

Translation Formulae for Non-repeatable Events

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I. THE PROBLEM

In Sweden period total fertility has risen continuously since the mid-1980s. Although similar trends can be found in other West European countries, the case of Sweden is remarkable: total fertility is relatively high (2.1 in 1990), and the increase has been fairly steep (in 1983 total fertility was 1.6). As Hoem has stated: '[the] rise in period fertility reflects a change in the time pattern of cohort fertility. Ultimate cohort fertility may eventually also rise as a result of this change....'¹ He concluded that recent trends in Sweden were to a large extent due to initial postponement of childbearing, followed by a period of 'making up', and he has analysed birth rates for the first three parities in some detail, controlling for age at previous birth.

Rodgers and Thornton studied U.S. birth cohorts between 1880 and 1965, and looked at the implications of period effects in age-specific marriage rates on the proportion ever-married. They found that peaks and troughs in these rates were barely reflected in the proportions of the cohort who ever-married.²

A common feature of these papers is that the authors investigated empirically summary period and/or cohort indices for variables that described non-repeatable events: parity-specific births and first marriages. An analytical treatment of the reciprocal relations between cohort and period indices, known as translation methods, was provided by Ryder some 30 years ago. His work has shed considerable light on the relationship between quantum and tempo for both cohort and period studies.³ These methods make it possible to isolate the impact on period quantum of changes in cohort tempo and changes in cohort quantum. One of Ryder's results (applied to fertility) is that when cohort timing is constant, total period fertility can be approximated by a linear function of completed cohort fertility.

Translation methods cannot be applied to study parity-specific fertility, mortality, or first marriage rates, because these methods were devised to investigate repeatable events, such as age-specific fertility irrespective of parity. The salient feature of rates relating to repeatable events is that they can be simply totalled (both when cohorts and periods

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¹ J. M. Hoem, 'Social policy and recent fertility change in Sweden', *Population and Development Review*, 16 (1990), pp. 735–748.

² W. L. Rodgers and A. Thornton, 'Changing patterns of first marriage in the United States', *Demography*, 22 (1984), pp. 265–279.

³ N. B. Ryder, 'The process of demographic translation', *Demography*, 1 (1964), pp. 74–82; 'Components of temporal variations in American fertility', in R. W. Hiorns (ed.), *Demographic Patterns in Developed Societies* (London: Taylor & Francis, 1980), pp. 15–54. See also G. Calot, 'Relations entre indicateurs démographiques longitudinaux et transversaux', *Population*, 47 (1992), pp. 1189–1240, for time-continuous expressions. We use the term 'quantum' to denote the proportion of individuals in a real or synthetic cohort who ever experience a vital event, and the term 'tempo' for the timing of this event during the life course.

are studied) to yield indices of quantum (completed cohort fertility, or total fertility, for example). Ryder was able to derive his translation formulae from this simple additive relationship. But age-specific rates for non-repeatable events cannot simply be summed in the same way to derive measures of quantum or tempo. Thus, when $m(x)$ stands for the first marriage rate at age x , and $n(x) = 2m(x)/(2 + m(x))$ is the corresponding one-year probability, the quantum of first marriages, i.e. the proportion ever-married in the absence of mortality and migration is found as

$$1 - \prod_x [1 - n(x)].$$

This multiplicative relationship between age-specific rates and the quantum index is characteristic of non-repeatable events. Translation formulae for such events are not known and it is the purpose of the present paper to derive them.⁴

II. QUANTUM AND TEMPO INDICES IN A LIFE TABLE

We begin by introducing some basic notions for the life table. The life table we use in this paper describes a situation in which individuals aged x may be in only one of two states: (1) not yet having experienced an event or (2) having experienced the event. The 'event' is therefore a transition from State 1 to State 2. Movement in the opposite direction is not permitted: State (2) is absorbing.

Let $l(x)$ [$x = 0, 1, 2, \dots, \omega$] represent the number of persons in a life table who have not yet experienced the event by age x . The radix $l(0)$ may be any convenient number. In the case of mortality $l(\omega) = 0$, for other types of events $l(\omega)$ is generally positive. Let $m(x)$ [$x = 0, 1, 2, \dots, \omega - 1$] be a series of occurrence-exposure (OE) rates, defined for the age group $(x, x + 1)$, for instance, first marriage rates, first birth rates, or death rates. Assume that the process under investigation is one for which the intensity is constant in the age interval. Thus, for the entire age range $(0, \omega)$, the intensities will be piece-wise constant, each being equal to the OE rate for the corresponding age group.

With these assumptions

$$l(x + 1) = l(x) \exp[-m(x)], \quad x = 0, 1, 2, \dots, \omega - 1, \quad (1)$$

so that

$$l(x) = l(0) \exp\left\{-\sum_0^{x-1} m(y)\right\}, \quad x = 1, 2, \dots, \omega. \quad (2)$$

⁴ Ryder has derived expressions for the impact that births of different orders may have on the quantum and tempo of period fertility (Ryder 1980), *loc. cit.* in fn. 3, pp. 47-52). However, the age and parity specific birth rates he used are calculated with denominators in which women are classified by age only, rather than by age and parity. Brass's approach is subject to essentially the same defect (see W. Brass, 'Cohort and time period measure of quantum fertility: Concepts and methodology', in H. A. Becker (ed.), *Life Histories and Generations*, Vol. 2, Utrecht, ISOR, 1990, p. 475). The rates that these authors have used are not proper occurrence-exposure rates (see R. Pressat, *L'Analyse démographique: Concepts, Méthodes, Résultats*, 4th edn, Paris, Presses Universitaires de France 1983, p. 87; A. J. Coale and D. R. McNeil, 'The distribution by age of the frequency of first marriage in a female cohort', *Journal of the American Statistical Association*, 67 (1972), pp. 743-749). To use such indices for translation may bias the results, e.g. the problems that arise when using the period total first marriage index (the sum of age-specific first-marriage rates) as an index of period quantum. See G. J. Wunsch and M. G. Termote, *Introduction to Demographic Analysis: Principles and Methods* (New York & London, Plenum Press, 1978), p. 55. In general, rates that relate the event in question (first birth, first marriage) to all individuals are easily distorted by shifts in the composition (by parity or marital status) of the individuals in the denominator. Cf. G. Feeney and W. Lutz, 'Distributional analysis of period fertility', in W. Lutz (ed.), *Future Demographic Trends in Europe and North America: What can we Assume Today?* (London, Academic Press, 1991), p. 193.

