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## **Sustainable Social Security: What Would It Cost?**

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## **Abstract**

Despite its current surplus and growing Trust Fund, the US Social Security system is in long-term imbalance, primarily due to future population aging ushered in by the retirement of the baby boom generations. Even with legislated increases in the normal retirement age in the coming decades, the Trust Fund will be exhausted in 2037 according to official projections (Board of Trustees 2000). The standard measure of long-run solvency, the 75-year Actuarial Balance, indicates that the imbalance could be removed by immediately and permanently raising the payroll tax rate by 1.89%, or by other changes in taxes and benefits with equivalent financial impact. However, a 1.89% tax increase would achieve solvency only through 2075. At that point, the Trust Fund would be falling rapidly, hitting zero a few years later. This would fix the system for 75 years, but would not put it on a sustainable basis.

This paper develops alternative measures of solvency, based on the cost of making the system sustainable over an infinite horizon. It proposes measures that attempt to reconcile the need for a longer policy horizon with the practical difficulties in making long-term projections. It considers three infinite horizon measures, and a fourth based on the simple criterion that the ratio of Trust Fund to Costs should be constant in 2075. We call this the Flat Fund Ratio criterion. This criterion is shown to be equivalent to one of the infinite horizon measures. It does not require any projection beyond the 75-year horizon, and it has common sense appeal. For these reasons, we propose its use for policy purposes. There is an infinite variety of sustainable policies, involving different changes in taxes and benefits, and with different allocation of costs and benefits across generations. A useful reference point is the immediate and permanent tax increase which would achieve balance over an infinite horizon. We call this the sustainable tax increase. It is a measure, not a prescription for policy.

We evaluate imbalance under the Intermediate Social Security projection assumptions, and also assuming more rapid mortality decline. The standard 75-year Actuarial Balance measure of the needed tax increase is 1.89%, which would be 2.4% with more rapid mortality decline. The Flat Fund Ratio sustainable tax increase is 3.1% under Intermediate assumptions or 4.2% with more rapid mortality decline (relative to the present value of payroll over an infinite horizon). Other infinite horizon measures indicate larger imbalances. We suggest that 4.2% is the most appropriate measure. It implies an imbalance more than twice as big as the standard Actuarial Balance measure (1.89%).

We also examine implications of policies maintained up to 2075 for the sustainable tax rate thereafter. Current policy would require a further 10.5% tax increase in 2075. A 1.89% tax increase through 2075 would require a 4.3% tax increase thereafter. An immediate increase of 3.1% would require no change in 2075. With a Pure Pay As You Go tax policy, payroll taxes in 2075 would be 6.2% higher than today, but no change would be needed immediately after 2075.

All these calculations are based on long-run projections which are very uncertain. Using Lee-Tuljapurkar stochastic simulations, we find that if the Flat Fund Ratio tax were

enacted today, there would be an 83% chance that in 2075, an adjustment of no more than 2% would be required. With the 2.4% tax increase indicated by the 75-year Actuarial Balance measure under rapid mortality decline, the corresponding probability would be only 19%. Despite this large improvement under the Flat Fund Ratio tax, it would be unrealistic to expect that any policy decision now could guarantee sustainability in the long-term. At best we can choose a policy that achieves sustainability in the “best guess” or median case, while expecting to adjust the policy in light of changing circumstances as time passes.

## Introduction

Despite its current surplus and growing Trust Fund, the US Social Security system is in long-term imbalance, primarily due to future population aging ushered in by the retirement of the baby boom generations. The Office of the Actuary of the Social Security Administration (SSA) projects that even with legislated increases in the normal retirement age in the coming decades, the Trust Fund will be exhausted in 2037 (Board of Trustees, 2000). Around the world, the financial outlook for unfunded public pension programs is bleak. The common underlying factors are increased life expectancy, low or falling fertility, and lower ages at retirement. Comparison of long-term funding imbalances in twenty OECD nations (Roseveare et al, 1996) reveals that the US imbalance is relatively small, with the typical imbalance two or three times as large. The US imbalance is nonetheless a prominent problem that attracts a great deal of attention from politicians, policy analysts, academics, and the general public<sup>1</sup>.

Indeed, in the US, the long-run financial problems of Social Security are once again in the news and on the political agenda. Why can't Social Security be fixed and stay fixed? In addition to issues of political will, there are two factors that make such a reform difficult: uncertainty about the population and economy far in the future, and the lack of appropriate measures of long-term financial balance.

Uncertainty arises because an assessment of the long-run finances depends on long-run projections. We don't know what the future will bring, and the passage of time yields new information that leads to revising/updating those projections. New projections require policy adjustments. This problem cannot be avoided, although automatic adjustments to new information can be built into policy. Stochastic projections, which we discuss later, can reveal the extent of this problem.

The most prominent measure of long-run solvency, and the most common test of proposed policies, is the 75-Year Actuarial Balance. Unfortunately, it has serious limitations for this purpose because it assesses solvency over a 75-year horizon, but takes no account of the situation thereafter. Fixing the system according to this criterion would leave it in serious imbalance after 75 years. Furthermore, the measure builds-in deterioration as time passes. One year from now, the end date of the 75-year horizon will have moved a year farther into the future period of deficit spending, so the newly calculated 75-Year Actuarial Balance will find even a "fixed" system to be newly out of balance. Unlike the first problem of uncertainty, this measurement problem can be solved by constructing measures that require sustainability beyond the 75-year horizon. In principle, the criterion should be balanced over an infinite horizon. However, due to uncertainty there is a natural reluctance to project even out to 75 years, let alone further.

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<sup>1</sup> Imbalance is measured as the present value of future taxes minus future benefits, plus initial assets, divided by GDP in 1994. Calculations are presented for real discount rates of 3, 5, and 7%, and rates of productivity growth of 1, 1.5, and 2%. Comparisons reported in the text were for a discount rate of 3% and productivity growth rate of 1%, corresponding to the assumptions of the Trustees Report (2000). The reported imbalance for the US is 62% of GDP. For Belgium, Norway, Portugal and Sweden it is over 200%, and for New Zealand and Denmark it is over 300%.

This paper proposes measures that attempt to reconcile the conceptual need for a longer policy horizon with the practical difficulties in making longer-term projections.

Infinitely many policies are consistent with sustainable Social Security. Some of these would raise taxes immediately and build up a large trust fund so as to avoid additional tax increases in the future. Others might raise taxes slowly, ending up with higher taxes for future generations than under the immediate tax increase policies. Still others might do nothing for now, leading to a large trust fund debt. This also could be a sustainable policy, provided that it includes a plan to raise taxes in the future, in order to stabilize the trust fund debt. These various policies would imply very different patterns of intergenerational transfers. We do not argue here for one or another of these. Our point is only that the choice of policy should be made with eyes wide open to the future. Any proposed policy should provide explicitly for the indefinite continuation of the system beyond the initial 75-year horizon. We will consider that a policy is “sustainable” if its future trajectory of taxes and benefits is balanced over an infinite horizon, according to current projections. We will consider that a tax is “sustainable” if its constant level would lead to balance over an infinite horizon under currently legislated benefits. Thus a sustainable tax increase, resulting in a constant level of future taxes, would be only one of many possible sustainable policies. Of course, benefits can change as well as taxes, and it is really the difference between tax revenues and benefit expenditures that matters. For simplicity of presentation, we will always couch our discussion in terms of tax changes.

Some reform proposals involve privatization and/or investment in equities. Other proposals maintain the current structure of the program, but make changes in taxes (raising the rate, removing the cap) or in benefits (reducing the COLA, raising normal retirement age, indexing benefits to life expectancy, making benefits need-based). We do not propose any specific policy. Instead, we propose measures of fiscal imbalance that must be addressed by policies of the second kind that keep the current structure of Social Security: a partially funded tax and transfer program, with the fund invested in US Treasury bonds.

## **Measuring the Imbalance: Limits of the 75-Year Actuarial Balance Measure**

How big is the imbalance in the long-term finances of the Social Security system (OASDI)? The most commonly used measure is the 75-Year Actuarial Balance (AB75). Virtually all the plans that have been proposed by politicians and analysts are based on this measure, and discussions in the media revolve around it as well. The AB75 equals the current amount in the trust fund, plus the present value of taxes over the next 75 years minus the present value of costs over the next 76 years, all expressed as a percentage of the present value of projected taxable payroll over the next 75 years. According to the most recent estimate by the Social Security Office of Actuaries, AB75 is 1.89%; the payroll tax rate would need to be raised by 1.89%, from its current level of 12.4% to 14.29%, in order to achieve long-term Actuarial Balance (Board of Trustees, 2000). If the payroll tax rate were immediately and permanently raised by this amount, then there would be sufficient funds to make all necessary expenditures over the next 75 years, and to have a trust fund equal to one year’s costs at the end of this period.

This measure indicates whether the system will remain solvent over the remaining lifetime of almost all participants currently contributing to the system. In 75 years, surviving contributors currently age 20 would be age 95. The difficulty is that even with the payroll increase of 1.89 % indicated by the AB75, after 79 years the Trust Fund would be exhausted and taxes would cover only 78% of projected costs. Then the payroll tax rate would have to be raised by a further 4.3 percentage points to 18.6% in order to cover costs, or benefits would have to be cut correspondingly (Board of Trustees, 2000: Table III.A.2, pp.171-2; note that taxes on benefits generate additional revenues for the system<sup>2</sup>). After 75 years, the system would be in a far more precarious situation than it is now. For this reason, some analysts have called for additional measures of the long-term soundness of the system (1999 Technical Panel:10, 28).

Figure 1 illustrates this problem. It plots the projected trust fund level under the current tax rate, and after a 1.89% increase, over the next 80 years. Since it would take about 4.5 years to exhaust the final balance equal to one year's costs ( $1/(1-.78) = 4.5$ ), the trust fund would hit 0 between 2079 and 2080, and would drop increasingly rapidly thereafter.

