-1755; and +.086 for 1756-1865 (Galloway, 1988). Mills (1994) finds with a VAR analysis that the same fertility measure we use here was positively influenced by the real wage in each subperiod. She does find significant structural changes over the 330-year period, but these have mainly to do with the differing responses of marital fertility and nuptiality to the real wage and mortality.[11] We believe it acceptable to treat the coefficients as constant over this period.

We also include a linear balancing equation:

(3.11) $$p_t = p_{t-1} + b_{t-1} - d_{t-1} + e_{t-1}$$

where e_{t-1} is defined so that the equation is true. Note that $p_t - p_{t-1}$, the difference between the natural logs of population size in adjacent years, equals the exponential rate of population growth, which must in turn equal the crude birth rate less the crude death rate plus the crude rate of net immigration. Thus e_t is primarily net immigration (which we believe to be very badly estimated, see Lee, 1985b and 1993b, and therefore do not seek to model as endogenous; see Stavins, 1988, for an alternative approach).

The intercepts for the birth and death rates, n_t and m_t, should change very slowly over time, and are certainly capable of exhibiting long swings. The intercept for both fertility and mortality are suspected of having large variance and cyclic tendencies with a very long period (2 or 3 centuries).

4. STATISTICAL METHOD

In order to estimate the 17 parameters above and the variance of each of the 6 noise terms (for 23 parameters total) as well as the unobserved series, we use the state space form (SSF) of these equations. The full SSF is described in Harvey (1989).

We assume measurement error is negligible and focus on the *transition equation* component of the SSF[12]:

$$\alpha_t = \mathbf{T}\alpha_{t-1} + \mathbf{c}_t + \eta_t \quad (4.1)$$

where here α_t is the 19x1 state vector containing the values of both the observed and unobserved series at particular times, **T** is a time-invariant 19x19 matrix containing the 17 parameters, $\mathbf{c}_t$ (distinct from the c_t in equation 3.3, the change in the demand for labor) is a 19x1 vector, and η_t is a 19x1 vector of the noise terms and zeroes. The nonzero elements of the vector η_t are serially uncorrelated with mean zero and time-invariant, diagonal covariance matrix **Q** containing the six unknown variances. The observed components of the state vector at time *t* are denoted by $\mathbf{y}_t$, and here they are related to α_t through the 3x19 matrix of zeroes and ones, **Z**, in the measurement equation component of the SSF:

$$\mathbf{y}_t = \mathbf{Z}\alpha_t \quad (4.2)$$

since measurement error is assumed away. In our case, $t = 1,\ldots,T$ where T is 327 years.

Once the equations are in this form, the Kalman filter may be used within a maximum-likelihood algorithm to obtain estimates of the 23 parameters. Given adequate initial values for the state vector and the variance-covariance matrix at time 0, the Kalman filter may be run *within* a likelihood maximization routine; that is, we run the Kalman filter for a given candidate set of parameters, calculate the log-likelihood, and iteratively adjust the candidate parameters/run the Kalman filter in an effort to maximize the likelihood. To do this mechanically, we used the *Splus* minimization function *ms( )* (see Chambers and Hastie, 1992) combined with a dynamically loaded FORTRAN routine containing the Kalman filter. To maximize over all 23 parameters, this routine typically makes several hundred passes through the Kalman filter algorithm in about five hours on a Sun SPARCstation 2+. Since the achieved maximum was not guaranteed to be global, we repeatedly ran the maximization routine with the initial parameter values randomly generated in search of any other possible maxima.

In this manner, we obtained maximum-likelihood estimates of the parameters along with estimates of the state vector over time, including those unobserved components,[13] from the last, likelihood-maximizing run of the Kalman filter. But as mentioned, each estimate of the state vector at time *t* is computed using the data only up to time *t*, as the Kalman filter is a real-time process. This yields estimates of the time series, which are not in fact consistent with a continual iteration of the transition equation; for example, the value of a series at time *t* could be estimated as two different values in two steps of the Kalman filter. Hence it is impossible to obtain a pure noise series from the real-time Kalman filter output, as this must be calculated by solving the transition with the estimated state vectors. This presented a problem, as we wanted to check the noise series for autocorrelation, as well as using them in a bootstrap simulation for the calculation of standard errors and confidence intervals. Thus, to obtain estimates of the state vector using all the data, we use the *fixed-interval-smoothing* algorithm defined in Harvey (1989:154). Standard errors of the parameter estimates may be estimated from the information matrix.