The proportion of the original number, $l(0)$, in the life table who have experienced the event by age ω is called the *quantum* of the process and written Q . As $Q = [l(0) - l(\omega)]/l(0)$ we see from Equation (2) that

$$Q = 1 - \exp \left\{ - \sum_0^{\omega-1} m(x) \right\}. \quad (3)$$

The number of individuals who experience the event during the interval $(x, x+1)$ is written $d(x)$, and as there is only a single event:

$$d(x) = l(x) - l(x+1). \quad (4)$$

The mean age at experiencing the event is written and defined as:

$$\mu = \sum x d(x) / \sum d(x). \quad (5)$$

The numerator of this fraction is:

$$1[l(1) - l(2)] + 2[l(2) - l(3)] + \dots + (\omega - 1)[l(\omega - 1) - l(\omega)], \quad (6)$$

which is equal to $\sum_{x=1}^{\omega-1} l(x) - (\omega - 1)l(\omega)$. The denominator of (5) may be written as $l(0) - l(\omega)$, or equivalently as $Ql(0)$. From (1) we find the mean age as

$$\mu = \frac{\sum_{x=1}^{\omega-1} \exp \left\{ - \sum_{i=0}^{x-1} m(i) \right\} - (\omega - 1)(1 - Q)}{Q}. \quad (7)$$

The mean age μ is one possible index of the *tempo* of the process; it indicates the average age at which individuals experience the event. A second possible index of tempo is the average age of the schedule of OE rates (denoted by \bar{x}).

$$\bar{x} = \sum_0^{\omega-1} x m(x) / \sum_0^{\omega-1} m(x) \quad (8)$$

For non-repeatable events $\bar{x} \geq \mu$, and in practice the strict inequality holds. The reason is that the number of survivors, $l(x)$, which is needed to compute the numerator of μ declines with age; but when \bar{x} is calculated its numerator will *increase* with age, relatively to the denominator. For repeatable events $\bar{x} = \mu$.

III. COHORT INDICES AS A FUNCTION OF PERIOD INDICES

Consider a set of age-specific rates given for a number of years. Changes in the rates over time will be reflected in the indices of both quantum and tempo. The problem is how to translate the time-dependent indices for the quantum and tempo of the process obtained from a period perspective into quantum and tempo indices for cohorts. This is the opposite of the path that is generally taken in translation studies. For instance, Ryder's formulae⁵ show the implications of changing cohort patterns for period quantum and tempo indices. But our formulation of the problem: ('what can we learn about cohort indices from observations of period indices?') would then be approached inductively, whereas we advocate a deductive approach. Given the observed period patterns, together with a number of assumptions which relate to their development in the long run

⁵ Ryder (1964), *loc. cit.* in fn. 3.

(constancy, linear falls or rises, etc.) the cohort patterns can be inferred unambiguously. The usual inductive approach would leave us with much greater uncertainty. Ryder's formulae imply that when cohort indices change in accordance with pattern A, then period indices will change necessarily in accordance with pattern B. In our case, when we observe pattern B, it is not unlikely that cohort indices have, indeed, changed in accordance with pattern A. But pattern B could also have been produced by a different cohort pattern, C. Our deductive approach will yield much firmer conclusions, but there is a price to be paid: the behaviour of period indices, and particularly of quantum indices, is much more irregular than that of corresponding cohort indices. This implies that the models we use when we follow a deductive procedure are more complex than when the procedure is inductive. For example, it is not unlikely that the proportion ever-married in different birth cohorts (the cohort quantum of first marriage) will decline linearly with time over a longish period. However, because of shifts in the age at first marriage in successive cohorts, the period quantum index (i.e. the proportion ever-married in the period life table) may not be capable of being approximated by a straight line, so that a polynomial of second or higher degree would be needed to describe it.

It should be noted that the analysis in this paper may also be carried out by a more traditional inductive approach. We argue in Section V that the change in perspective only requires that some plus signs in our basic Equation (13) be changed into minus signs.⁶

Let $m(t, x) \geq 0$ be an OE rate which depends not only on age x , but also on time, t . All the expressions for quantum and tempo indices derived in Section II now become time-dependent. We write $Q_p(t)$ and $\mu_p(t)$ for time-dependent quantum and tempo indices derived from a period life table (aggregation over x , with t fixed), and $Q_c(g)$ and $\mu_c(g)$ for time-dependent quantum and tempo indices derived from a cohort life table (aggregation over x , with fixed $g = t - x$ for cohort g). We write

$$\sum_{x=0}^{\omega-1} m(t, x) = A(t)$$

for the period sum of rates and

$$\sum_{x=0}^{\omega-1} m(g+x, x) = B(g)$$

for the cohort sum of rates.

The definitions of $A(t)$ and $B(g)$ imply that the period and cohort quantum may be written respectively as

$$Q_p(t) = 1 - \exp[-A(t)] \quad \text{and} \quad Q_c(g) = 1 - \exp[-B(g)].$$

When the sum of rates is small, it is approximately equal to the quantum index.⁷ In this case, translation formulae for non-repeatable events are identical with those for repeatable events, as we show in Section III.1 (Eqn (12)). However, quantum indices for

⁶ The fact that we look for cohort patterns to be inferred from period data does not imply that we are of the opinion that period analyses are less valuable than cohort analyses. Both have their merits. In this paper we are primarily interested in the *arithmetical* relationships between quantum and tempo in cohorts and periods. For *substantive* views on the relative values of period and cohort perspectives, see J. Hobcraft, 'Data needs for fertility analyses in the 1990s', in A. Blum and J.-L. Rallu (eds.), *European Population*. Vol. 2. *Demographic Dynamics* (Paris, John Libbey Eurotext), pp. 447-460, and M. Ní Bhrolcháin, 'Period paramount? A critique of the cohort approach to fertility', *Population and Development Review*, 18 (1992), pp. 599-629.

⁷ See Calot, *loc. cit.* in fn. 3, p. 1224.

most non-repeatable events lie between 0.50 and 1.00 (for instance, first and second births, and first marriages). For such events it would not be appropriate to approximate the quantum index by the sum of rates. Since $A(t) = -\log(1 - Q_p(t))$, we find for the first two derivatives of the sum of period rates with respect to time:

$$A'(t) = Q'_p(t)/[1 - Q_p(t)], \quad (9a)$$

and

$$A''(t) = Q''_p(t)/[1 - Q_p(t)] + [Q'_p(t)/(1 - Q_p(t))]^2. \quad (9b)$$

We introduce the age-specific proportions $a(t, x)$ and $b(g, x)$ of the sum of period rates $A(t)$ and of cohort rates $B(g)$ by:

$$m(t, x) = A(t)a(t, x) = B(g)b(g, x), \quad g = t - x \quad (10)$$

and

$$\sum_{x=0}^{\omega-1} a(t, x) = \sum_{x=0}^{\omega-1} b(g, x) = 1.$$

We thus obtain the fundamental identity for the sum of cohort rates:

$$B(g) = \sum_{x=0}^{\omega-1} m(g+x, x) = \sum_{x=0}^{\omega-1} A(g+x)a(g+x, x).$$

This expression states that the sum of cohort rates $B(g)$ is a function of the sums of period rates $A(t)$, and age-specific proportions $a(t, x)$ for all possible ages (and periods) that individuals who belong to cohort g pass through as they age. The formulae will serve as a base for further analyses. The idea is to expand both $(A(g+x) = A(t)$ and $a(g+x, x) = a(t, x)$ in a Taylor series about $t = g$, which will result in an expression for $B(g)$ as a function of $A(g)$ and $a(g, x)$ and their derivatives with respect to time (Section III.1). Next, we shall analyse some special cases on the assumption that the period quantum $Q_p(t)$ and the age-specific proportions $a(t, x)$ are constant, or change linearly with time (Sections III.2 and III.3). In Section III.4 we expand $m(g+x, x)$ in a Taylor series in order to find expressions for the cohort tempo.