<< Figure 1 around here >>

Here is a different way to look at the same point: Each year, the Trustees issue a new Report which updates the AB75. Each new AB75 includes one more year of deficit far in the future, when expenditures exceed tax revenues, and the previous year's Actuarial Balance grows by the interest rate. For this reason, referred to as "change in the valuation period", each year the new AB75 tends to be worse than the previous<sup>3</sup>. The most recent report indicates that the "change in the valuation period" since the previous year's report resulted in a worsening of the AB75 by .07% (Board of Trustees, 2000: Table II.F.21, p.130). Under the Trustees Report's assumptions, this source of change is a certain and completely predictable result of the passage of an additional year, and is a consequence of choosing a finite horizon. Although the size of the annual change is small, it is repeated systematically year after year, and so it cumulates. Based on the last eleven Trustees Reports, the cumulated change since 1989 due to valuation period is .67%. The size of the changes has increased steadily over that period, and will presumably be larger in the coming decade. Since this deterioration in the AB75 is predictable, it should be possible to construct a measure which reflects it in advance, and therefore does not change over time as the horizon is shifted farther into the future.

The most recent Technical Advisory Panel for Social Security endorsed this kind of concern with "sustainability" over the longer term. Stating that "emphasis on the 75-year Actuarial Balance is misleading," the panel noted that "many designers of reform try to

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<sup>2</sup> The payroll tax rate is currently set at 12.4%. However, taxes on benefits generate additional revenues, so that the current tax income is 12.65% of payroll rather than 12.4%. By 2075, this is projected to rise to 13.34% (all numbers based on Board of Trustees, 2000: Table III.A.2). The calculations reported in this paragraph reflect this increased revenue from taxes on benefits.

<sup>3</sup> Since there are other changes, which also alter the AB75, the net result from all causes may be either improvement or deterioration.

reach balance simply by targeting their plans only at the 75-year actuarial deficit ... [and] ... usually end up in a situation where their reforms only last a year before being shown out of 75-year balance again” (1999:28). However, the panel also recognized that “there is no way to demonstrate long-term patterns of sustainability....” (1999:28). This paper attempts to fill this gap.

## **A Simple Measure of Social Security Sustainability: The Flat Fund Ratio Tax**

As a first step to address the sustainability of Social Security, we suggest a simple common sense criterion: At the end of the 75-year projection horizon, the ratio of the Trust Fund to costs should be constant, or equivalently the Fund should be growing at the same rate as costs. We call this the “Flat Fund Ratio” (FFR) criterion. It has a number of appealing features. We will show that any system that is in balance over an infinite horizon, and which has reached a steady state by 2075 (the key assumption), must satisfy the Flat Fund Ratio criterion. If this ratio is constant, then interest payments on the Fund will be a constant proportion of the system’s costs each year. The criterion can be applied using information already contained in the Trustees annual reports; no projection beyond the 75-year horizon is necessary. It provides a test of sustainability for any proposed policy, including changing trajectories of taxes and benefits over the 75-year period. We can calculate the imbalance as a present value of the shortfall. This, relative to the present value of payroll, gives the immediate and permanent tax increase, or “sustainable tax”, which would satisfy the Flat Fund Ratio criterion. Additionally, for any policy of changing taxes and benefits up to 2075, we can readily calculate the tax increase in 2075 that would be needed to achieve sustainability. (Details are given in Appendix A, equations 8 through 14).

Suppose that a given policy implies a trust fund balance in 2075 of  $F(2075)$ , and expenditures and payrolls during that year of  $C(2075)$  and  $W(2075)$ , respectively. Appendix A shows that the sustainable tax rate thereafter must equal  $C(2075)/W(2075) - F(2075)/W(2075) * (\text{interest rate} - \text{growth rate of costs})^4$ . Similar equations are given for the infinite horizon Actuarial Balance evaluated in 2000, and other quantities. To illustrate, suppose that current policy were maintained through 2075. To achieve sustainability thereafter, the necessary payroll tax rate starting in 2075 would be 23%, a 10.5% increase. Alternatively, consider a policy based on the AB75, to increase the payroll tax immediately and permanently by 1.89%, with benefits as under current law. Then to achieve sustainability after 2075, the payroll tax rate would have to be 18.4%, requiring an additional tax rate increase of 4.1%, on top of the original 1.89%. By contrast, if there were an immediate payroll tax increase of 3.1%, then sustainability would require no change in 2075. Put differently, the sustainable tax rate in 2000, according to the FFR criterion, is 3.1%. This value was found using equation A.9. These results will be revisited in a later section.

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<sup>4</sup> Based on the Intermediate assumptions in the Trustees Report (2000),  $C(2075)/W(2075)$  (which is the cost rate in 2075) is 19.53%,  $W(2075)$  is 10.17 trillion in 1999 dollars, and the growth rate of costs in 2075 is 1.45% per year.

## Infinite Horizon Measures of Sustainability

Although the Flat Fund Ratio criterion has a common sense appeal, it is ad hoc and the horizon is arbitrary. In this section we will consider three strategies for carrying out infinite horizon calculations of balance, and show that the Flat Fund Ratio criterion corresponds to one of these strategies.

But why should we require balance over an infinite horizon, rather than the 75-year horizon currently used by the Actuaries? Actually, prior to 1965 the Actuaries did assess solvency over an infinite horizon, but the 1965 Advisory Council recommended they switch to the current 75-year horizon (see Appendix B for a discussion of this and other aspects of the Actuaries' procedures, based on Goss, 1999). Social Security is structured in a very particular way. Financing current benefits largely out of taxes on current workers automatically creates new obligations to pay benefits in the future. The young generation that pays benefits to the old generation must itself be supported in its old age by the next young generation. Social Security is built on the concept of a "chain letter" of benefits and contributions across generations over an infinite time horizon. Under such a system, the net obligations at any moment are the system's implicit debt and the program could not be shut down without either canceling the implicit debt – a policy that no one recommends – or providing for its repayment. Thus, as long as one considers policies to continue Social Security in its current form, assessment and balance over an infinite horizon are not only appropriate but necessary<sup>5</sup>. In this regard, our approach here is quite similar to Generational Accounting (see Appendix C for a discussion of similarities and differences between the two approaches).

The most direct approach to the infinite horizon assessment is to continue the 75-year projection farther into the future, say for 300 or 500 years, when the power of discounting would render further extension irrelevant. This extended projection can be achieved by simply continuing the long-run rates of covered wage growth and real interest rates assumed for the 75-year horizon. For the demography, the values of fertility, mortality, and immigration assumed at the end of the 75-year horizon can likewise be continued. This is the approach used in Generational Accounting, which also requires very long forecast horizons (see Auerbach et al, 1999). Goss (1999) reports an infinite horizon Actuarial Balance of  $-4.7\%$  based on the assumptions of the 1996 Trustees Report<sup>6</sup>. The Infinite Horizon imbalance in the system would then be  $4.7\%$  of the present value of payroll, in contrast to the  $2.19\%$  of payroll indicated by the AB75 calculation in the 1996 report.

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<sup>5</sup> Some policy alternatives such as privatizing Social Security and replacing it with a system of funded private retirement accounts do not require an infinite time horizon since they generally include a plan for repayment of this implicit debt.

<sup>6</sup> Although details are not given, it appears that Goss calculated the 500-year Actuarial Balance from extended projections of this sort. We have replicated this result by extending our population projection out for 300 years based on the 1996 Trustees Report assumptions, and then after year 75, assuming that the tax income per person 20-64 continues to grow at the same exponential rate as observed at 75 years, and that the benefit per person 65+ likewise continues to grow at the same exponential rate thereafter.

Although this approach is straightforward, it is problematic to extend the forecasting horizon for even a few decades, let alone to infinity. Many people already believe that the 75-year horizon is too distant to be meaningful. We will return to this topic later, but first we will explore alternative approaches. One appealing way to proceed is simply to assume that after 75 years (or some other interval), the system has settled into some kind of steady state, so that rates of growth of costs and tax revenues are thereafter constant. In this case, we can work from the situation at the end of the 75-year projection, without extending the projection horizon further. We implement this approach for two different kinds of steady state, according to whether the rates of growth of costs and tax revenues are equal.

First consider the Unstable case, in which the rates of growth of costs and tax revenues may differ. These rates can be calculated from the last two years of the 75-year projection period, indicating that for the US system, costs grow significantly more rapidly than payroll. One cause of this discrepancy is continuing mortality decline, which in turn leads to an ever-rising old-age dependency ratio. Differences in the growth rates of payroll and costs could also arise from an unstable population age distribution at younger ages, or from assumed changes in fertility or immigration. We will call the resulting calculation of imbalance the Unstable Infinite Horizon imbalance.

In the Stable case, it is assumed that the growth rates of payroll and costs are equal. Tax revenues cover only two thirds of costs in 2075, and this proportional gap remains constant thereafter under current policy. We will call the resulting calculation of imbalance the Stable Infinite Horizon imbalance. We show in Appendix A, that if the stable assumption holds, then any system that is balanced over an infinite horizon will satisfy the Flat Fund Ratio criterion. This puts the Flat Fund Ratio criterion on a solid theoretical footing, and clarifies its meaning.

## **Demography in the Long-Run and the Three Evaluation Strategies**

### **Long-Run Demography**

Let us consider the suitability of the three strategies (Flat Fund, Unstable Infinite Horizon, and Stable Infinite Horizon) from a demographic perspective. Most long-term demographic projections assume that fertility will trend toward some level and then remain constant, and that the annual flow of immigration and emigration will be constant. Mortality, however, is generally projected to continue to decline indefinitely.