Finally, in order to obtain reasonable starting values for the initial estimated state vector at time 0 and an initial variance-covariance matrix, we started with reasonable guesses at the unknown elements of the state vector and generated a diagonal variance-covariance matrix randomly. The estimated series typically converged to some trend within ten steps of the filter; or else they remained stationary around zero where expected. Then the smoothing algorithm was applied backwards to produce another set of starting values, and these were in turn fed into the Kalman filter. We worked this process iteratively, and the starting values converged to the same values before and after the backward smoothing within several passes.

5. ESTIMATES OF THE WAGE-POPULATION EQUATION: TECHNOLOGICAL PROGRESS AND THE DEMAND FOR LABOR SCHEDULE

5.1. Wage-Population Elasticity

The maximum-likelihood estimates for all parameters together with their standard errors are shown in Table 1. We first consider the estimated parameter β, which indicates how population growth affects wages. Our estimate of β is close to 1, implying that a 10 percent increase in the population level would result in a 10 percent decrease in the real wage, all other factors held constant. The standard error of the estimate is very small, at .0066. Inspection of the residual plot (not included) shows that population size accounts mainly for long-term change in wages, but not for their short-term volatility. This was expected, since the p_t series is quite smooth. The large short-term fluctuations in the wage rate must be explained by other variables, most likely weather induced shocks. The large, long-term dips and peaks in the real wage are largely due to changes in the supply of labor, arising from prior fluctuations in fertility and mortality, although longer-term fluctuations in the growth of the demand for labor played some role as well.

Comparable estimates for England in the literature range from .99 to 2.7. Within this range, estimates based on the annual data series from Wrigley-Schofield used here have found larger (in absolute value) elasticities, which increase when correction is made for autocorrelated disturbances (Lee, 1980). Estimates based on longer time units, such as 10, 25 or 50 years, lead to smaller estimated elasticities. For example, Lee (1973) found 1.1 with 50-year data from 1250 to 1800. Weir (1991) made pooled time series cross-section estimates for five European countries, including England, based on fifty-year periods from 1500 to 1800. For the five different models he estimated (with and without country specific intercepts, and period dummies, etc.), the range of estimated elasticities was from .99 to 1.46. The estimates on longer time units are lower because simultaneous equation bias is greater for the short-run component of covariance than for the long-run component, as a frequency domain analysis of the model shows (not presented here) and as was discussed earlier. The low estimate in this paper is striking because although it is based on annual data and allows for autocorrelated disturbances, it nonetheless falls at the lower and more plausible end of the range.

Lee (1980) developed a two-sector model, in which labor shifted endogenously between agriculture and industry. Drawing on that model, the elasticity of substitution of labor and land in agriculture would have to be about .5 to account for the estimated wage-population elasticity of -1.04 found here. This is substantially lower than most, but not all, estimates for contemporary Third World populations.

5.2. Level and Rate of Change of Demand for Labor

The log of the level of the demand for labor is estimated by a_t which is plotted in the top panel of Figure 5. The rate of change of the demand for labor, or the rate of technological progress, is given by first differences of a_t, denoted c_t, which are plotted in the bottom panel.[14] Inspection of the plot of the demand for labor, a_t, suggests that we distinguish three periods. From 1540 to 1600, the demand for labor underwent an increase and then decrease in a larger fluctuation than occurs elsewhere in the series. Across this wide swing, the average rate of change was .36 percent per year. Over the next two centuries from 1600 to 1800 the rate of change was remarkably steady at .49 percent per year. Finally, after 1800 there was a dramatic acceleration and the rate nearly quintupled from .49 to 2.4 percent per year. An alternative periodization from 1795 to 1870 avoids the worst distortions from trend, associated with the Napoleonic wars, yielding a somewhat slower growth rate of 2.01 percent. Either way, the 19th century witnessed a sudden and sustained increase in the rate at which labor could be absorbed by the economy without depressing wages. This finding is consistent with recent revisionist views that the generalized increase in the rate of economic growth in England did not occur until the 19th century (Crafts and Harley, 1992; Pol and Voth, 2000).