To expand OE rates in a Taylor series implicitly assumes that the rates may be written as a polynomial function of time, with one polynomial for each age.⁸

III.1. The general case

Taylor series approximations for $A(g+x)$ and $a(g+x, x)$ yield

$$\begin{aligned} A(g+x) &= A(g) + xA'(g) + \frac{1}{2}x^2A''(g) + \dots, \\ a(g+x, x) &= a(g, x) + xa'(g, x) + \frac{1}{2}x^2a''(g, x) + \dots \end{aligned}$$

where derivatives of $A(t)$ and $a(t, x)$ are taken with respect to time. $A(g)$ and $A(g+x)$ represent the values of $A(t)$ at time $t = g$ and $t = g+x$ respectively, and similarly for $a(g, x)$ and $a(g+x, x)$, and the derivatives of both $A(t)$ and $a(t, x)$. The latter formulae imply that $B(g)$ may be written as

$$\begin{aligned} B(g) &= \Sigma\{A(g) + xA'(g) + \frac{1}{2}x^2A''(g) + \dots\}\{a(g, x) + xa'(g, x) + \frac{1}{2}x^2a''(g, x) + \dots\} \\ &= A(g)\{1 + \Sigma xa'(g, x) + \Sigma \frac{1}{2}x^2a''(g, x) + \dots\} \\ &\quad + A'(g)\{\Sigma xa(g, x) + \Sigma x^2a'(g, x) + \Sigma \frac{1}{2}x^3a''(g, x) + \dots\} \\ &\quad + A''(g)\{\Sigma \frac{1}{2}x^2a(g, x) + \Sigma \frac{1}{2}x^3a'(g, x) + \Sigma \frac{1}{4}x^4a''(g, x) + \dots\} + \dots \end{aligned} \quad (11)$$

⁸ Ryder, *loc. cit.* (1964) in fn. 3, p. 75.

Next, define the k 'th moment of the distribution $a(t, x)$ as $M_k(t) = \Sigma x^k a(t, x)$, and the i 'th derivative (with respect to time) of the k 'th moment as $M_k^{(i)}(t) = \Sigma x^k a^{(i)}(t, x)$, so that the first moment $M_1(t)$ will be equal to the mean age of the distribution (\bar{x}) defined in (8). Substitution in (11) yields:

$$B(g) = A(g) \{1 + M_1'(g) + \frac{1}{2}M_2''(g) + \dots\} + A'(g) \{M_1(g) + M_2'(g) + \frac{1}{2}M_3''(g) + \dots\} \\ + A''(g) \{\frac{1}{2}M_2(g) + \frac{1}{2}M_3'(g) + \frac{1}{4}M_4''(g) + \dots\} + \dots \quad (12)$$

This is the general translation formula for the cohort quantum in the case of repeatable events, when $A(t)$ and $B(g)$ serve as indices for the period and cohort quantum respectively.

Substitution in Equation (12) of the sum of rates $A(t)$ and $B(g)$ and of the derivatives of $A(t)$ given in (9) results in:

$$-\log \{1 - Q_c(g)\} = -\{\log(1 - Q_p(g))\} \{1 + M_1'(g) + \frac{1}{2}M_2''(g) + \dots\} \\ + \frac{Q_p'(g)}{1 - Q_p(g)} \{M_1(g) + M_2'(g) + \frac{1}{2}M_3''(g) + \dots\} \\ + \left\{ \frac{Q_p''(g)}{1 - Q_p(g)} + \left(\frac{Q_p'(g)}{1 - Q_p(g)} \right)^2 \right\} \left\{ \frac{1}{2}M_2(g) + \frac{1}{2}M_3'(g) + \frac{1}{4}M_4''(g) + \dots \right\} + \dots$$

When we solve for the cohort quantum $Q_c(g)$ in the above expression, we finally obtain the following translation formula:

$$Q_c(g) = 1 - \{1 - Q_p(g)\}^{E_1(g)} \exp \{-E_2(g)\} \quad (13)$$

where we write in short $E_1(g) = 1 + M_1'(g) + \frac{1}{2}M_2''(g) + \dots$, and

$$E_2(g) = \frac{Q_p'(g)}{1 - Q_p(g)} \{M_1(g) + M_2'(g) + \frac{1}{2}M_3''(g) + \dots\} \\ + \left[\frac{Q_p''(g)}{1 - Q_p(g)} + \left\{ \frac{Q_p'(g)}{1 - Q_p(g)} \right\}^2 \right] \left\{ \frac{1}{2}M_2(g) + \frac{1}{2}M_3'(g) + \frac{1}{4}M_4''(g) + \dots \right\} + \dots$$

Calot has derived a special case of our more general expression (13) in which $E_1(g) = 1 + M_1'(g)$ and $E_2(g) = 0^9$. Thus, his formula is a first-order approximation to quantum (assuming $Q_p(t)$ is constant over time), and a second-order approximation to tempo (assuming that $M_k(t)$ varies linearly with time).

Equation (13) shows that there are two distinct factors that govern the relation between cohort quantum, and period quantum and tempo of a non-repeatable event. The first is the exponent $E_1(g)$, which shows the impact of changes in the period index's tempo. When period tempo is constant over time, all derivatives of the moments $M_k(t)$ are zero, which implies that $E_1(g) = 1$. When the period tempo changes linearly, all, but the first, derivatives vanish. Assume that $M_1'(g) > 0$, i.e. the mean age of the age-specific schedule of rates increases with time. *Ceteris paribus* (in particular the value of $E_2(g)$), this translates into an increase in the cohort quantum $Q_c(g)$, since $0 \leq Q_p(g) \leq 1$.

The second factor is a mixture of period quantum and period tempo. When period quantum is constant, its derivatives vanish, which implies that $\exp \{-E_2(g)\} = 1$, and the

⁹ Calot (1993), *loc. cit.* in fn. 3, p. 1226.

second factor exerts no influence on the cohort quantum. When the period quantum increases linearly (so that $Q'_p(t) = Q'_p > 0$ for all values of t), and period tempo is constant ($M_k(g) = M_k$),

$$E_2(g) = M_1 \left\{ \frac{Q'_p}{1 - Q_p(g)} \right\} + \frac{1}{2} M_2 \left\{ \frac{Q'_p}{1 - Q_p(g)} \right\}^2 + \dots$$

which is positive when $Q'_p > 0$. In this case $\exp\{-E_2(g)\} < 1$, and $E_1(g) = 1$ by hypothesis. This implies that the cohort quantum for cohort g is larger than the period quantum at $t = g$. This is obvious. In Section III.2 we shall investigate a case of more practical interest, the situation when we wish to use information from period data to apply to the cohorts that are at the ages when the risk of e.g. giving birth to a first child, a first marriage, etc. is highest, rather than to the cohorts whose members have not yet experienced this event.

To return to the general case, it is clear from Equation (13) that when the period quantum is close to unity, so will the cohort quantum be, almost irrespective of the pattern of period tempo. This agrees with expectation. Take, for example, the extreme case of mortality. Since everyone dies, both the period and the cohort quantum come to 1. On the other hand, a period quantum which is very near zero may result in the cohort quantum exceeding the period quantum: the difference will depend on the changes in period quantum (derivatives of $Q_p(t)$ contained in $E_2(g)$) and on the period tempo (contained in $E_2(g)$) and the changes therein (both in $E_1(g)$ and $E_2(g)$).