Consider the population up to age 65. In the long run, survival to age 65 will approach 100%. The US Total Fertility Rate is projected by the Actuaries to approach 1.95, below replacement, which would lead eventually to population decline if there were no net immigration. Net immigration is assumed to remain constant at roughly 1 million per year. As time passes, the population up to age 65 will increase or decrease until the net immigration flow is just sufficient to offset the natural decrease due to low fertility. At that point, the population below age 65 will reach a steady state age distribution, with constant size.

Now consider the population above age 65,  $Pop(65+)$ . Each year, it will be augmented by an inflow equal to the number of 64-year olds in the previous year, a number that eventually will be constant from year to year, call it  $Pop(65)$ . Assume that all net immigrants arrive or depart before the age of 65. The size of  $Pop(65+)$  will approximately equal this inflow times the life expectancy at age 65,  $e_{65}$ :  $Pop(65+) = Pop(65) * e_{65}$ . While  $Pop(65)$  will eventually become constant,  $e_{65}$  will continue to increase. Vaupel (1986) has shown that if age-specific death rates continue to decline at a constant proportional rate over the long term, then  $e_{65}$  will eventually rise at a constant linear rate. In particular, if mortality at all ages over 65 declines at 1% per year, then  $e_{65}$  will rise at approximately one year per decade. A constant linear increase in  $e_{65}$  will imply a declining proportional rate of increase in  $e_{65}$ , eventually approaching zero as the centuries pass<sup>7</sup>.

We can conclude that for a long time, the growth rate of the elderly population, and therefore of costs, will exceed the growth rate of the working-age population and therefore of covered wages and tax revenues. Therefore the stable assumption is initially wrong, and will underestimate the pressures on the system. In the very long run, the assumption of equal rates of change of taxes and expenditures is appropriate, but this convergence occurs at a slower rate than observed initially (say in 2075), and during the transition to the long run, a larger gap between taxes and expenditures opens up due to the slowing of population growth, increasing  $e_{65}$ , and consequent population aging. These points are confirmed by a demographic projection over a 300-year horizon.<sup>8</sup>

We will present results calculated according to each of these strategies. However, our general view is that there is little value in spinning out over several centuries the implications of assumptions that are so fragile. Later we will explicitly model uncertainty in the forecasts. The appeal of the stable and unstable assumptions is that they can be implemented from existing projections, and they are simple. We expect that the stable assumption will lead to an underestimate of the imbalance, while the unstable assumption will lead to an overestimate.

### **How Fast Will Mortality Decline?**

Many analysts believe that the Intermediate mortality assumption of the Trustees Report represents an unrealistically slow rate of decline of mortality (Stoto and Durch, 1993; Technical Panel Report, 1999: 3, 22, 64-67). Lee and Carter (1992) developed a new

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<sup>7</sup> By arguments given in Vaupel (1986), we know that  $e_{65}(t)$  will be increasing by roughly .1 years for each one year increase in  $t$ . After 300 years,  $e_{65}$  will be roughly 30 years greater. Since  $e_{65}(2000)$  is about 18 years, an increment of .1 years per year initially contributes a growth rate of  $.1/18 = .55\%$ . In 2075,  $e_{65}$  may be about 25 years, and the growth rate will have fallen to  $.40\%$ . After 300 years,  $e_{65}$  may be 48 years, and the growth rate will then be only  $.1/48 = .2\%$ . Thus the elderly population will continue to grow after the working-age population has ceased to grow, and its growth rate will initially (in 2075) exceed that of the working-age population by about  $.4\%$ , which will be halved after another 225 years.

<sup>8</sup> The projection shows the rate of growth of the 65+ population declining from  $.69\%$  per year in 2075, to  $.14\%$  per year in the year 2300. Over this same period, the growth rate of the working-age population declines from  $.14\%$  per year to  $.05\%$  per year. Neither remains constant, obviously, and the difference between the two decreases from  $.55\%$  initially to  $.09\%$  in 2300.

method for forecasting mortality, which projected life expectancy gains at almost double the rate projected by the Social Security Actuaries at that time. [The Lee-Carter forecasts are incorporated in new stochastic and deterministic population forecasts by Lee and Tuljapurkar (1994 and forthcoming).] Although the Actuaries have raised their life expectancy forecast slightly in the latest Trustees Report (Board of Trustees, 2000), their forecast of gains is still substantially below the Lee-Carter forecast. An updated application of the Lee-Carter method forecasts life expectancy in 2075 of 85.9 years, for sexes combined. If mortality follows this path, then the system imbalance will be greater in 2075, with a larger deficit and a greater trust fund debt. In addition, the elderly population will be growing more rapidly than under the Actuaries' projections. The population 65 and over is projected by the Actuaries to grow at .49% per year in 2075, while the corresponding figure for the Lee-Carter mortality projection (with the same fertility and immigration assumptions used by the Actuaries) is .74% per year, or higher by .25 percentage points (.74% - .49% = .25%).

### **Estimates of Imbalance According to Various Measures**

All the information needed to compute the various sustainable tax rate measures under projection assumptions of the Social Security Actuaries is given in the annual Trustees Report<sup>9</sup>. Appendix A provides the equations used for the actual calculation of the flat Fund Ratio or infinite horizon Stable (A.9) and Unstable (A.7) cases.

Table 1 presents the calculations for the 75-Year Actuarial Balance, plus four different measures of imbalance relative to sustainability. These measures are expressed relative to the present value of payroll, so they can be interpreted as the size of an immediate and permanent increase in the tax rate necessary to achieve balance, or the corresponding reduction in costs of benefits relative to payroll. They can also be interpreted in dollar amounts. [The first column shows the estimate based on the Trustees Report (Board of Trustees, 2000) demographic and economic projections.]

<< Table 1 about here >>

The first column gives the projections based on the assumptions of the Social Security Actuaries, taken from the Trustees Report (2000). The first row is the standard 75-year Actuarial Balance, rounded up from .0189 to .019. The Flat Fund Ratio measure, which is equivalently the Stable Infinite Horizon measure, raises this to .031, or by 1.2%. Under the Unstable Infinite Horizon measure this rises to .037, and with the extended projection to a 300-year horizon, it is .035. Measures of the imbalance based on sustainability criteria indicate a much larger imbalance than the standard AB75 measure. As expected,

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<sup>9</sup> The report does not explicitly provide the projections of the fund once it turns negative. However, it can be estimated from other data in the report. Annual income from the payroll tax and interest on the funds and annual costs for the remaining years were estimated by multiplying taxable payroll by income and cost rates, respectively. The information necessary for this estimation, including annual adjusted CPI, taxable payroll, and OASDI income and cost rates, were obtained from more detailed versions of Tables III.A2 and III.B1. The tables are available from SSA's web sites (<http://www.ssa.gov/OACT/TR/TR98/lr3A2-2.htm>; <http://www.ssa.gov/OACT/TR/TR98/lr3B1-2.htm>).

the imbalance under the Stable or Flat Fund Ratio measure is considerably less than under the Unstable or Extended Projection measures.

The second column reflects more rapid mortality decline as in the Lee-Carter forecast, but otherwise with the same projection assumptions as the first column. These estimates were made using the MVR-Berkeley Social Security simulation program in deterministic mode (unpublished runs; see Lee and Tuljapurkar, forthcoming). This second column shows how much the imbalance increases with more rapid mortality decline. Note that the increase, which is .5% for the AB75 measure, is larger for the sustainability measures, rising to 1.1%, then 2.0%. The sustainability criteria attach more weight to events far in the future compared to the 75-year measure, although they discount the future at the same rate.

Which of these measures of imbalance should be used for planning? We have already discussed the problems with the 75-year Actuarial Balance. However, we are reluctant to base our estimates on forecasts that go past the 75-year limit, and therefore we are inclined to favor one of the measures based on the steady state assumption. Of the two, the Stable or the Flat Fund Ratio measure seems the more appealing. Its rationale is intuitive and consistent with common sense. It is a natural extension of the current actuarial practices, and does not require invocation of the infinite horizon. It provides a lower-bound estimate of the long-term imbalance, since it assumes that costs grow no faster than tax revenues after 2075. A lower-bound estimate is probably desirable in this context. We also believe the evidence that mortality will decline more rapidly than the Actuaries project. Therefore, we suggest that the long-term imbalance in the Social Security system be put at around 4.2% of the present value of payroll, rather than at 1.89%. According to this reasoning, the imbalance is more than twice as great as indicated by the standard 75-Year Actuarial Balance measure.

## **Future Implications of Some Current Policies**

We can use the measures based on the Flat Fund Ratio or Stable Infinite Horizon concepts to evaluate and compare the long-run fiscal implications of policy options. In each case, we will characterize the policy up to 2075, present the implied fund balance in 2075, and present the subsequent sustainable tax rate, the unique constant payroll tax rate that would balance the system over the remaining infinite horizon under the Stable assumption.<sup>10</sup> We will consider five policies: continuing current policy until 2075; immediately raising the payroll tax by AB75 rate; immediately raising the payroll tax by Flat Fund Ratio/Stable rate; a payroll tax that rises linearly from its current level now, to a sustainable level in 2075 (this uniquely defines that level); and a pure Pay-As-You-Go (PAYGO) in which once the trust fund is exhausted, the payroll tax rate is raised each year so that tax revenues exactly equal costs. These are very stylized policies, and are presented to illustrate the approach, which could be applied to any time path of tax rates and benefit costs specified as far into the future as desired.

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<sup>10</sup> The sustainable tax rate is a better measure for this purpose than the long horizon Actuarial Balance in 2075, since the measures differ in the present value of payroll.