The estimated rate of absorption (ρ/β) in percent per year was:

	State Space	OLS
1540-1600	.34	.34
1600-1795	.47	.43
1795-1870	2.01	n.a.

Because the rate of increase in the demand for labor appears, from our state space estimates, to have been so regular over the interval 1600 to 1800 (see Figure 5), and indeed was not much different on average from 1540 to 1600, no great violence is done by assuming the rate of increase to be constant or linearly increasing over this period, as in Lee (1980, 1985a). For comparison sake, we have shown estimated values of the rate of absorption from earlier work (Lee, 1980). The Weir study mentioned earlier found a rate of absorption for England of .29 to .36 percent per year for 1500 to 1800, slightly lower than the other results reported here, probably because the estimate of the population-wage elasticity was based on data from five European countries, not just England.

6. ESTIMATES OF FERTILITY AND MORTALITY IN RELATION TO THE WAGE: THE PREVENTIVE AND POSITIVE CHECKS

The lagged effects of wage variation on fertility and mortality are described by the estimated values of μ_i and δ_i in Table 1. These coefficients are plotted against lag in Figure 6 with 95 percent confidence intervals indicated by horizontal lines for each estimate, and with the coefficients reported in Lee (1981) for comparison.[15]

Consider first the fertility-wage relation. The shape of the lag-response is similar to that of the earlier study, but the coefficients are about twice as large: The new estimates indicate that wage variation has a substantially stronger effect on the timing of fertility than in the old estimates. The shape of this timing effect is largely due to the natural periodicity of birth intervals, as explained earlier. The results are closer to the typical pattern of response for Europe than were the original Lee (1981) estimates (see Galloway, 1988, and Lee, 1990 and 1993b).

We find a similar difference for the mortality coefficients shown in the lower panel of Figure 6: The wage appears to have a stronger effect on the timing of mortality in these new estimates, although the lag pattern has a similar shape. A puzzling feature of the old results is repeated here and indeed amplified: The strongest effect occurs two years after a variation in wages. We had expected the main effect of the real wage on mortality to be contemporaneous. We can speculate *ex post* about possible explanations: Higher wages may have induced increased migration and urbanization which caused higher mortality later on; the effects of poor nutrition (as opposed to starvation, which was rare) emerge only slowly;

or perhaps the result is artifactual, due to the coincidental alignment of some large exogenous epidemic and a major harvest failure.

Another interesting feature of the lag pattern for mortality is the apparent rebound effect: the first three coefficients sum to -.00796, and the last two sum to +.00592. This might reflect the selectivity of crisis mortality, if the incremental deaths were of the weakest and most vulnerable members of the population, who would soon have died in any case.

It is also interesting to consider the cumulative effect of a wage variation, independent of its effect on the timing of the vital rates, by summing all the lag coefficients. For purposes of comparison to other estimates, we first convert these coefficients to elasticities by multiplying by 1/(average CBR) = .034 and 1/(average CDR) =.027, respectively. The summed elasticities are presented in Table 2, along with other sums for comparison. The lag sum for fertility is positive, consistent with a Malthusian preventive check, and very close in value to the other estimates for England and for Europe. The lag sum for mortality is negative, consistent with a Malthusian positive check, but it is quite close to zero, much closer than the other two estimates.

Very little of the long-term variation in either fertility or mortality appears to be explained by variations in wages. This can be seen by visually comparing the movement of the intercept series n_t and m_t for fertility and mortality, shown in the bottom panel of Figure 7, with the smoothed birth and death rates plotted in Figures 1 and 2. Most of the influence of the wage on the vital rates appears to be short-term in nature .[16]

7. OTHER SPECIFICATIONS

In addition to the basic model described earlier, we estimated some variants. We focused in particular on the problem of covariation between the disturbances in the fertility and mortality equations. Earlier research (Lee, 1981) established a strong negative association of short-run variations of fertility and mortality, with lowest fertility occurring nine or ten months after peak mortality, followed by a strong rebound of fertility. There is no plausible causal path through which mortality variation could cause this pattern, and it is most likely due to the causal influence of unobserved variations in health status (morbidity) on both fertility and mortality. We experimented with adding some common error components to the fertility and mortality equations, but were unable to satisfactorily identify their variances.