Finally, we are often interested not only in the cohort quantum but also in changes therein. Can we conclude from period indices that the cohort quantum is rising or falling? A formula for the first derivative of $Q_c(g)$ as a function of period cohort and tempo and changes therein may be obtained in a relatively straightforward manner from Equation (13), although the resulting expression is complex. The derivative of the cohort quantum can be obtained more easily for special cases of Equation (13), which are based on simplifying assumptions (Sections III.2 and III.3).

III.2. Constant period tempo and linear change in period quantum

In this section we assume that period tempo is constant, and period quantum changes linearly with time. This implies that all derivatives of $M_k(t)$, and all derivatives other than the first of $Q_p(t)$ are zero. As before we shall write $Q'_p(t) = Q'_p$ and $M_k(t) = M_k$ for all values of t . In addition, we assume that Q'_p is so small that terms of third and higher orders can be ignored. In our discussion of Equation (13) we already noted that these assumptions lead to

$$Q_c(g) = 1 - \{1 - Q_p(g)\} \exp \left[-M_1 \frac{Q'_p}{1 - Q_p(g)} - \frac{1}{2} M_2 \left\{ \frac{Q'_p}{1 - Q_p(g)} \right\}^2 \right]. \quad (14)$$

Here the cohort quantum for cohort g is expressed as a function of the level of the period quantum for time $t = g$, the slope of the straight line that describes the time path of the period quantum, and the first two moments of the distribution $a(t, x)$. The situation when there is no translational distortion (i.e. no upward or downward shift in the cohort quantum compared to the period quantum as a result of the time trend in the OE rates) may be evaluated by setting $Q_c(g) = Q_p(g)$ in Equation (14). This leads to the conditions $Q'_p = 0$; $Q_p(g) = 1$, and $Q_p(g) = 1 + \frac{1}{2} Q'_p M_2 / M_1$. When any one of these three conditions holds, there will be no translational distortion, and the period quantum will be an

accurate estimator of the cohort quantum (given the assumptions of this section). Since the values of $Q_p(g)$ are restricted to the interval $[0,1]$, the third condition will hold only provided $-\frac{1}{2}(M_1/M_2) \leq Q'_p \leq 0$. Because all moments are non-negative¹⁰, a negative slope of the period quantum ($Q'_p < 0$) is a necessary condition for the third case.

To analyse the relation between the quantum for a cohort g , and the tempo and quantum for *any* time T (not just $t = g$) we shall use the concept of 'dating',¹¹ and write for the period quantum:

$$Q_p(T) = R + ST \quad 0 \leq Q_p(T) \leq 1.$$

Here the slope S is equal to $Q'_p(T) = Q'_p$, and the value of R was set arbitrarily at $Q_p(g)$. (This means that T is measured relative to g , and that negative values of T represent calendar years before year g .) Then, for any time T (with $-R/S \leq T \leq (1-R)/S$ when the slope S of the period quantum is positive, and $(1-R)/S \leq T \leq -R/S$ when the slope is negative), $Q_p(T) = Q_p(g) + Q'_p T$, and thus $Q_p(g) = Q_p(T) - Q'_p T$. Substituting the latter expression for $Q_p(g)$ in Equation (14), together with our assumptions, leads to:

$$Q_c(g) = 1 - \{1 - Q_p(T) + Q'_p T\} \exp\{-E_2(T)\} \quad (15a)$$

with

$$E_2(T) = M_1 \frac{Q'_p}{1 - Q_p(T) + Q'_p T} + \frac{1}{2} M_2 \left[\frac{Q'_p}{1 - Q_p(T) + Q'_p T} \right]^2. \quad (15b)$$

Equation (15) shows the relationship between the quantum for cohort g and that for a period T years later than g , on the assumption of a constant period tempo and a linear change in period quantum with time. The movement in cohort quantum is clearly non-linear. Under certain conditions which are defined by the parameters $Q_p(T)$, Q'_p , M_1 and M_2 , there may be a point in time (say T^*), when the cohort quantum for the cohort born in year g equals the period quantum for the calendar year $g + T^*$. When such a time T^* exists, it may be evaluated by requiring that $Q_c(g) = Q_p(T^*)$ in Equations (15), and solving for T^* . This yields:

$$T^* = (1/Q'_p) \{1 - Q_p(T^*)\} \{1 - \exp(E_2(T^*))\} \quad (16)$$

where $E_2(T^*)$ follows from Equation (15b). An obvious iterative procedure for obtaining T^* when the values of the other parameters are given is to start with some initial value for T^* , evaluate the right hand side of Equation (16), re-enter this value on the right hand side, and repeat this procedure until the process converges. Convergence was very fast in those cases that we have investigated (when a value T^* existed). A similar procedure may be used to determine the cohort for which the quantum is a certain factor q times the period quantum, by requiring that $Q_c(g) = qQ_p(T^*)$ $0 \leq q \leq 1/Q_p(T^*)$, and solving for T^* .

III.3. Constant period quantum and linear change in period tempo

Assume that the period tempo changes linearly with time, and that the period quantum is constant in some interval. In other words $M'_k(t) = M'_k$, and all derivatives, other than the first, of the k 'th moment are 0. In addition, we have $Q_p(t) = Q_p$, while, at the same time, all derivatives of $Q_p(t)$ vanish. In that case the general equation (13) becomes:

$$Q_c(g) = 1 - \{1 - Q_p\}^{1+M'_1}. \quad (17)$$

¹⁰ In Section III.1, the k 'th moment was defined as $\Sigma x^k a(t, x)$. Since both x and $a(t, x)$ are non-negative, so are the moments.

¹¹ Ryder (1964), *loc. cit.* in fn. 4, p. 77.

This implies that the complement of the cohort quantum for cohort g will equal the complement of the period quantum, raised to the power $1 + M'_1$, where M'_1 is the (constant) slope of the mean age of the schedule with time. Under the assumptions of this section, therefore, the cohort quantum will be constant. The exponent $1 + M'_1$ expresses the distributional distortion, i.e. the translational distortion (upward or downward shift in the cohort quantum) caused by a shift in the period tempo. In this *multiplicative* model for non-repeatable events the *exponent* $1 + M'_1$ plays the same role as the *factor* $1 + M'_1$ does in the *additive* model for repeatable events.¹²

When the mean age of the schedule of rates is declining, $M'_1 < 0$ and, therefore, $Q_c(g) = Q_c < Q_p$. Thus, when events take place at ever younger ages, the period quantum will overestimate the cohort quantum. If $M'_1 > 0$ the opposite result holds.