Figure 2 plots the Fund Ratio under six different tax rate regimes from 2000 to 2099, using the Lee-Tuljapurkar projections with more rapid mortality decline. Under the current legislation scenario, the Fund Ratio peaks in 2013 and declines to zero in 2034, a little sooner than under the Actuaries' assumptions. With a tax increase of 2.4%, which is the 75-year Actuarial Balance under the Lee-Tuljapurkar projections, the Ratio would peak around 2020 and decline to zero in 2078. With a tax increase of 4.2% to meet the Flat Fund Ratio criterion of balance, the ratio would peak in 2074-2075. By construction, it is flat between these two dates. If the Stable assumption were true, then the fund ratio would remain constant thereafter, but Figure 2 shows that it is projected to decline slightly after 2075. If the tax is raised above 4.5%, as under the 300-year Actuarial Balance measure (4.6% increase) and the Unstable Infinite Horizon measure (5.7% increase), then the fund ratio continues to rise throughout the 21<sup>st</sup> century.

<< Figure 2 about here >>

Figure 2 also shows that under the linear increase policy, the fund ratio would remain remarkably flat throughout the century, particularly after 2045, tracking the increase in costs quite closely, at a ratio of about 3.5. At this ratio, interest from the fund covers about 10% of annual costs.

Table 2 describes additional implications of the five different policies. The first column labels the current tax policy under consideration, and the second column gives the corresponding tax rate assumed to hold between 2000 and 2075. The third column gives the implied fund balance in 2075 (in 1999 dollars). The numbers in this column are consistent with the ratios in Figure 2 for 2075. Given this fund balance in 2075, equation (A.14) in Appendix A is used to calculate the sustainable tax thereafter. Under Social Security Intermediate assumptions, GDP in 2075 is 28.5 trillion in 1999 dollars. We see that under current legislation, the fund would be in debt by 2075 for an amount roughly equal to GDP, were this permitted by law. The Flat Fund tax would generate a fund roughly equal to three fourths of GDP. Funds of these latter sizes are unrealistic, and would be inconsistent with the independent projection of interest rates.

<<Table 2 about here >>

The table shows that the various policies differ in how they allocate the pension costs of population aging to future versus current generations. Under current policy, workers in 2075 would face a 23% tax, while with an immediate Flat Fund Ratio 3.1% tax hike, their taxes would be only 15.5%, with a much higher share of the load born by current workers. Pure PAYGO, the linear increase, and an AB75 increase all fall between these two extremes. Of these three, both the PAYGO and linear increase policies impose gradual increases in tax rates, while the AB75 policy imposes a discontinuous large jump in 2075 as we have modeled it.

Table 3 is identical to Table 2, except that it gives results under the more rapid mortality decline projected by Lee-Carter (1992). Under more rapid mortality decline, not only are the initial tax rate increases greater, but the sustainable payroll tax after 2075 is greater

under the current law and the AB75. Thus under the current law, more rapid mortality decline would require an additional tax increase of 2.3% for sustainability in 2075, for a total tax rate then of 25.2%. The immediate tax increase required by the AB75 criterion would be .5% larger with more rapid mortality decline, with the gap widening to nearly 2% by 2075, requiring a 5.4% payroll tax increase in 2075 for subsequent sustainability.

## Uncertainty

All the various measures of long-run imbalance are necessarily based on long-run projections. Because we cannot know what the future will bring, we cannot know what policies would actually achieve fiscal balance and sustainability. Is it worth trying to fine-tune current policies to achieve sustainability according to projections which themselves contain a wide margin of error? Stochastic projections of Social Security finances by Lee-Tuljapurkar (1998a,b, and forthcoming) provide a means for estimating probability distributions of outcomes under different current tax policies. In these projections, four inputs are generated from stochastic time series models for each year: fertility, mortality, productivity growth, and the real interest rate on US Treasury Bonds.

Figure 3 shows probabilities for the date of Trust Fund exhaustion under different tax rate regimes, based on the Lee-Carter mortality decline. The possible tax rate regimes are located on the horizontal axis, extending from current policy, with a zero percent increase in the payroll tax rate, up to an 8 percent increase in the tax rate at the far right. In between lie the increases indicated by the 75-year Actuarial Balance criterion, the Flat Fund Ratio criterion, and the Unstable Infinite Horizon criterion. The associated probabilities are given on the right or the left vertical axis.

One line, sloping downward toward the right, shows the probability of system insolvency (Trust Fund exhaustion) by 2075 on the left vertical scale. With a tax increase of 1.89%- the AB75 tax rate increase under the SSA Intermediate assumptions- there is a 55% chance of fund exhaustion by 2075. An increase of 2.4%, corresponding to the AB75 under the more rapid mortality decline of the Lee-Carter model, reduces this probability to 40%. With the FFR increase of 4.2%, this probability drops approximately to 10%, and with the Unstable Infinite Horizon tax (an increase of 5.7%) it drops to 2%.

<<Figure 3 about here>>

The other lines on Figure 3 show, on the right hand scale, the median date of exhaustion with a 95% probability interval. With a tax increase of 1.89%, the median date of exhaustion is 2070; with a 2.4% increase, it is 2085. Anything above a 3% increase pushes the median date of exhaustion past 2100. With the Flat Fund Ratio increase of 4.2%, there is only a 2.5% chance of exhaustion before 2075.

However, the purpose of these proposed measures of imbalance is not to minimize the risks of fund exhaustion, but rather to achieve sustainability for the system, and to avoid the continual revisions that come from a changing valuation period. Figure 3 does not really help us to evaluate these aspects of their performance. A more appropriate test,

therefore, would be to ask how the situation would look in 2075 if the Flat Fund Ratio tax were enacted today.<sup>11</sup>

In this spirit, suppose that the sustainable tax (according to the Flat Fund Ratio/Stable Infinite Horizon criterion) is instituted in 2000, based on our best estimate of what the future will hold. We then do 1000 stochastic simulations, holding that tax rate constant up until 2075. In 2075 we reassess the situation, by recalculating what the best guess sustainable tax would be from 2075 into the future. We will do this for each of the 1000 stochastic outcomes in 2075. If all is well, then the median of the sustainable tax rates calculated for 2075 forward will be the same as the sustainable tax rate estimated in 2000, since both are supposed to work over an infinite horizon.

However, for any particular stochastic outcome in 2075, the stochastic Trust Fund balance will be different than expected, due to the randomness of the four inputs, fertility, mortality, productivity growth and interest rates, between 2000 and 2075. Consequently, for the typical stochastic trajectory, the sustainable tax rate after 2075 will be different than expected. Furthermore, we must imagine the Actuaries making their new projections in 2075. They might estimate exactly the same models for the four inputs, so that their forecasts in 2075 have the same long-run mean behavior as the forecasts made in 2000. We prefer not to constrain their projection models in this way, but rather to assume that they extrapolate into the future the growth rate of costs that were observed over the previous five years, from 2070 to 2075. For each stochastic trajectory, this extrapolated growth rate will reflect the random particularities of the past five years, leading to further differences of the new sustainable tax rate from its expected or median value. Finally, the new sustainable tax rate depends on the particular stochastic values of payroll and costs in 2075. The equation for the sustainable tax rate at any time  $T$  after the stable assumption holds, is given by equation A.14:

$$\tau^* = \frac{(\beta - r)F(T) + C(T)}{W(T)}$$

where  $F$  is the fund level,  $C$  is costs,  $W$  is payroll,  $r$  is the interest rate, and  $\beta$  is the rate of growth of costs and by the stability assumption, the rate of growth of payroll, as well.  $T$  is taken to 2075 for our purposes.

Figure 4 plots the distribution of the change in the sustainable tax in 2075; that is, the new stochastic sustainable tax minus the old fixed one. The distribution is sharply peaked, and lies almost entirely between  $-.05$  (a 5% reduction in the tax in 2075) and  $+.05$  (a 5% increase in the tax in 2075). Note that the sustainable tax rate is just the ratio of the Infinite Horizon Actuarial Balance in 2075 to the present value of payroll, so we can also interpret the distribution is showing the size of the infinite horizon imbalance relative to payroll.

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<sup>11</sup> Such a test is somewhat arbitrary for two reasons: First, immediate enactment of a constant tax rate is only the simplest of any number of sustainable policies, and second, there is nothing special about the 75-year period to re-assessment, other than convenience.

Figure 4 also shows a similar distribution for the case in which the payroll tax follows current law, and for the case in which the tax increase indicated by the 75-year Actuarial Balance, AB75, is immediately enacted. For these cases, the figure shows the distribution of the difference between the sustainable tax in 2075 and the constant tax rate for 2000 to 2075. These distributions are located farther to the right, indicating the greater probability of necessary tax increases. The medians for the three distributions are indicated on the horizontal axis.

<<Figure 4 about here>>

It is also useful to plot the cumulative probability distributions that correspond to the frequency distributions shown in Figure 4. These cumulative distributions are shown in Figure 5, and interpreted in Table 4. If a tax according to the Flat Fund Ratio criterion were enacted today, then there would be only a 47.2% chance that the Actuarial Balance would be negative, necessitating a tax increase for sustainability. If taxes were raised according to the AB75 criterion, then there would be a 94% chance that the balance would be negative, and under current policy, the probability would be 99.7 %.

<<Figure 5 and Table 4 about here>>

The last column of Table 4 indicates that with the Flat Fund Ratio tax, there would be an 83% chance that the Actuarial Balance in 2075 would fall within 2% of the present value of payroll, or equivalently that the new sustainable tax in 2075 would be within 2% of that in place from 2000 to 2075. For the AB75 tax regime, this probability would be far lower at 19%. The contrast between these two distributions summarizes the advantage of the Flat Fund Ratio criterion over the current Actuarial Balance criterion.

Although the Flat Fund Ratio Tax performs quite well in this stochastic context, with better than an 80% chance of ending within 2% of perfect balance in 2075, it is nonetheless clear from Figures 4 and 5 there is substantial uncertainty. The future is unknown and unknowable. We can only point our policies in what seems today to be the right direction. Fortunately, there will be many opportunities in the coming decades to alter policies in the light of new knowledge as the future becomes history. Policies could be redesigned in coming decades, or alternatively, adaptive policy rules could be specified now in such a way that the response to future contingencies would be made clear in advance. Indexation of benefit level or age at retirement to future life expectancy would be one such policy.