8. SIMULATIONS

In order to see how well our fitted model could capture key aspects of the evolution of the interrelated historical series, we carried out a stochastic simulation for a 300-year period. The results of this are not shown here, but can be summarized as follows: The simulated population size appeared qualitatively similar to the actual, in terms of smoothness and propensity to fluctuate slowly about an upward trend. The general upward trend in population size and the demand for labor result from the initial value for the rate of drift in the demand for labor, which was taken to be positive as estimated from the data. Wages also appeared to vary in appropriate ways over the longer run, but in the short run were less variable than the actual. The simulated mortality series appeared to be qualitatively very similar to the actual, but the simulated fertility series, like the wage series, had too little short-term variation. The reason for these discrepancies between the behavior of the simulated fitted system and the actual historical one are not yet clear.

Lee (1993a) showed that OLS estimates of the influence of population size on wages, β, would be biased upwards, and that correction for autocorrelated residuals would significantly worsen the problem. For the simulated data, we knew the true value of this coefficient, so we could test this finding by using OLS to estimate β from these data. When we did so, we estimated the absolute value of β to be 1.22, more than two standard deviations higher than the true value of β, 1.04. In the case of the lag coefficients for fertility and mortality in relation to wages, however, all the estimates were close to the "true" values.

9. HOMEOSTASIS IN A STOCHASTIC SETTING

The estimated equations describe a Malthusian system in which fertility and mortality react to deviations of the wage from its equilibrium level in such a way as to move the wage, and the population size, toward their equilibrium levels. The estimated disturbance terms characterize the stochastic context, in which external influences continuously buffeted the vital rates and wages, disrupting this equilibrating process. The positive and preventive checks can be viewed as filters which determine the response of the system to different kinds of shocks in the short run and in the long run (see Lee, 1993a). For example, in the long run, shocks to the demand for labor have no effect on wages but are fully passed on to population, in accordance with the Iron Law of Wages.

The key parameters for understanding the behavior of the system are, first, the lag sum for fertility minus the lag sum for mortality, call this α (which is the derivative of the population growth rate with respect to the natural logarithm of wages, abstracting from effects that are purely on timing of births and deaths), and second, minus the elasticity of wages with respect to population, which is β. From the state space estimates, α=.0062 and β=1.045, so their product, $\alpha\beta$, is .0065. In Lee (1993a) it is shown that this product determines the rate of convergence towards equilibrium of the economic demographic system. Put differently, it tells us how rapidly the influence of a shock to the system wears off: at the exponential rate $\alpha\beta$ = .0065 per year. Thus a shock (such as an epidemic unrelated to wage levels) which reduced population size by 10 percent T years ago would have an influence on the current state of the system given by 10 percent times $e^{-.0065T}$. The half-life of a shock, therefore, is a bit more than a century, or 107 years (found by solving for T the equation .5 = $e^{-.0065T}$). These estimates imply somewhat slower convergence than earlier estimates by Lee, discussed in Lee (1987, 1993a), where $\alpha\beta$=.01 and the half-life was 70 years. Such weak equilibration and slow convergence means that population size and living standards were strongly affected by the accumulation of shocks to fertility, mortality, and the demand for labor. This means that even though fertility, mortality and wages were all endogenous in a Malthusian system, each nonetheless moved with sufficient independence that it could exert a strong exogenous causal force on the others, even over periods measured in centuries.

10. CONCLUSION

State space models and methods have enabled us to overcome some longstanding econometric problems in the estimation of these relationships in English economic demographic history, and to produce a coherent set of estimates for quantities of interest. The estimated strengths of the preventive and positive checks, and the distinctive timing with which each of these operates, were found to be similar to those found by previous research. Unfortunately, the estimates of these relations appear to be based largely on short-run covariance, with the long-run movement of the birth and death rate series largely determined by their random intercepts. This reduces confidence in the interpretation we give them in terms of a homeostatic system.

The elasticity of the real wage with respect to the size of the population was found to be very close to -1, substantially lower than previous estimates using these annual data from Wrigley and Schofield, but in line with earlier estimates using data averaged over long time periods, and also in line with pooled cross-section time series estimates for a number of European countries. While the large size of this elasticity still poses problems of interpretation, it is nonetheless considerably closer to theoretical expectations of about -.5 than the earlier estimates of -1.5 to -2.7 based on annual data.