III.4. Cohort tempo

In order to derive an expression for indices of cohort tempo, we develop $m(g+x, x)$ in a Taylor series:

$$m(g+x, x) = \sum_{i=0}^{\infty} \frac{x^i}{i!} m^{(i)}(g, x).$$

Then we calculate the mean age at which members of cohort g experience the event ($\mu_c(g)$), and the mean age of the schedule of OE rates ($\bar{x}_c(g)$):

$$\mu_c(g) = \frac{\sum_{x=1}^{\omega-1} \exp \left\{ - \sum_{i=0}^{x-1} \sum_{j=0}^{\infty} \frac{x^j}{j!} m^{(j)}(g, i) \right\} - (\omega-1)(1-Q_c(g))}{Q_c(g)}$$

$$\bar{x}_c(g) = \frac{\sum_{x=0}^{\omega-1} xm(g+x, x)}{\sum_{x=0}^{\omega-1} m(g+x, x)} = \frac{\sum_{x=0}^{\omega-1} x \sum_{j=0}^{\infty} \frac{x^j}{j!} m^{(j)}(g, x)}{\sum_{x=0}^{\omega-1} \sum_{j=0}^{\infty} \frac{x^j}{j!} m^{(j)}(g, x)} = \frac{\sum_{x=0}^{\omega-1} x \sum_{j=0}^{\infty} \frac{x^j}{j!} m^{(j)}(g, x)}{-\log\{1-Q_c(g)\}}.$$

Unfortunately, these expressions do not provide much new insight. The problem with $\mu_c(g)$ is that the partial sum (from $i=0$ to $i=x-1$) in the numerator defies further analysis. The expression for $\bar{x}_c(g)$ may be simplified, for example by restricting the Taylor expansion to the first two or three terms and substituting for the corresponding moments $M_k(t)$ and their derivatives. This would lead to expressions similar to those found by Ryder for the case of repeatable events.¹³ Moreover, the disadvantage of $\bar{x}_c(g)$ is that it has no straightforward demographic interpretation (unlike the mean age at which the event is experienced, and unlike the mean age of the schedule $\bar{x}_c(g)$ in the case of *repeatable* events). Probably the only way to approach the translation problem for tempo indices that are easy to interpret is by extensive simulation, in which a relationship between the time path of $M_k^{(i)}(t)$ and $\mu_p(t)$ is established (and similarly for the corresponding cohort indices).

However, instead of looking at the cohort tempo, the cohort quantum between two exact ages, y and z , may be fruitfully analysed, where $0 \leq y < z \leq \omega$. In the study of mortality in particular, it may be important to know what proportion of the cohort

¹² See Equation (12) and Ryder (1964), *loc. cit.* in fn. 3, p. 76.

¹³ Ryder (1964), *loc. cit.* in fn. 3, p. 76.

survives to the 70th birthday, given certain developments in the period proportion who survive to that age, and the various moments of the schedule of age-specific mortality for ages between 0 and 70. By choosing various values for y and z , detailed insight into the cohort tempo may be obtained.

IV. ILLUSTRATIONS

Observed first marriage rates for males aged 15–59 in Norway for the period 1961–90 are given in the Appendix. These rates were used to compute the sum of rates $A(t)$, the period quantum $Q_p(t)$, and the first three moments $M_k(t)$. To illustrate the consequences for the cohort quantum of a change in period quantum and tempo we used the following values for $Q_p(t)$, $M_1(t)$ and $M_2(t)$: $0 \leq Q_p(t) \leq 1$; $M_1(t) = 26, 30, \text{ and } 34$; $M_2(t) = 800, 1000, \text{ and } 1200$, respectively.

Table 1 shows that, during the 1970s and 1980s, the average annual drop in the period quantum amounted to approximately 1.4 percentage points. It therefore seemed reasonable to choose values for Q'_p between -2 and $+2$ percentage points annually for the illustrations. Period tempo, expressed by the mean age of the schedule of age-specific first marriage rates at first fell by 0.22 years per calendar year, but, by the beginning of the 1970s, the trend was reversed and the mean age increased by 0.38 years per calendar year at the end of the 1980s. Values of $M'_i(t)$ between -0.4 and $+0.4$ were selected to show the consequences of changes in period tempo.

The choices described above are probably broad enough to cover most cases of interest when first marriages are considered. Figure 1 illustrates Equation (14), and shows cohort quantum $Q_p(g)$ as a function of period quantum for the year $t = g$ [$Q_p(g)$], when the period quantum changes linearly with time. The slope of the line (denoted by Q' in the Figure) varies between -2 and $+2$ per cent per year. Period tempo is constant, and the first and second moments are 30 and 1000 respectively. The solid straight line between the points (0, 0) and (1.0, 1.0) denotes the situation when the period quantum is constant ($Q'_p = 0$). In that case, there is no translational distortion: cohort and period quantum are equal. The Figure clearly shows that the degree of distortion generally increases *ceteris paribus* when the period quantum declines or its slope become steeper. However, this is not always the case for negative slopes in the period quantum (cf. below). When the period quantum is 0.90, the translational distortion is already considerable: the excess of the cohort over the period quantum amounts to at least eight percentage points, irrespective of the period quantum's slope. The distortion is much larger for relatively low levels of the period quantum. Figure 1 shows that when $Q_p = 0.65$ (which is probably close to the level of first marriages in Norway at the beginning of the 1990s), the cohort quantum is 20 percentage points lower when the period quantum declines by one per cent per year, and 25 per cent higher when it increases by that amount each year. Most striking perhaps is the result that the degree of translational distortion is smaller for a fall of two per cent in the period quantum than for a fall of one per cent. The point of zero translational distortion is given by the condition $Q_p(g) = 1 + \frac{1}{2}Q'_p \cdot M_2/M_1$ (Section III.2), which in Figure 1 leads to $Q_p(g) = 0.67$ for $Q'_p = -0.02$, and $Q_p(g) = 0.83$ for $Q'_p = -0.01$. This change is counterintuitive. From Figure 1 it may be concluded, in general, that the steeper the slope in the period quantum, the larger the error in estimation caused by obtaining the cohort quantum from the period quantum. But, surprisingly, this does not hold in the case of a negative slope when the period quantum is high: on the interval between $Q_p(g) = 1 + \frac{1}{2}Q'_p \cdot M_1/M_2$ and $Q_p(g) = 1$, the period quantum *underestimates* the cohort quantum, and the effect is stronger the steeper the downward slope. The effect

Table 1. Rate sum, period quantum, and moments of the schedule of age-specific first marriage rates, males, Norway

	1961-65	1966-70	1971-75	1976-80	1981-85	1986-90
$A(t)$	2.47	2.56	2.22	1.74	1.37	1.15
$Q_p(t)$	0.92	0.92	0.89	0.82	0.75	0.68
$M_1(t)$	30.16	29.17	28.76	29.53	30.51	32.40
$M_2(t)$	936	905	878	925	986	1108
$M_3(t)$	32581	30102	28696	30888	33924	40042

Source: computed on the basis of table 5.7 in 'Befolkningsstatistikk 1992 Hefte III' (Statistisk Sentralbyrå, Oslo-Kongsvinger, 1991). Marriage rates for five-year intervals were assumed to apply to each one-year interval within the broader interval.

disappears as $\frac{1}{2}Q'_p \cdot M_2/M_1$ approaches zero, because the zero distortion point then moves towards $Q_p(g) = Q_c(g) = 1$. We cannot give a satisfactory explanation for this counter-intuitive cross-over.