## **Conclusion**

The 75-year Actuarial Balance (AB75) measure, on which policy discussions are currently based, measures solvency over the lifetimes of current workers, but not for the lifetimes of the workers in coming decades who will pay for the retirement of current workers. We must assess system solvency for these as well.

The sustainability and balance measures developed in this paper fill this need. Unlike the standard AB75, they should require revision only when new information becomes

available, and not because of the predictable change in valuation period as time passes. In particular, we propose the Flat Fund Ratio measure (which is equivalent to the Stable Infinite Horizon measure), as the basic measure of long-run solvency. Calculation does not require projections beyond the current 75-year horizon, and it has a strong common sense appeal. It probably underestimates the long-run imbalance, for reasons discussed, but it would move us most of the way in the right direction. We suggest that this measure would be a more suitable basis for policy discussions and public debate. Under the 2000 Trustees Report assumptions, the Flat Fund Ratio measure indicates that the system is 3.1% out of balance, and with more rapid mortality decline (which we believe more likely) the imbalance rises to 4.2%. These figures are substantially higher than the 75-Year Actuarial Balance measure of 1.89%.

Our estimates of the standard AB75 measure and the Sustainable measures all discount at the projected real rate of interest earned on the Trust Fund, or 3%. At this rate, a dollar in 75 years counts only a tenth as much as a dollar today. Nonetheless, because the Sustainable measures depend on the rates at which costs and income are growing after 75 years, they are considerably more sensitive to slowly developing changes such as the consequences of more rapid mortality decline. Under the Trustees' assumptions, the elderly population in 2075 will be growing at .52% per year (Trustees Report, 2000:Table II.H.1), while under the Lee-Tuljapurkar assumptions of more rapid mortality decline, it will be growing at .73% per year, or .21% faster. This difference at the end of the valuation period strongly affects the Flat Fund Ratio measure, but does not explicitly affect the AB75 calculation.

It is convenient to interpret various sustainability measures in terms of the immediate and permanent tax-hike necessary to achieve balance, which we call the sustainable tax, but this is not a prescription for policy. Many kinds of changes in taxes or benefits, with different timing, could be used to achieve balance, with differing intergenerational consequences. For comparing the long-term fiscal implications of various Social Security policy proposals, we suggest that the sustainable tax increase evaluated in 2075 should provide a useful tool. In any event, the long-term future is highly uncertain, and we can at best hope to aim towards the middle of a wide distribution of possible outcomes, or to design adaptive policies. No matter what policies are implemented today, we should expect to modify them as the future unfolds.

Although there is a serious imbalance in the long-run finances of the US Social Security system, the size of the imbalance is certainly manageable in relation to GDP. The Actuaries' measure of 1.89% of payroll amounts only to .76% of GDP. According to the Flat Fund Ratio measure, with more rapid mortality decline, the 4.2% increase in the payroll tax rate would correspond to only 1.7% of GDP<sup>12</sup>. That is, if taxes were immediately and permanently raised by 1.7% of GDP today, the system would be put on a sustainable path. Recall, however, that the median imbalance in OECD countries is two and a half times as great as in the US, and that some countries have imbalances six times the size of that in the US. We have not been able to estimate the Flat Fund Ratio Tax for these other countries, but there is every reason to expect that as for the US, it would

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<sup>12</sup> See Table III.C2 of the Board of Trustees Report (2000).

indicate greater problems than are apparent from the AB75 measure reported in Roseveare et al (1996). For these countries, with their more generous pensions, longer life, lower fertility and earlier retirement, the fiscal problems will be far more painful to confront and resolve.

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## Appendix A: Analytics of Sustainable Tax Measures

We define the following variables, all measured in real terms, net of inflation:

$F(t)$ : the level of the Social Security fund in year  $t$

$W(t)$ : the taxable payroll in year  $t$

$C(t)$ : the cost in year  $t$  for benefits and administration

$I(t)$ : the tax income in year  $t$  (payroll tax plus tax on benefits, but not including interest)

$r$ : the interest rate, which is taken equal to the rate earned on the Trust Fund.

$\tau$ : the current tax rate relative to payroll

$\delta$ : the size of a one time immediate and permanent increase in the payroll tax rate needed to achieve a stated goal, such as 75-year Actuarial Balance ( $\delta^{75AB}$ ), Flat Fund Ratio ( $\delta^{F75}$ ), Stable Infinite Horizon ( $\delta^{S\infty}$ ) or Unstable Infinite Horizon ( $\delta^{U\infty}$ ).

$T$ : the time horizon for an evaluation, which can be finite or infinite

Additional notation will be introduced as necessary.

### 1. Actuarial Balance

Define the present values of income from taxes and of costs over a time horizon of  $T$  years. This presupposes a policy which specifies the tax rate and the costs of benefits over the horizon  $T$ .

$$(A.1) \quad PV(I, T) = \int_0^T e^{-rs} I(s) ds$$

$$(A.2) \quad PV(C, T) = \int_0^T e^{-rs} C(s) ds$$

The Actuarial Balance over horizon  $T$ , relative to this policy, is defined as:

$$(A.3) \quad AB(T) = PV(I, T) + F(0) - PV(C, T+1)$$

This is usually expressed as a proportion of the present value of taxable payroll over the same horizon,  $PV(W, T)$ . We will call this proportion  $-\delta^{AB(T)}$ , where  $\delta$  can be interpreted as the size of the immediate and permanent increase in the payroll tax rate (relative to the tax rate of the policy) which would achieve balance, or set  $AB(T) = 0$ , leaving a fund at time  $T$  equal to the next year's costs. In these expressions,  $T$  can take any positive value, including infinity. In the calculations of the Actuaries,  $T=75$ .

### 2. Actuarial Balance with an Infinite Time Horizon

It will be useful to divide each infinite integral into two parts, the first over the time period for which a detailed projection is available, which is 75 years in the case of the Social Security system, and the second from this point to infinity, for which detailed projections are not available and a simplifying assumption must be made. The equation for the infinite horizon Actuarial Balance, given above with  $T=\infty$ , can be rewritten as:

$$(A.4) \quad AB^\infty = F(0) + \int_0^T e^{-rt} [I(t) - C(t)] dt + \int_T^\infty e^{-rt} [I(t) - C(t)] dt$$

Inspection of the first two terms on the right indicates that this is the fund balance at time  $T$ , discounted to time 0, or  $e^{-rT} F(t)$ . (Note that this is almost identical to  $AB(T)$ , differing only by the exclusion of the present value of one year's costs at the end of the period,  $e^{-rT} C(T+1)$ ). This can be evaluated using data contained in the Trustees Report.<sup>13</sup> The infinite integral requires additional assumptions, as discussed in the text.

1. Unstable Assumption: Assume that from  $T$  on,  $I(s)$  and  $C(s)$  grow at constant exponential rates  $\alpha$  and  $\beta$ , with both less than  $r$ . Substituting and simplifying, we find:

$$(A.5) \quad AB^\infty = e^{-rT} F(T) + e^{-\alpha T} I(T) \int_T^\infty e^{(\alpha-r)t} dt - e^{-\beta T} C(T) \int_T^\infty e^{(\beta-r)t} dt$$

$$(A.6) \quad AB^\infty = e^{-rT} \left[ F(T) + \frac{I(T)}{r-\alpha} - \frac{C(T)}{r-\beta} \right]$$

Relative to the present value of payroll, this is:

$$(A.7) \quad \delta^{AB(\infty)} = - \frac{F(T) + \frac{I(T)}{r-\alpha} - \frac{C(T)}{r-\beta}}{e^{rT} PV(W, T) + \frac{W(T)}{r-\alpha}} \quad (0.1)$$

Note that the  $AB$  and  $\delta$  are defined relative to a baseline policy, here taken to be continuation of current policy, and  $F$ ,  $I$ ,  $C$  and  $W$  all likewise refer to this baseline policy.

2. Stable Assumption: Both  $I(S)$  and  $C(S)$  grow at the constant exponential rate  $\beta$  less than  $r$ . In this case, the expressions derived above become:

$$(A.8) \quad AB^\infty = e^{-rT} \left[ F(T) + \frac{I(T) - C(T)}{r-\beta} \right]$$

$$(A.9) \quad \delta^{AB(\infty)} = - \frac{F(T) + \frac{I(T) - C(T)}{r-\beta}}{e^{rT} PV(W, T) + \frac{W(T)}{r-\beta}}$$

### **3. The Flat Fund Ratio Criterion**

Consider the stable infinite horizon case. Assuming balance, so that  $AB^\infty=0$ , we have:

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<sup>13</sup> There are complexities related to tax income derived from taxation of benefits rather than taxation of payroll. We assume that income from the taxation of benefits in year  $s>T$ ,  $\theta(s)$ , represented as a fraction of expenditures, remains constant. When growth rates of payroll and expenditures differ,  $\theta(s)$ , expressed in terms of an additional tax rate levied to payroll, will also grow at a constant rate of  $\beta-\alpha$ . While incorporated in actual calculations, the representation of  $\theta(s)$  was not included in this appendix to simplify the mathematical presentation.

$$(A.10) \quad F(T) = \frac{I(T) - C(T)}{(\beta - r)}$$

The trust fund evolves according to

$$(A.11) \quad dF/dt = rF + I(t) - C(t).$$

Combining these we get:

$$(A.12) \quad dF(T)/dt = \beta F(T)$$

Since, by assumption, C is also growing at the exponential rate  $\beta$ , the Fund Ratio will be constant or flat after T, including the case where  $F(T)=0$ . This establishes that the Flat Fund Ratio after T is a necessary condition for balance over an infinite horizon, provided that the system is stable after T.

So a sustainable policy that achieves steady state will have the fund growing at the same rate as costs, and therefore the Flat Fund Ratio criterion will be satisfied after T.