Direct and unconstrained estimates of the trajectory of the demand for labor are also of considerable interest. The relative constancy of its rate of increase from 1600 to 1800 is surprising, since the inception of the Industrial Revolution in England is believed to lie early in the eighteenth century, and one would have expected an acceleration of the demand for labor during that period. Instead, there was a very sharp and discontinuous rise in the rate of increase starting around 1810.

A number of extensions of this analysis are called for. Instead of treating the real wage as a single variable, it could be modeled as the nominal wage divided by the price level, following the approach in Lee, 1985a. Employers of labor then adjust their nominal wage offers in line with their expectations of the selling price of their outputs, which depends on their expectations about the future price level. This approach allows rapid inflation to temporarily depress the real wage. The strong negative association of fluctuations in fertility and mortality, most likely due to the influence of unobserved health conditions on both, can be modeled explicitly as arising from shared components of variance in the disturbance terms. Similarly, marriage rates could be introduced, with appropriate treatment of a strong association of fluctuations in marital fertility and nuptiality, most likely arising from shared variance components in their disturbances (see Lee, 1981). Data on variations in weather could be introduced as influences on fertility, mortality and the wage levels. Dummies for wars and political disturbances could be included. While these refinements would make the model more realistic, they would be unlikely to change the main results of interest, and they would substantially complicate what is already quite a complicated model.

The new approach taken here puts on a solid statistical footing the estimation of economic-demographic relationships in an historical Malthusian system. Our new estimates indicate that the elasticity of wages with respect to population size was lower than suggested by previous work based on the Wrigley and Schofield data series. Our new explicit estimates of growth in the demand for labor show that it increased at a remarkably steady rate until around 1800, and then accelerated dramatically. As for the vital rates, we have found that most of the long-term change in fertility and mortality was non-Malthusian in origin (that is, unrelated to changes in wages), and instead was a response to other influences such as weather, disease, or institutional change.

Bibliography

Bailey, R.E. and M.J. Chambers (1994) "Long-term Demographic Interactions in Pre-Census England," *Journal of the Royal Statistical Society* (Series A), 157, part 1.

Boserup, Ester (1965) *The Conditions of Agricultural Progress*, Allen and Unwin Press.

Boserup, Ester (1981) *Population and Technological Change*: A Study of Long-Term Trends, University of Chicago Press.

Chambers, M.J. and Hastie, T. J. (1992) *Statistical Models in S.* Pacific Grove, CA: Wadsworth & Brooks.

Crafts, N.F.R. and C. K. Harley (1992) "Output Growth and the Industrial Revolution: A Restatement of the Crafts-Harley View," *Economic History Review* 45, 703-730.

Fogel, Robert (1994) "Economic Growth, Population Theory, and Physiology: The Bearing of Long Term Processes on the Making of Economic Policy," *American Economic Review* v.84 n.3 (June), 369-395.

Galloway, Patrick (1988) "Basic Patterns in Annual Variations in Fertility, Nuptiality, Mortality, and Prices in Pre-Industrial Europe," *Population Studies* v.42 n.2, 275-302.

Habakkuk, H. J. (1965) "The Economic History of Modern Britain," in D.V. Glass and D.E.C. Eversley, eds., *Population in History* (Chicago: Aldine), 147-158.

Hamilton, James D. (1994) *Time Series Analysis*, Princeton University Press, Princeton NJ.

Harvey, A. C. (1989). *Forecasting, Structural Time Series Models and the Kalman Filter,* Cambridge, Cambridge University Press.

Hayami, Yujiro and Vernon W. Ruttan (1985) *Agricultural Development*, The Johns Hopkins University Press, Baltimore and London.

Lee, Ronald D. (1973) "Population in Preindustrial England: An Econometric Analysis," *Quarterly Journal of Economics* 87, no. 4 (November), 581- 607.

Lee, Ronald D. (1975) "Natural Fertility, Population Cycles, and the Spectral Analysis of Births and Marriages," *Journal of the American Statistical Association*, 70, no. 350 (June), 295-304.