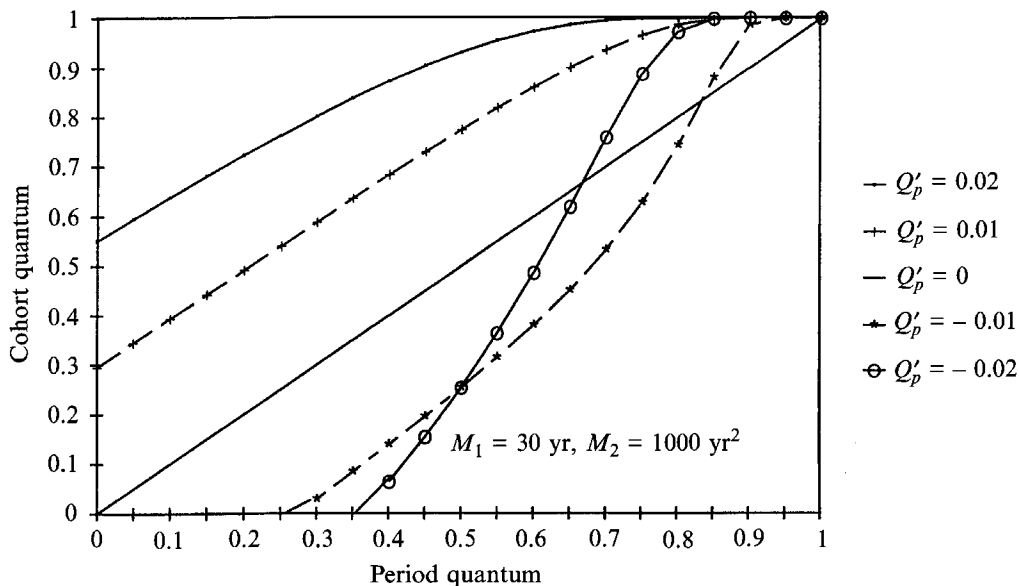


Figure 1. Cohort quantum as a function of time-constant period tempo, and a linear change in period quantum with slope Q'_p

In addition to the values used in Figure 1, we computed Equation (14) for two other values of M_1 , 26 and 34 years respectively. The other parameters (Q'_p and M_2) were the same as in Figure 1. The results are not shown here, but the pattern is similar to that in Figure 1 (underestimation of the cohort quantum by the period quantum when the latter's slope is positive, and a crossover for negative slopes). The extent of translational distortion increases with increasing values of M_1 . For instance, when the first moment is 26 years and $Q_p(g) = 0.5$ we found $Q_c(g) = 0.36$ and 0.92 for values of Q'_p of -0.2 and $+0.2$ respectively. When the first moment was 34, the corresponding values were 0.12 and 0.94 (and 0.25 and 0.93 when $M_1 = 30$, see Figure 1).

It should be noted that Equation (14) yields negative values for the cohort quantum, when the level of the period quantum is low, and when the period quantum decreases with time. Such negative values are more likely, the larger the value of M_1 . There are two explanations for this. First, a linear fall in the period quantum cannot be sustained for

Table 2. Cohort quantum Q_c as a function of the slope in the period quantum (Q'_p) and the second moment of the age pattern of rates (M_2) – results of Equation (14) with $Q_p = 0.65$ and $M_1 = 30$

	-0.02	-0.01	$\frac{Q'_p}{0}$	0.01	0.02
M_2					
800	0.47	0.41	0.65	0.89	0.98
1000	0.62	0.45	0.65	0.90	0.99
1200	0.72	0.49	0.65	0.91	0.99

very long when the period level is already low. And, the higher the mean age the higher will age ω be (in other words, the longer will be the period for the cohort to complete its life course), given a particular value for the second moment. The lowest value of $Q_p(g)$, which yields a non-negative value of $Q_c(g)$ can be found by iteratively solving for $Q_p(g)$ in the equation:

$$Q_p(g) = 1 - \exp \left\{ \frac{Q'_p}{1 - Q_p(g)} \left(M_1 + \frac{\frac{1}{2} Q'_p M_2}{1 - Q_p(g)} \right) \right\}$$

which follows immediately from (14), by setting $Q_c(g) = 0$. Given a starting value for $Q_p(g)$, together with values of Q'_p , M_1 and M_2 , the right-hand side of this expression may be evaluated, and this results in a second value of $Q_p(g)$. This value is again entered on the right hand side and so on. The iterative process converges within a few steps to a unique value in all cases that we have investigated.

The second reason why negative values occur is that terms of third and higher orders were excluded from the Taylor expansion that resulted in Equation (14). The latter expression is more sensitive to this omission when the value of the period quantum is low than when it is high. Addition of a third-degree term to (14) results in somewhat higher values for the cohort quantum (though negative values still appear).¹⁴

Table 2 is a variation on Figure 1: it contains cohort quantum values for three values of the second moment: 800, 1000, and 1200. A value of 0.65 was chosen for the period quantum $Q_p(g)$. Table 2 shows that low values of the second moment tend to reduce the cohort quantum, and that this effect is stronger the lower the value of the first derivative of the period quantum. When values for the first derivative of the period quantum are low, the amount of translational distortion is very sensitive to changes in the second moment. For instance, when $Q'_p = -0.02$ the translational distortion changes from -3 (62 per cent minus 65 per cent) to $+7$ (72–65) percentage points when M_2 moves from 1000 to 1200. However, a fall in M_2 to a level of 800 results in a large translational distortion of -18 (47–65) percentage points.

Figure 2 illustrates Equation (17); it indicates the impact of a constant period

¹⁴ Assuming a linear change in the period quantum we found the i 'th derivative of the sum of rates $A(t)$ as $A^{(i)}(t) = (i-1)!(A'(t))^i$, where $i \geq 1$ and $A'(t)$ is given by Equation (9a). It may be verified that addition of a third-degree term to the Taylor series approximation results in an additional term in the expression for $E_2(g)$ in (14):

$$E_2(g) = M_1 \left\{ \frac{Q'_p}{1 - Q_p(g)} \right\} + \frac{1}{2} M_2 \left\{ \frac{Q'_p}{1 - Q_p(g)} \right\}^2 + \frac{1}{6} M_3 \left\{ \frac{Q'_p}{1 - Q_p(g)} \right\}^3,$$

where M_3 is the third moment. Without the third-degree term, Equation (14) yields a negative value of -0.024 for the cohort quantum in Fig. 1, when $Q_p(g) = 0.25$ (for the case when $Q'_p = -0.01$). Adding the third-degree term leads to a value of $+0.0000028$ and values of $Q_p(g)$ below 0.25 still resulted in negative $Q_c(g)$ -values (M_3 was chosen as 30,000, approximately the value for Norwegian males after 1960. See Table 1).