Now consider a policy defined up to T, but not thereafter. Assuming that costs and payroll are both growing at the same constant rate  $\beta$ , we can find the constant tax rate  $\tau^*$  to apply after T to achieve sustainability. The condition for sustainability in steady state (at time T) is:

$$(A.13) \quad F(T)(\beta - r) + C(T) = I(T)$$

Divide both sides by payroll in year T,  $W(T)$ , noting that  $I/W$  is the desired tax rate  $\tau^*$ .

$$(A.14) \quad \tau^* = \frac{(\beta - r)F(T) + C(T)}{W(T)}$$

## **Appendix B. How the Social Security Actuaries Have Measured Imbalance**

Goss (1999) provides an excellent summary and discussion of the evolution of the methods used to assess the solvency of the Social Security program. Prior to 1965, the solvency of the system was evaluated over an infinite horizon, or “in perpetuity”. The methods used are described in Meyers (1959). Projections at that time were based on the assumption of constant age-specific earnings and benefits. The 1958 projections were carried out through 2050, after which it was assumed that the costs would be constant. This assumption permitted the calculation of the infinite horizon tax rate for a system in steady state. After 1965, the infinite horizon was replaced by a 75-year horizon on the recommendation of the Advisory Council. According to Goss (1999:19) this had a relatively small effect on the long-run cost projections at that time, because costs were projected to remain flat in any case, rather than rising exponentially as they do now.

Starting in 1973, the projections began to assume a changing time path for earnings and benefits, since new legislation linked benefits to past earnings. Currently, a variety of measures is employed. While the date of fund exhaustion projected under intermediate assumptions and the 75-year Actuarial Balance are best known and most influential, many other figures for the short, medium and long run are calculated and presented. However, no current measure is similar to the former infinite-horizon constant tax rate

necessary to meet projected obligations. While discontinuation of the practice in 1965 apparently had little effect on the estimated solvency of the system at the time at the time, that would not be true today.

## **Appendix C. Relation of These Methods to Generational Accounting**

Our approach shares many features of Generational Accounting (Auerbach et al, 1991; Auerbach et al, 1999). Like Generational Accounting, we start with the premise that the budget must be balanced over an infinite horizon, and consider the kinds of policy changes that would be required to achieve this balance. However, our approach also differs in a number of ways. Generational Accounting is based on generations, while we focus on periods. Generational Accounting assesses the sustainability of current policy if it were constant for all time. If it is not sustainable, then a new constant policy applied to all future generations is calculated and used to assess the extent of generational imbalance. For us, a policy is a time path of taxes and benefits stretching into the future, and a constant policy is a special case. Generational Accounting is applied in a comprehensive way to government budgets, whereas we only examine Social Security finances. This limited focus on Social Security can be misleading, since other government programs might have offsetting effects. These differences arise from the practicalities of implementation rather than from important conceptual differences. Also, because there is a long tradition of actuarial projection and accounting over a long time horizon for Social Security, but not for government budgets in general, our task is easier.

Table 1. Estimates of Various Measures of Imbalance under the SSA(2000) Intermediate and Lee-Tuljapurkar Projections

Measure of Imbalance as Proportion of Present Value of Payroll	Imbalance as % PV of Payroll by Mort Assump		Imbalance in Trillions '99 \$	
	SSA	L-T	SSA	L-T
75-year Actuarial Balance (AB75)	1.9	2.4	3.08	3.94
Flat Fund Ratio	3.1	4.2	6.90	9.65
Stable Infinite Horizon	3.1	4.2	6.90	9.65
Unstable Infinite Horizon	3.7	5.7	8.37	12.81
300-Year Projection	3.5	4.6	7.76	10.11

Note: The column labeled SSA is based on data in the Trustees Report (2000). The column labeled L-T is based on median values from the Lee-Tuljapurkar stochastic projection for Social Security finances, with the long run means of key input variables constrained to equal the Intermediate assumptions of the Trustees Report (2000) except for mortality, which follows the Lee-Carter style mortality projections, leading to a median life expectancy of 86 for 2075. AB75 is measured relative to PV of payroll over 75-year horizon, and others are measured relative to an infinite or 300-year horizon. See text for definitions of measures.

Table 2. Future Implications of Various Current Policies: Fund Balance in 2075 and Sustainable Tax Rate Thereafter (Evaluation Based on SSA Mortality Decline)

<b>Tax Policy</b>	<b>Percent of Payroll Tax up to 2075</b>	<b>Fund Bal in 2075 (trillions 1999 \$)</b>	<b>Percent of Sustainable Payroll Tax After 2075</b>	<b>Percent of Required Payroll Tax Increase in 2075</b>
<b>Current Law</b>	12.4	-28.33	22.9	10.5
<b>Raise by AB75</b>	14.3	+1.45	18.4	4.1
<b>Raise by FFRT/Stable <math>\infty</math></b>	15.5	+20.62	15.5	0
<b>Linear increase to sustainable level in 2075</b>	12.4 rises to 17.7	+6.04	17.7	0
<b>Pure PAYGO</b>	12.4 rises to 18.6	0	18.6	0

Note: Assumed values for input variables:  $r=0.03$ ,  $\beta = .0145$ ,  $W(2075) = 10.17$  Trillion,  $Cost\ rate(2075) = 19.53\%$ . The figures given for payroll tax rates do not include the additional revenue from taxation of benefits, which amounts to .25% of payroll in 2000, and .94% by 2075.

Table 3. Future Implications of Various Current Policies: Fund Balance in 2075 and Sustainable Tax Rate Thereafter (Evaluation Based on Lee-Carter Mortality Decline)

<b>Tax Policy</b>	<b>Percent of Payroll Tax up to 2075</b>	<b>Fund Bal in 2075 (trillions 1999 \$)</b>	<b>Percent of Sustainable Payroll Tax After 2075</b>	<b>Percent of Required Payroll Tax Increase in 2075</b>
<b>Current Law</b>	12.4	-36.30	25.2	12.8
<b>Raise by AB75</b>	14.8	+1.53	20.3	5.4
<b>Raise to FFRT/Stable <math>\infty</math></b>	16.6	+29.16	16.6	0
<b>Linear increase to sustainable level in 2075</b>	12.4 rises to 19.3	+7.34	19.3	0
<b>Pure PAYGO</b>	12.4 rises to 20.5	0	20.5	0

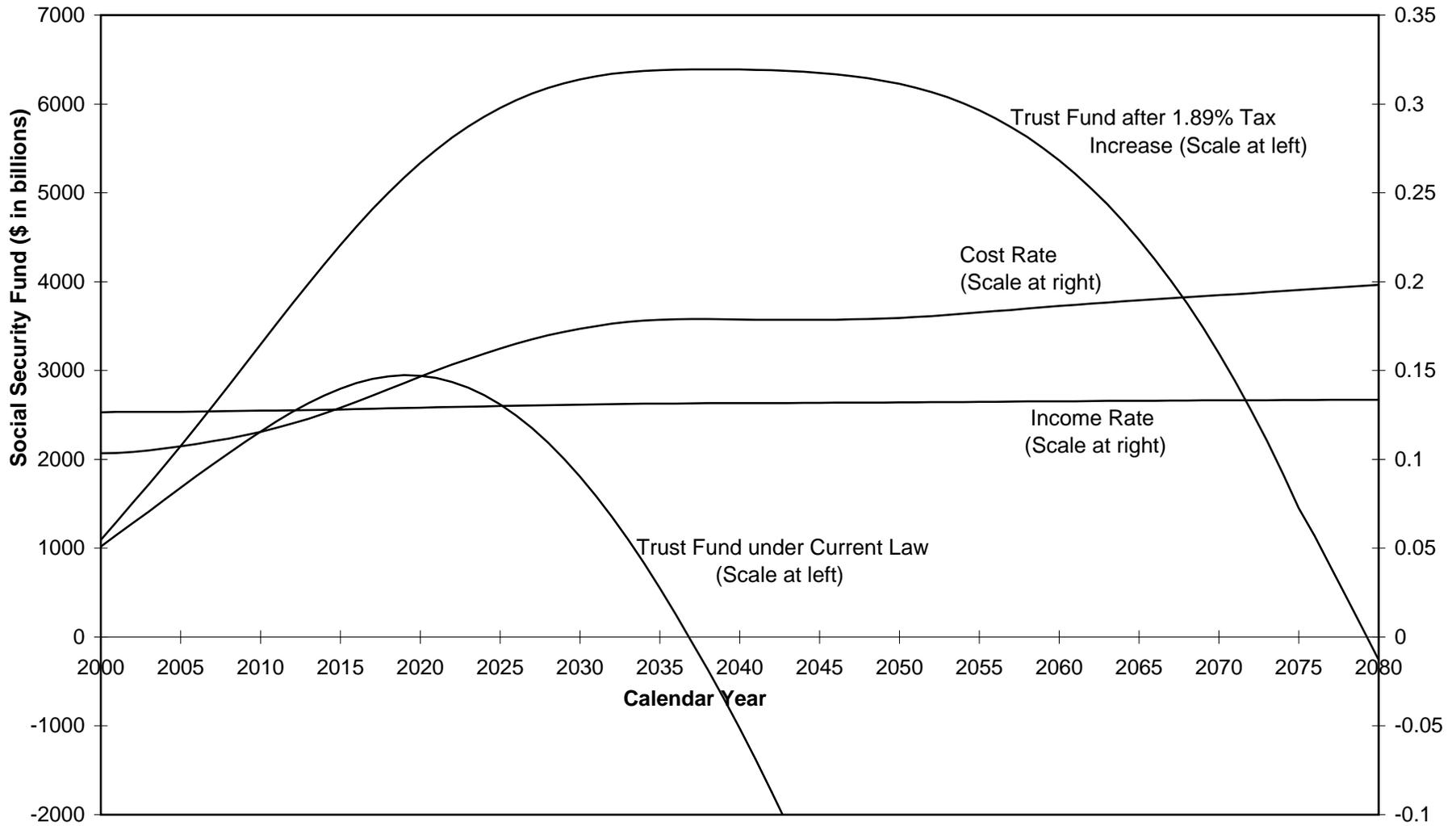
Note: Assumed values for input variables:  $r=0.03$ ,  $\beta = .0169$ ,  $W(2075) = \$10.145$  trillion, Cost rate (2075) = 21.39%. The figures given for payroll tax rates do not include the additional revenue from taxation of benefits, which amounts to .25% of payroll in 2000, and .94% by 2075.