Lee, Ronald D. (1980) “A Historical Perspective on Economic Aspects of the Population Explosion: The Case of Preindustrial England,” in R. A. Easterlin, ed., *Population and Economic Change in Developing Countries*, Chicago, University of Chicago Press, 517-566.

Lee, Ronald D. (1981) “Short-Term Variation: Vital Rates, Prices and Weather,” in E.A. Wrigley and R.S. Schofield, *The Population History of England 1541-1871*: *A Reconstruction*, Harvard University Press, Cambridge, Mass., 356-401.

Lee, Ronald D. (1985a) “Population Homeostasis and English Demographic History,” *Journal of Interdisciplinary History*, XV: 4 (Spring), 635-660.

Lee, Ronald D. (1985b) "Inverse Projection and Back Projection: Comparative Results and Sensitivity Tests for England," *Population Studies*, (July), 233-248.

Lee, Ronald D. (1987) "Population Dynamics of Humans and Other Animals," *Demography* v.24, n.4 (November), 443-466.

Lee, Ronald D. (1988) "Induced Population Growth and Induced Technological Progress: Their Interaction in the Accelerating Phase," *Mathematical Population Studies*, v.1 n.3, 265-288.

Lee, Ronald D. (1990) "The Demographic Response to Economic Crisis in Historical and Contemporary Populations," *Population Bulletin of the United Nations*, no. 29, 1-15.

Lee, Ronald D. (1993a) "Accidental and Systematic Change in Population History: Homeostasis in a Stochastic Setting," *Explorations in Economic History* v.30, 1-30.

Lee, Ronald D. (1993b) "Inverse Projection and Demographic Fluctuations: A Critical Assessment of New Methods," in David Reher and Roger Schofield, eds., *Old and New Methods in Historical Demography*, Oxford University Press, 7-28.

Lindert, Peter H. (1983) "English Living Standards, Population Growth and Wrigley-Schofield," *Explorations in Economic History*. v.20, 131-155.

Lindert, Peter H. (1985) "English Population, Wages and Prices: 1541-1913," *Journal of Interdisciplinary History*, v. 115, n.4, 609-634.

Mills, Judith Woerner (1994) *New Evidence on the Demographic Transition in England*, unpublished dissertation in Economics at the State University of New York at Stony Brook.

Phelps-Brown, E. H., and S. V. Hopkins (1957) "Wage Rates and Prices: Evidence for Population Pressure in the Sixteenth Century," *Economica* v.24 (November), 289-306.

Pol, Antras and Hans-Joachim Voth (2000) "Productivity Growth During the English Industrial Revolution: A Dual Approach," Working Paper No. 495 (October), Universitat Pompeu Fabra, Economics & Business, Barcelona, Spain.

Schofield, Roger (1985) "English Marriage Patterns Revisited," *Journal of Family History*, 10, 2-20.

Stavins, Robert (1988) "A Model of English Demographic Change: 1573-1873," *Explorations in Demographic History*. v. 25, 98-116.

Tsoulouhas, Theofanis C. (1992). "A New Look at Demographic and Technological Changes: England, 1550-1839," *Explorations in Economic History* v. 29, n. 2 (April), 169-203.

Weir, David (1984a) "Life Under Pressure: France and England, 1670-1870," *Journal of Economic History* v.44 n.1, 27-47.

Weir, David (1984b) "Rather Never Than Late: Celibacy and Age At Marriage in English Cohort Fertility: 1541-1871," *Journal of Family History*, no. 9, 341-355.

Weir, David (1991). “A Historical Perspective on the Economic Consequences of Rapid Population Growth,” *Consequences of Rapid Population Growth in Developing Countries*, Taylor and Francis, 41-66.

Wrigley, E.A., R.S. Davies, J.E. Oeppen, and R.S. Schofield (1997) *English Population History from Family Reconstitution 1580-1937,* Cambridge University Press, Cambridge.

Wrigley, E.A. and R.S. Schofield (1981) *The Population History of England 1541-1871: A Reconstruction*, Cambridge, Mass., Harvard University Press.

Yule, G. Udney (1906) “Changes in the Marriage and Birth Rates in England and Wales During the Past Half Century,” Journal of the Royal Statistical Society v.69, 18-132.

Footnotes

---

[1] If the short-run elasticity of labor supply with respect to the wage is not zero, then β will combine the elasticities of labor demand and supply.