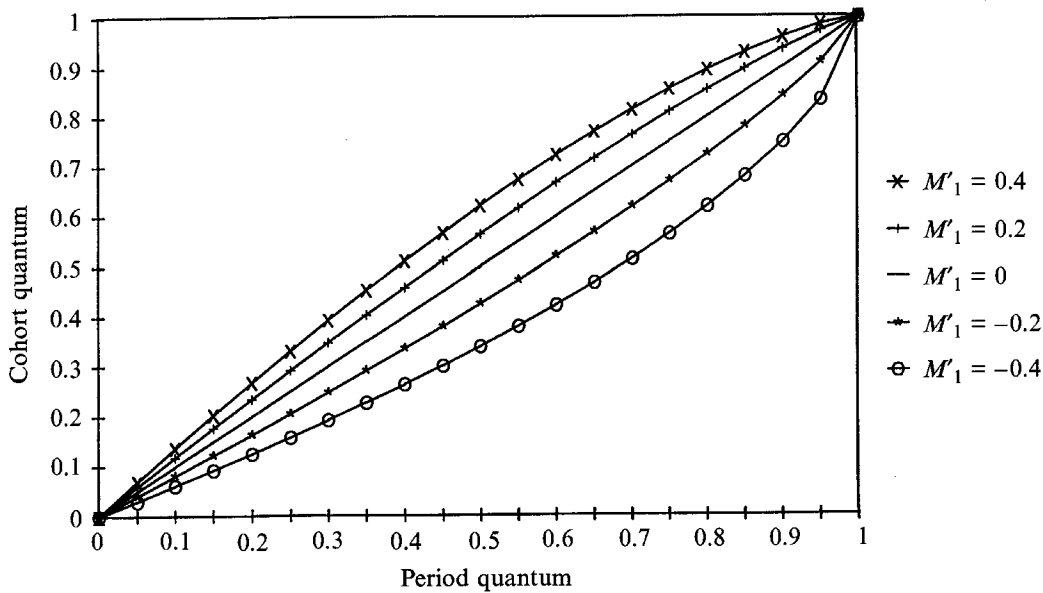


Figure 2. Cohort quantum as a function of period quantum and a linear change in period tempo with slope M'_1

quantum and a period tempo which is a linear function of time, on cohort quantum. The slope of the first moment varies between -0.4 and $+0.4$ years per calendar year. Deviations from the unmarked solid line $M'_1 = 0$ give the amount of distributional distortion. For instance, if period quantum is constant at 0.75 and the mean age of the schedule increases by 0.4 years per calendar year, the cohort quantum is 0.86 , or 11 percentage points higher than that of an undistorted cohort. The largest distributional distortion in Figure 2 occurs for $Q_p = 0.55$, together with $M'_1 = 0.4$ ($+12.3$ percentage points) and for $Q_p = 0.70$, together with $M'_1 = -0.4$ (-18.6 percentage points). Note that the lines in Figure 2 are not symmetric around the line of zero distributional distortion. Distortions caused by a negative slope of the first moment are stronger than those caused by a positive slope of equal absolute value. It is straightforward to prove that the condition $0 \leq Q_c(g) \leq 1$ is equivalent to $M'_1 \leq 1$ for this model. We believe that in practice the latter condition which implies a rise in the mean age of the rate schedule amounting to at most one year per calendar year covers all cases.

The figures in Table 1 show that from the beginning of the 1970s the period quantum of first marriages for Norwegian males fell more or less linearly, and that the first moment also increased approximately linearly. If we assume that movement in both the period quantum and the tempo can be represented by a straight line, the cohort quantum may be inferred from Equation (13) by setting all derivatives of order two or higher equal to zero. This implies that $E_1(g) = 1 + M'_1$ and that to a second-order approximation:

$$E_2(g) = \frac{Q'_p}{1 - Q_p(g)} \{M_1(g) + M'_2\} + \frac{1}{2} \left\{ \frac{Q'_p}{1 - Q_p(g)} \right\}^2 \{M_2(g) + M'_3\} \quad (18)$$

If the linear trends in period quantum and period tempo were to continue for long enough, the proportion ever-married for the five-year cohort consisting of those who were 15 years old in 1976–80 (i.e. the birth cohort 1961–65) may be computed by using the observed values $Q_p = 0.82$, $Q'_p = -0.014$, $M_1 = 29.53$, $M'_1 = 0.18$, $M_2 = 925$, $M'_2 = 10.8$ and $M'_3 = 523$. The values of the first derivatives of the period quantum and the three moments in 1976–80 were estimated at one-tenth of the difference between the

values of the corresponding indices in adjacent periods. This yields $E_1(g) = 1.18$ and $E_2(g) = 1.2424$, resulting in $Q_c = 0.962$. It is not surprising that period quantum underestimates cohort quantum in this example, since both Figure 1 (assuming a constant period tempo and linear change in period quantum), and Figure 2 (assuming a constant period quantum and a linear change in period tempo) suggest that the cohort quantum is higher than its period analogue. But the degree of translational distortion (14 percentage points, leading to an estimate of 96 per cent ever-married at age 60 in the birth cohort 1961–65) is perhaps higher than would have been expected on the basis of intuition. Clearly, the straight-line extrapolations that underlie the calculations are too crude, and the use of polynomials of higher order may lead to lower values of Q_c . For instance, a second-degree polynomial for M_3 describes the data rather accurately for the entire period 1961–1990. This results in $M'_3 = 408$ in 1976–80, which in turn yields a value of 0.946 for the cohort quantum. If, in addition, the slope in the period quantum were estimated by a polynomial of the second degree, we find $Q'_p = -0.012$ and a cohort quantum of 0.90.

To illustrate the concept of partial quantum which may be used to gain insight into the tempo of the process, consider Table 3, which is similar to Table 1. The only

Table 3. *Rate sum, period quantum, and moments of the schedule of age-specific marriage rates for ages 20–29, males, Norway*

	1961–65	1966–70	1971–75	1976–80	1981–85	1986–90
$A(t)$	1.39	1.55	1.41	1.02	0.73	0.46
$Q_p(t)$	0.75	0.79	0.75	0.64	0.52	0.37
$M_1(t)$	25.02	24.90	24.86	25.05	25.28	25.70
$M_2(t)$	634	628	626	635	647	667
$M_3(t)$	16260	16032	15957	16313	16738	17493

Source: see Table 1.

difference is that the range of ages for which the various indices are computed is restricted to 20–29. Thus, on the basis of a synthetic first-marriage table for the period 1981–85, 52 per cent of the non-married at age 20 would marry before their 30th birthday. We note that this proportion falls steeply after the end of the 1960s. The first moment of the distribution of age-specific marriage rates in the age group 20–29 was 25.3 in 1981–85, and there is a weak U-turn in this index over the period 1961–1990. Now consider the period 1981–85. On the assumption of a linear decline in the partial quantum of 2.7 percentage points per year (one-tenth of the difference between adjacent periods), and annual increases of 0.07, 3.2, and 118 respectively in the first three moments, Equation (18) yields a partial quantum of 0.322 for the ‘cohort’ 1981–85 ($E = 1.07$, $E_2 = -0.3918$). Thus, based on the assumption of a linear change in partial quantum and the first three moments of the distribution in the age group 20–29, we expect for the cohort born 1961–65 that the probability of never-married men aged 20 marrying before their 30th birthday will be 0.32, roughly speaking one-third of the estimated total quantum (ages 15–59) for this birth cohort.