Table 4. Probability (%) Under Different Tax Policies That Social Security is Fixed in 2075 (i.e. that Actuarial Balance is near 0 in 2075)

<i>Tax Policy (size of tax increase)</i>	<i>Percent Probability that Actuarial Balance over Infinite Horizon Falls in Indicated Range in 2075</i>				
	$AB^\infty < 0$	$AB^\infty < -.02 * PV(W)$	$AB^\infty < -.05 * PV(W)$	$AB^\infty > .02 * PV(W)$	$AB^\infty$ within $\pm .02 * PV(W)$
Current (0%)	99.7	97.5	89.6	0	2.5
AB75 (2.4%)	94.0	80.3	50.9	0.4	19.3
FFRT (4.2%)	52.8	10.3	0.1	7.0	82.7

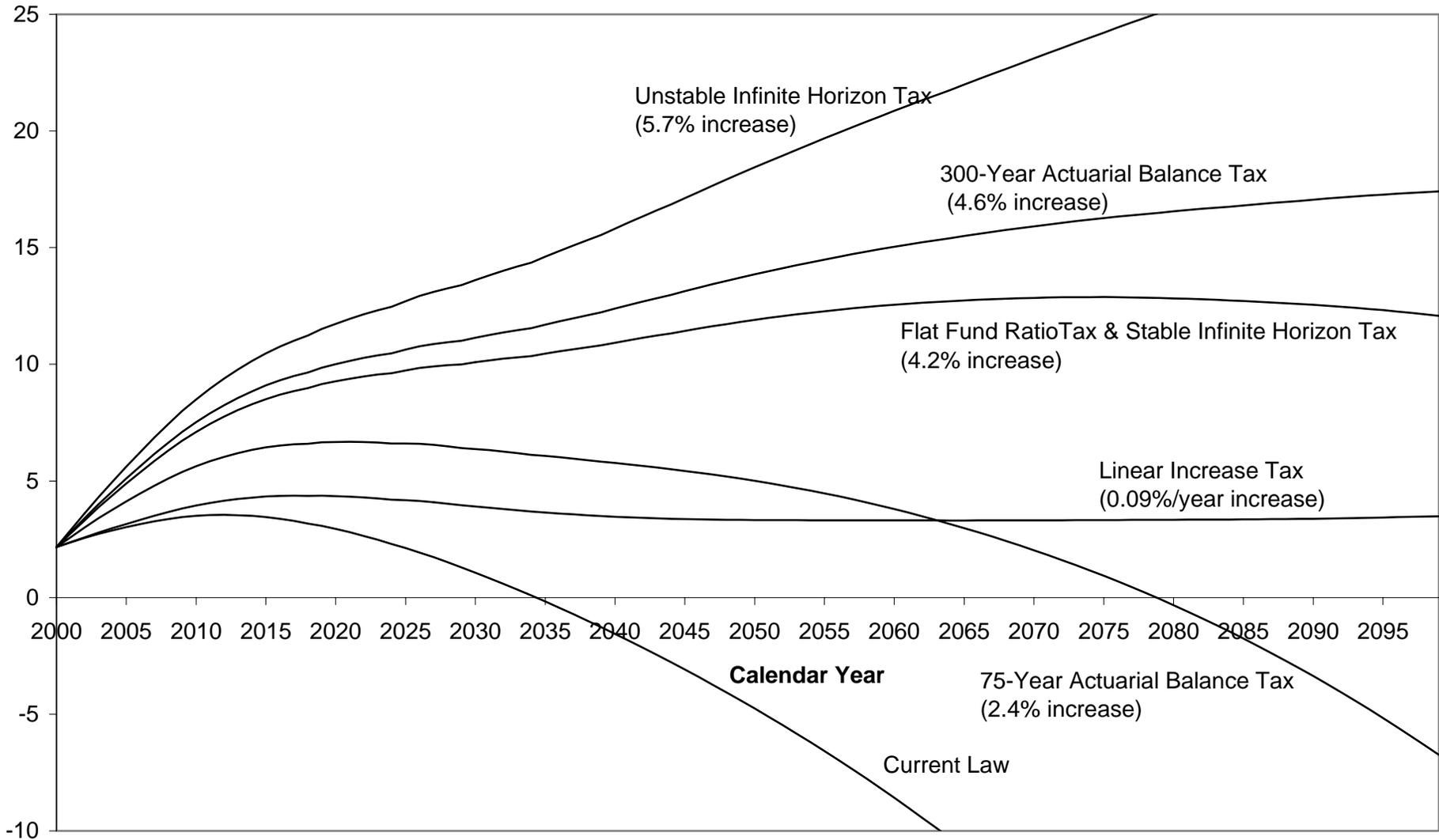
Note. The Actuarial Balance in 2075 is calculated for the stable case, continuing the increased tax rates forever, and assuming that the rate of increase in costs observed between 2070 and 2075 continues forever. PV(W) is the present value of future payroll evaluated in 2075. Calculations include revenues from taxes on benefits in proportion observed in 2075.

**Figure 1. Social Security Fund Projections Under Current Law and After a 1.89% Payroll Tax Increase, 2000-2080**



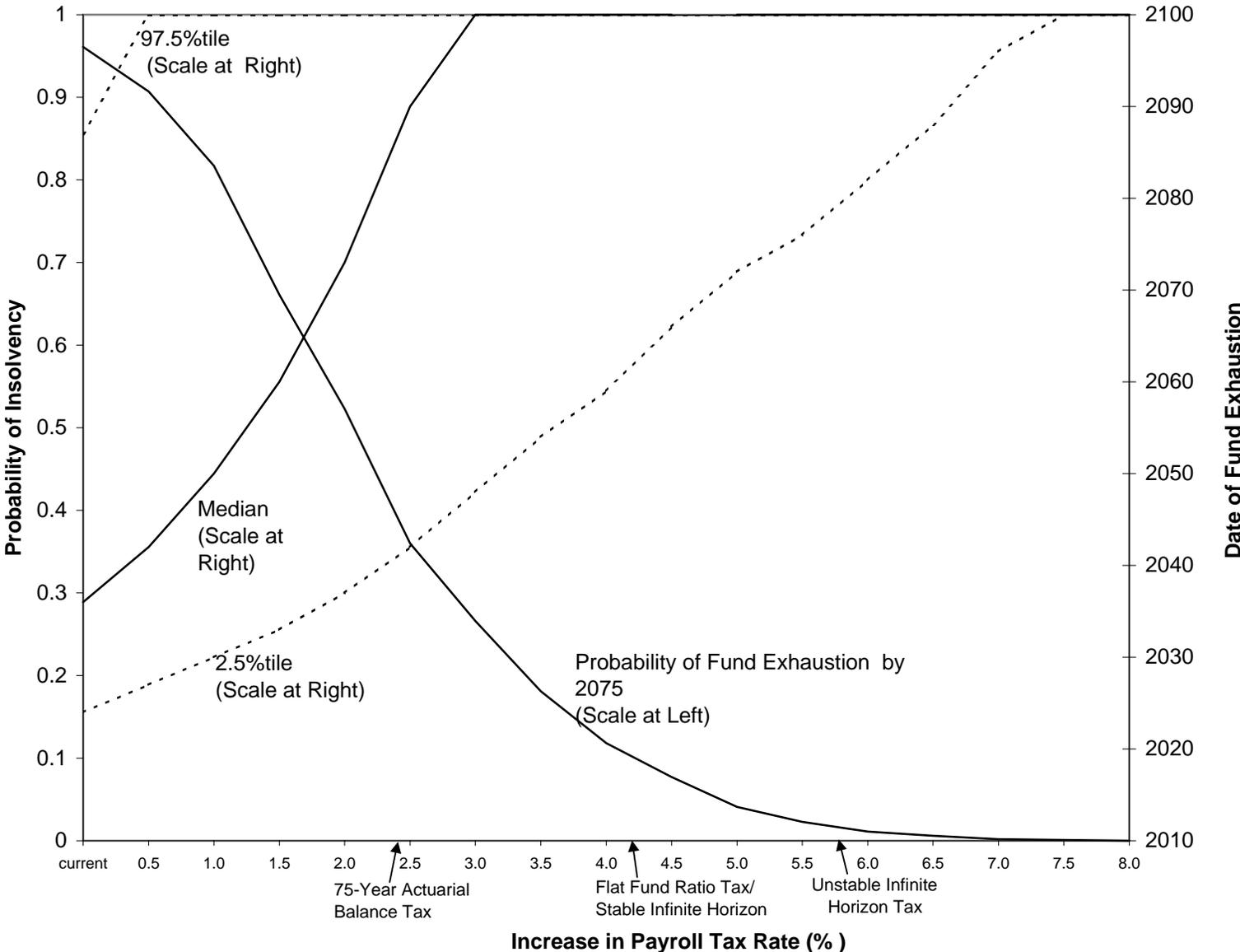
Note: Trust Fund Projection: For 2000 - 75, authors' calculation based on the information from the 2000 Trustees' Report (see footnote 4 in the text or detail). For 2076-80, the figures are based on linear extrapolation of those between 2070 and 2075. Income and Cost Rates are under current law, and are taken from Table III.A.2: Table III.A.2, 2000 Trustees Report.