[2] Hayami and Ruttan (1985:144-145, 149) present estimates of the coefficients on labor, land, and other inputs from eight cross-national estimates of a Cobb-Douglas type agricultural production function for 1960, 1970 and 1980. The coefficients on labor range from .20 to .45, suggesting that non-labor inputs accounted for coefficients of .55 to .80. However, most of these other inputs would be expected to vary over the long run as population grew. If we restrict our attention to the coefficients on land, those range from 0 to .40. One could find support in these studies for a wage-population elasticity of anything from 0 to -.8. In their own, new estimates, based on larger samples for the same years, they find labor elasticities for less developed countries of around .55, and land elasticities ranging from -.05 to +.05.

[3] Hayami and Ruttan (1985:159) provide tests of the assumption of a unitary elasticity of substitution in cross-national agricultural production functions for 1960, 1970 and 1980, and are unable to reject the assumption.

[4] To quote Hamilton (1994:445) on this point: "…for any unit root process there exists a stationary process that will be impossible to distinguish from the unit root representation for any given sample size T."

[5] The population on June 30 of each year is based on Table A3.3 (p.531), and deaths and births totals for the year starting July 1 are based on Table A2.3 (p.496). We interpolate to get the population on Jan. 1, and divide this series into the deaths and births to obtain the vital rates. The real wage is based on Table A9.2 (p. 642). Data have been adjusted to refer to harvest years.

[6] Since the change in population size equals the sum of births, deaths, and net migrants, the question arises of how migration estimates can be inadequately measured at the same time that the other variables are reasonably reliable. The answer is that while births and deaths are given, back projection estimates the number of net migrants jointly with the change in the population size. Back projection is the method used in Wrigley and Schofield (1981). Lee (1985b, 1993b) has shown, however, that the back projection method is fundamentally flawed, and its estimates of migration and population size are unfounded. Inverse projection, from which back-projection was derived, is well founded but more modest in its goals. It takes other sources of information about population size as a basis for the rest of the estimation procedure, and does not attempt to make independent estimates of migration and population size. Using it, Lee (1985b) found essentially the same b and d, and TFR and e_0, as Wrigley and Schofield, under the assumption of constant rates of net migration in subperiods between observed population sizes. From this we conclude that birth rates, death rates and population size are adequately estimated for our purposes, but net migration is not.

[7] Smoothed using the *Splus* function *supsmu( )*, not the fixed-interval smoother.

[8] It has been shown to be very similar (proportionately) to an agricultural wage series for the lengthy period in which they overlap.

[9] We had also fit the model with u_t modeled as noise, but significant autocorrelation showed up as expected.

[10] In the short run, marital fertility was highly variable, and drove short-run variations in fertility (see Lee, 1975). Nuptiality was also variable in the short term, but the effect on fertility in the short term was dispersed across the stock of marriages. Marriage was more important for longer-run changes in fertility. From Wrigley et al (1997) we know that the average age at first marriage declined by about three years in the 18th century. Unfortunately, family reconstitution cannot tell us much about proportions never marrying. It is likely that rising proportions never married contributed to the fall in fertility from 1550 to 1650 (Weir,

1984b; Schofield, 1985). Probably important in the 17th century. Long-term variations in marital fertility were also important, as found by Wrigley et al (1997). Wrigley and Schofield (1981), which was based on aggregate data, treated marital fertility as constant over time, and ascribed variations in fertility to changes in nuptiality. Their more recent work, Wrigley et al (1997), is based on family reconstitution data that can support a more detailed investigation. In this work they report that the Total Marital Fertility Rate (over ages 20 to 49) underwent a long swing of about .7 children per woman or by 10% (Wrigley et al, 1997:355). This swing was due to various documented factors including the length of the average inter-birth interval, the prevalence of premarital conceptions, and varying sterility at older ages.

[11] For example, she finds that the short-term relationship of marital fertility to wages changed from positive to negative around 1700, but her strongest coefficient, at lag 1, is significantly positive in all subperiods, and no test is given for the hypothesis that the lag sum is different from zero, so in our view this evidence of a sign change in the relationship is inconclusive. In any event, for overall fertility, rather than marital fertility, no sign change in this relation is found.