V. SUMMARY AND CONCLUSIONS

In this paper we have derived expression for the quantum of non-repeatable events experienced by members of a cohort, as a function of the indices for quantum and tempo observed for synthetic cohorts from a period perspective. These new expressions

complement Ryder's translation formulae for repeatable events. By adopting a life-table perspective based on piecewise constant age-specific intensities, both the period and the cohort quantum can be expressed as a simple logarithmic transformation of the period (or cohort) sum over all ages of OE rates. A Taylor expansion was applied to the sum of rates for a given cohort and terms were re-classified into contributions from the period sum of rates and the moments of the age-specific distributions of period rate schedules, together with their derivatives with respect to time. Finally, the sum of rates for period and cohort was transformed back into period and cohort quantum indices.

Mathematically, we can summarize our results concisely.¹⁵ Let $V_k(t)$ and $W_k(g)$ denote the absolute moments of the schedule of age-specific rates for period t and cohort g , $m(t, x)$ and $m(g+x, x)$ respectively:

$$V_k(t) = \sum_x x^k m(t, x)$$

$$W_k(g) = \sum_x x^k m(g+x, x).$$

Assuming that the rates follow a trend which is polynomial in time, Taylor expansions for $m(g+x, x)$ and $m(g, x) = m(t-x, x)$ lead to:

$$m(g+x, x) = \sum_i \frac{x^i}{i!} m^{(i)}(g, x) \quad m(t-x, x) = \sum_i \frac{(-x)^i}{i!} m^{(i)}(t, x).$$

It follows that

$$W_k(g) = \sum_i \frac{1}{i!} V_{k+i}^{(i)}(g) \quad V_k(t) = \sum_i \frac{(-1)^i}{i!} W_{k+i}^{(i)}(t).$$

The expression of $W_k(g)$ takes period moments and their derivatives as given and translates them into cohort moments. This is the approach followed in this paper. Traditional translation studies were focused on period indices as a function of cohort moments and their derivatives, i.e. the second expression. By inserting minus signs in the appropriate position, and replacing cohort moments by period moments it is possible to change from one perspective to the other.

For the cohort sum of rates, denoted by $B(g)$, we have:

$$B(g) = W_0(g) = \sum_i \frac{V_i^{(i)}(g)}{i!}. \quad (19)$$

Next the cohort quantum equals $Q_c(g) = 1 - \exp\{-B(g)\}$. By including sufficient period moments and their derivatives in the expression for $B(g)$, the cohort quantum may be evaluated to any desired level of precision. Finally, it may be useful to factor the period moment $V_k(t)$ into a factor $A(t) = V_0(t)$ which represents the sum of period rates and a factor $\sum x^k a(t, x) = M_k(t)$ which represents the moments of the period distribution of the age-specific rates.

The cohort mean age of the schedule of rates ($W_1(g)/W_0(g)$) may also be computed from period indices, but the results are difficult to interpret demographically. Each derivative of $V_k(t)$ should then be written as an appropriate function of the derivatives of $A(t)$ and $M_k(t)$.

Alternatively, the mean age at which members of the cohort experience the event (see Equation (7)) leads to a complex formula which does not provide much insight. This would imply that the expressions derived in this paper might be used fruitfully in the

¹⁵ See also for the case of repeatable events Ryder (1964) *loc. cit.* in fn. 3, p. 76.

analysis of such non-repeatable events as first marriages, births by parity, emigration etc., but *not* for deaths: the quantum of mortality, by definition, is 1.00, and therefore attention is only given to the tempo aspects of this phenomenon. However, as a second best to indices of tempo, the notion of a partial quantum may be applied by restricting the age interval of the OE rates to some ages that are central for the process (for instance, ages between 20 and 30 for first marriages or first births, or 70 to 80 years for mortality, and analysing the partial quantum for the cohort on the basis of the 'partial' period indices for the relevant age interval.

We investigated in some detail three special cases of Equation (13) (summarized in (19): (i) constant period tempo and linear change in period quantum (see Equation (14)); (ii) constant period quantum and linear change in the period age distribution of age-specific rates (Equation (17)); and (iii) a linear change in both period quantum and the period age distribution (Equation (18)). More complex models which involve polynomials of degree two or higher may also be evaluated.

A problem with the present approach is that polynomials may yield an accurate description of observed trends for period indices, but that extrapolations may easily lead to unrealistic values, for instance a period quantum outside the range (0, 1). To overcome this problem, alternative specifications of the trend have been proposed in the context of translation formulae for repeatable events, for instance a logistic or a periodic function.¹⁶ This approach may also be applied to non-repeatable events. But even with the present approach, based on polynomials, it would be advisable (though not always sufficient) to try and fit a polynomial to the sum of period rates $A(t)$, instead of to the period quantum $Q_p(t)$. Equation (3) restricts the period quantum value to the interval (0, 1), irrespective of the extrapolated value for the sum of period rates (provided that the sum remains positive).

Numerical illustrations were computed for first marriages of Norwegian males for the period 1961–90. We showed that the proportion of 82 per cent ever married for the period 1976–80 underestimates the proportion married in the birth cohort 1961–65 by between 8 and 14 percentage points, assuming a linear decline in the period quantum of between 1.2 and 1.4 per cent. Approximately one-third of the cohort quantum would be located in the age interval 20–29.

There are at least two types of problem to which the expressions derived here may be applied fruitfully. The first is when information about the quantum and the first two or three moments of the process is available for a number of years, and we want to analyse the implications for recent cohorts whose members have not yet completed their life course. Provided we assume that period indices follow some polynomial of low degree as a function of time, the translation formulae provide the wherewithal to infer the quantum and partial quantum for these recent cohorts. The second type of application consists in formulating assumptions in population projections for parity-specific fertility. Projection assumptions have often been formulated by means of a few summary indices (for instance, the proportion childless, or the mean age at first childbirth) and are only broken down into age-specific rates at a later stage. Translation formulae may be used to investigate the implications of extrapolated period indices for cohort behaviour and conversely.

¹⁶ J. de Beer, 'Translation analysis as a device for extrapolating fertility rates', Netherlands Central Bureau of Statistics. Unpublished paper, 1982.

APPENDIX: AGE-SPECIFIC MARRIAGE RATES FOR NEVER MARRIED MALES,
NORWAY, PER 1000

	1961-65	1966-70	1971-75	1976-80	1981-85	1986-90
15	0	0	0	0	0	0
16	0.2	0.2	0.1	0.0	0	0.0
17	2.5	2.3	1.3	0.5	0.2	0.1
18	12.1	11.7	9.8	4.5	2.4	1.6
19	36.3	37.5	34.3	18.1	9.6	3.0
20-24	109.9	130.7	120.6	79.8	49.7	23.9
25-29	168.0	180.1	160.6	125.0	95.3	68.3
30-34	99.9	96.7	80.7	73.0	64.0	62.7
35-39	50.4	46.5	36.3	33.9	32.3	37.1
40-44	27.6	23.0	18.1	16.3	15.8	19.2
45-49	15.8	12.6	9.7	8.3	8.3	9.6
50-54	7.9	7.1	5.4	5.0	4.7	4.8
55-59	4.6	4.1	3.0	2.6	2.3	2.7

Source: Table 5.7 in 'Befolkningsstatistikk 1991 Hefte III' (Statistisk Sentralbyrå, Oslo-Kongsvinger, 1991).