**Figure 2. Trust Fund/Cost Ratios For Different Tax Rate Regimes,  
Based On Lee-Tuljapurkar Projections With More Rapid Mortality Decline, 2000-2099**



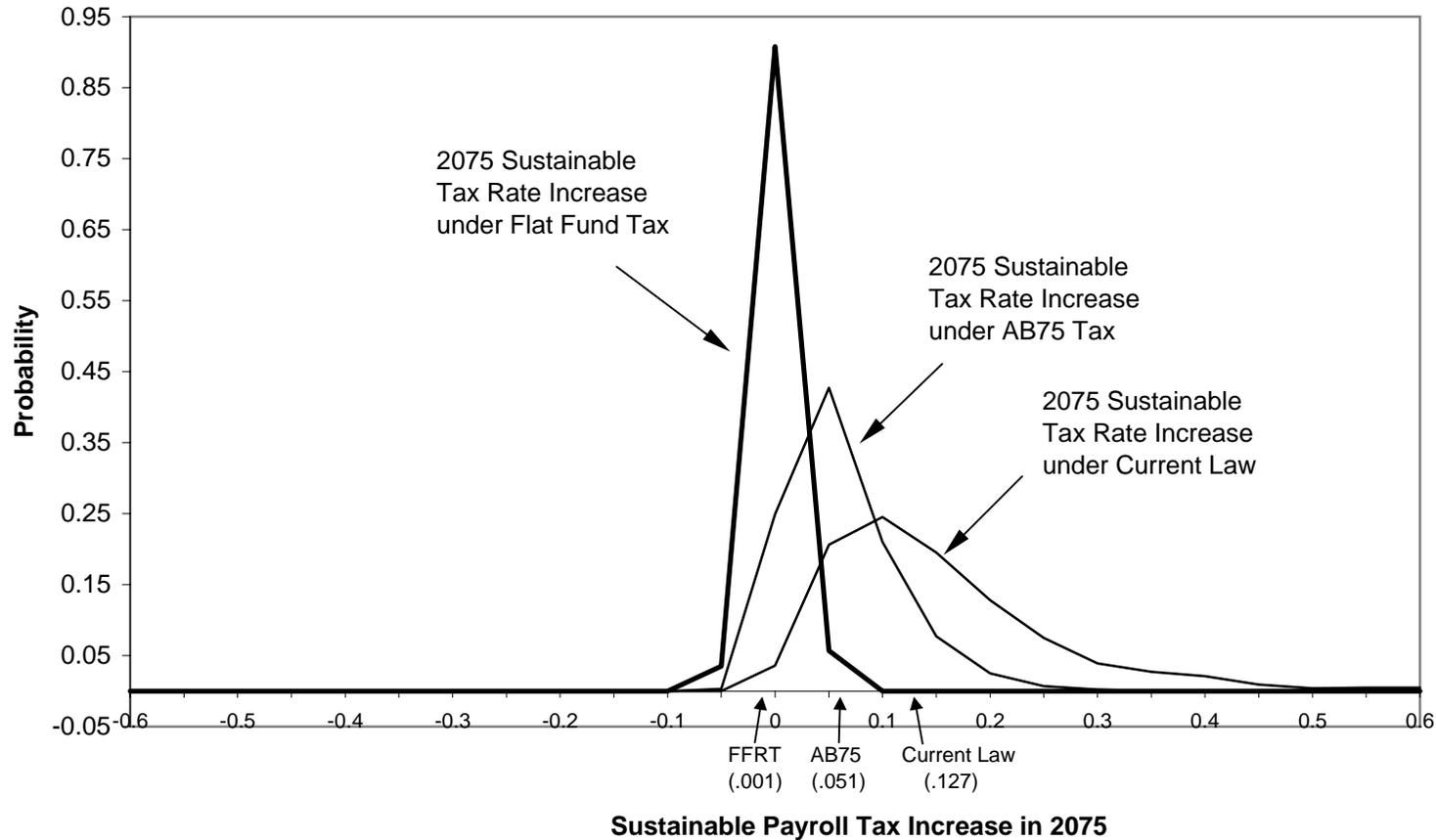
Note: Fund/Cost Ratios are based on the authors' calculation using the deterministic Lee-Tuljapurkar projection (with SSA's Intermediate Assumptions and Lee-Carter mortality decline) on payroll tax contributions, expenditures, balance and miscellaneous income as the base and applying appropriate tax rate increases to calculate increased tax contributions. Fund/Cost ratio, or trust fund ratio, is the ratio of the assets at the beginning of the year to the expenditures during that year (Trustees' Report 2000, page 222). See text for definition of the tax rate regimes

**Figure 3. Probability of Fund Exhaustion by 2075 and Probability Distribution of Dates of Exhaustion for Different Tax Rate Increases (under Lee-Tuljapurkar Forecasts)**



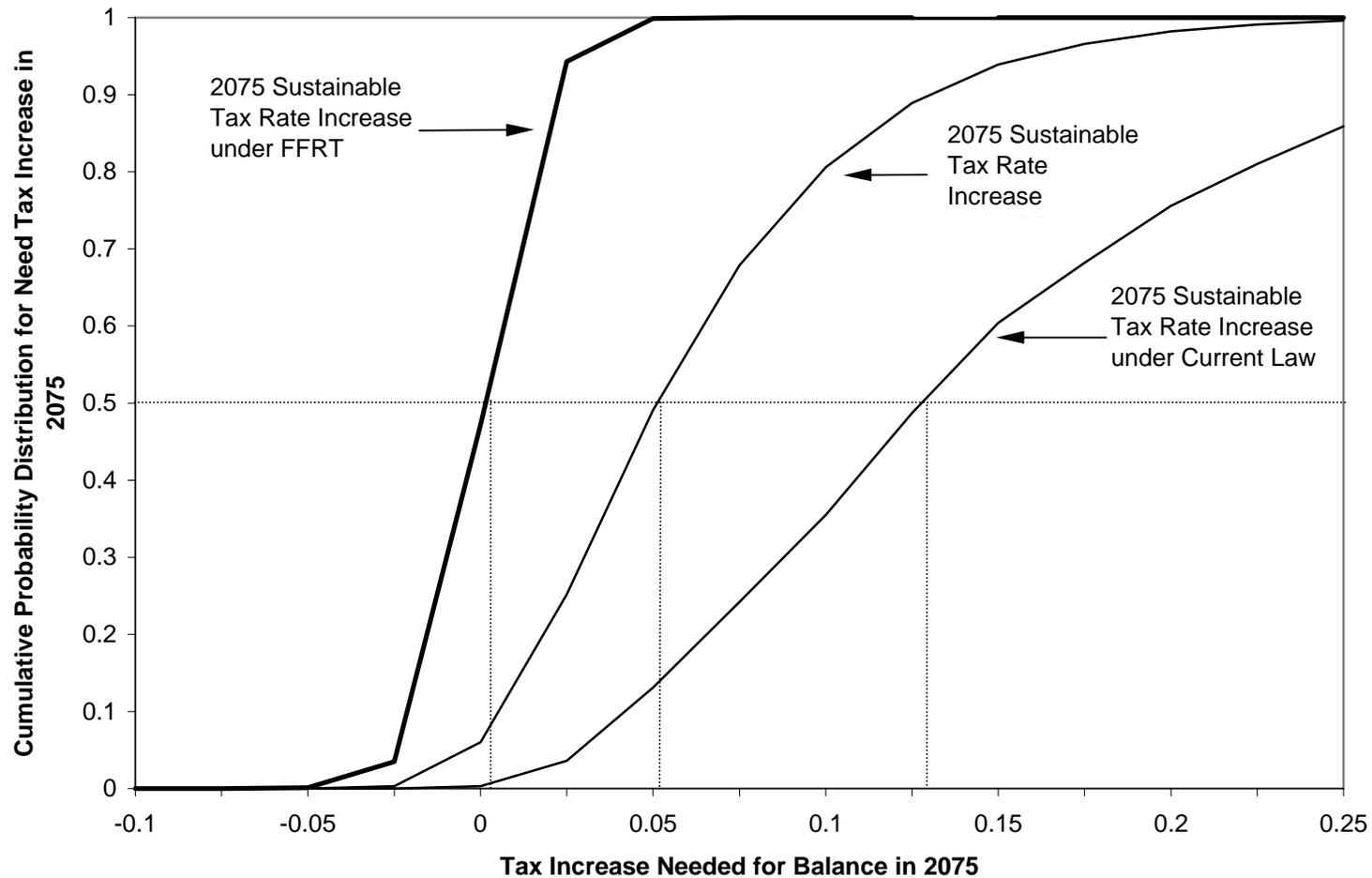
Note: Authors' calculation based on 1,000 stochastic simulations with the SSA's Intermediate Assumptions and Lee-Carter mortality decline at different payroll tax rate increases.

**Figure 4. Probability Distribution of Sustainable Payroll Tax Increase In 2075 under Different Tax Regimes Up To 2075 (Lee-Tuljapurkar Projections with More Rapid Mortality Decline and Fixed Interest Rate at 3%)**



Note: Authors' calculation based on 1,000 stochastic simulations with the SSA's Intermediate Assumptions and Lee-Carter mortality decline. For the calculation of tax rate increase in 2000, a 5-year annual average growth rate for expenditures (2070-75) and a fixed interest rate at 3.0% were used. Additionally, the median tax rate increase necessary to meet the criterion across all simulated paths (AB75: .0218 and FFRT: .0405) was used to calculate the sustainable tax rate increase in 2075.

**Figure 5. Cumulative Probability Distribution of Trust Fund Sustainability In 2075 under Different Tax Regimes Up To 2075 (Lee-Tuljapurkar Projections with More Rapid Mortality Decline and Fixed Interest Rate at 3%)**



Note: Authors' calculation based on 1,000 stochastic simulations with the SSA's Intermediate Assumptions and Lee-Carter mortality decline. For the calculation of tax rate increase in 2000 and 2075, see note on Figure 6.