[12] There are two reasons to set measurement error to zero. The first is that proper treatment of measurement error would require a more careful and structured approach, in which the mode of construction of the data series, such as the use of constant adjustment factors, and the specific accounting identities, was taken into account to allow for structural aspects of the errors such as persistence over time. The second is that measurement error has not been incorporated in prior specifications in the literature, and we would like our results to be comparable.

[13]Of course the observed components of $\mathbf{a}_t$ will simply be equal to the observed population and vital rates since we have assumed measurement error away.

[14] The c_t series shows a likely structure for a random walk, but the differences of this series are a bit too highly serially correlated to be noise. The short-term fluctuations in the w_t series are largely captured by the AR(2) s_t series. The estimated noise term ε_t which makes up s_t does indeed look a great deal like noise, except for a slight decrease in variance over time. The plots of s_t and of ε_t are not shown in this paper.

[15]These are taken from Table 9.7, p.372, which contains elasticities of mortality on prices; to obtain comparable estimates, multiply the parameters by the means of the birth rate and death rate, respectively, and reverse the signs.

[16] Based on visual inspection of the plot of the short-term AR(2) disturbance term u_t (not included in this paper), it is not obvious that this series forms an AR(2) term, but it is certainly stationary, and the noise term ρ_t which makes up u_t looks reasonably like pure noise except for a notable decrease in the variance of the series over time. We checked the noise terms by plotting the raw and smoothed power spectrum of each; spectrums departing significantly from the line at $\sigma^2/2\pi$ fail the noise test. We checked the normality assumption with a simple quantile-quantile plot. The distribution of the disturbances is clearly non-normal, with much heavier tails, particularly the right tail. This is to be expected, since positive shocks to mortality, as we are thinking of them, can typically raise mortality a great deal, but negative shocks to mortality are constrained by some lower limit at zero, if not higher.

TABLE I.

Coefficient Estimates for England, 1540-1870

Item	Description	Estimate	Stand. Err. of Estimate	t Statistic
Wage-Pop Elasticity	Elasticity of wage wrt pop			
β		-1.0446	0.00659	-58.5
Fert-Wage Coeffs	Effect of lnW on Fert			
μ1	at lag 0 yr	.00409	0.000930	4.4
μ2	at lag 1 yr	.00622	0.000959	6.5
μ3	at lag 2 yrs	-.00547	0.000998	-5.5
μ4	at lag 3 yrs	.00214	0.00105	2.0
μ5	at lag 4 yrs	-.00280	0.000985	-2.8
Mort-Wage Coeffs	Effect of lnW on Mort			
δ1	at lag 0 yr	.000198	0.00199	.1
δ2	at lag 1 yr	-.00173	0.00192	-.9
δ3	at lag 2 yrs	-.00642	0.00190	-3.4
δ4	at lag 3 yrs	.00219	0.00191	1.1
δ5	at lag 4 yrs	.00372	0.00196	1.9
Variances	Var of Innovation in			
Var(ε)	wage AR disturbance term	7.59e-03	6.21e-4	12.2
Var(ν)	drift of rate of tech prog	4.89e-06	2.29e-6	2.1
Var(ψ)	AR fert disturbance term	1.91e-06	2.49e-7	7.7
Var(σ)	fert intercept random wk	2.51e-07	1.30e-7	1.9
Var(ρ)	mort AR disturbance term	1.03e-05	8.76e-7	11.8
Var(ϕ)	mort intercept random wlk	1.77e-07	1.25e-7	1.4
AR Coeffs	AR coeff for disturbance in			
γ1	wage	.751	0.0547	13.7
γ2	wage	-.211	0.0497	-3.9
κ1	fert	.472	0.0876	5.4
κ2	fert	.0929	0.0693	1.3
λ1	mort	.599	0.0599	10.0
λ2	mort	-.111	0.0576	-1.9

TABLE II.

Comparisons of Estimated Lag Sums of Elasticities of Responses of Fertility and Mortality to Variations in Wages

	State Space (England)	OLS (England)	Europe (median of fourteen countries)
fertility and wages	.123	.14	.14
mortality and wages	-.076	-.10	-.16

Note: The OLS estimates are from Lee (1981), and refer to England, 1548-1834. The numbers for Europe are median values of estimates for 14 populations for varying time periods from 1540 to 1870, calculated from Galloway (1988).