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Measuring Change and Continuity in Parity Distributions

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Procedures are developed to allocate the change in mean fertility to the change in specific parities or groups of parities. One procedure uses the proportion at each parity and another uses parity progression ratios. Both are based on the delta method for approximating change in a function of several variables. Drawing on an analogy to survival in a life table, the relational logit model is applied to parity progression. This method allows several parity distributions to be synthesized and to have differences summarized with two parameters. The three procedures are applied to successive cohorts of white U.S. women who completed their childbearing between 1920 and 1980.

Summary measures of fertility, although useful condensations of large amounts of information, are liable to mask important aspects of the process that they summarize. To avoid oversimplification, several methods have been developed to adjust or control for variations in the risk of childbearing. Thus some methods take account of the effect of age and sex composition on the crude birth rate (CBR), either by disaggregating into a set of age-specific fertility rates or by standardizing against a given age distribution. Change in the CBR can be expressed in terms of change in the age, sex, and marital status composition (Das Gupta, 1978; Kim and Strobino, 1984). Such methods adjust for variation and change in the risk of childbearing across population subgroups.

Summary measures such as the total fertility rate (TFR) can also be broken down into components to measure the effect of the proximate determinants. Here attention is shifted to factors affecting the risk of childbearing by women in the fertile ages. Decomposition according to the proximate determinants was developed for aggregates by Bongaarts (1978) and adapted to individual-level data by Hobcraft and Little (1984). It too has been adapted to the study of change so that change in fertility rates can be expressed as changes in marital exposure, contraceptive use, and breastfeeding practices (Pullum, Casterline, and Shah, 1987).

This article develops another type of decomposition. Rather than assessing the importance of—or adjusting for—the value of some determinants of risk, here the outcome itself will be the basis of the disaggregation. The mean parity of a group of women will be expressed in terms of the distribution across specific parities, and more important, the change in mean parity will be partitioned into terms that refer to the specific parities. A model for summarizing change relative to a synthetic parity distribution will also be presented. Results will be illustrated with data from the United States, and we shall indicate how our findings relate to those of other researchers who applied different methods to these data.

Analysts often focus on specific parities, particularly 0 (Tolnay and Guest, 1982), 1 (Polit and Falbo, 1987), and 2 (David and Sanderson, 1987). Ryder (1969, 1982), in particular, brought out the character of fertility as a transition across successive parities, a notion recently extended significantly by David et al. (1988). Retherford (1985) described how a small shift in the dominant parity can have a major impact on the mean as fertility falls. Apart from the calculation of order-specific fertility rates and parity progression ratios, however, there are few analytical methods that truly use the fact that children come in integers. The methods to be developed here are motivated by a perspective on childbearing as a sequence of transitions across specific parities, terminating in specific parities.

Expressing Change in Terms of Proportions at Each Parity

One behavioral model for family formation, particularly in settings of high control over fertility, assumes that women or couples have a target family size that is established early in the process of family formation, even though it is subject to subsequent adjustments (Lee, 1980). Under this model, it is sensible to describe fertility and its changes in terms of the proportions who eventually terminate at specific numbers of children.

Let M be the mean parity of a cohort of women and p_i the proportion of women at each parity i; thus

$$M = \sum_i ip_i. \tag{1}$$

This formula simply states that M is a weighted average of the integers $i = 0, 1, 2, \ldots,$ with weights p_i (which sum to unity). M is exactly the same as the cumulative fertility of the cohort, that is, the (unweighted) sum of the age-specific fertility rates f_x of the cohort up to its present age y:

$$M = \sum_x^y f_x. \tag{2}$$

When the cohort is observed at the highest age of childbearing, or later, both formulas for M will be identical to the TFR. One can examine the proportions p_i as well as the age-specific fertility rates f_x to obtain a more complete picture of family formation than is possible with M alone, with the essential difference that the parity distribution expresses the variability in completed numbers of children, whereas the age-specific rates indicate how the mean arose over time and age.

The value of analyzing the full parity distribution is well established, so we turn immediately to the use of equation (1) to partition *change* in M. We are aware of only one effort in the demographic literature to achieve this.

Cutright and Shorter (1979) adapted the technique of direct standardization to describe the impact on M of change in the parity distribution. They applied a modification of well-established methods for partitioning change in a rate into terms for change in specific rates and change in composition. To paraphrase their approach, they asked this hypothetical question: What would be the mean fertility of a cohort if its observed mean fertility for parity groups 0–1 and 2+ were combined with the sizes of those groups as observed in a second cohort?" The difference between the actual and standardized means for the first cohort was then expressed as a ratio to the difference between the actual means for the two cohorts.

Although this approach may appear to be a direct parallel of the decomposition of change in the CBR into terms for fertility rates and age–sex composition, it is not valid in this context.[1] Standardization is inappropriate for decomposing mean parity because the mean of a distribution across the integers is a property of that distribution and does not involve any other rates or means as in, for example, the mean fertility of different ethnic or educational groups.

We shall describe an alternative decomposition, which is a very simple application of the familiar delta method with a side condition. The delta method is based on the chain rule for the differentiation of functions of many variables (e.g., Courant, 1937:vol. 1, pp. 472–476). In a general context, if y is a function f of several variables, labeled $x_1, \ldots, x_k$, then the total differential of y is given by

$$dy = \sum_i (\partial y / \partial x_i)\, dx_i, \tag{3}$$

leading to the finite approximation

$$\Delta y = \sum_i (\partial y / \partial x_i)\, \Delta x_i, \tag{4}$$

in which the differentials dy and $dx_1, \ldots, dx_k$ are replaced by finite differences Δy and $\Delta x_1, \ldots, \Delta x_k$. If a side condition is satisfied by the x variables, of the form $g(x_1, \ldots, x_k) = 0$, then the differential is obtained from the partial derivatives of $f + Cg$ rather than f. The constant C, a Lagrange multiplier, is specified by some boundary condition, such as the value of the differential when all x variables are 0. (In statistical rather than mathematical terminology, C represents the loss of 1 df, because of the side condition, and the boundary condition is required for identifiability.) Thus the quantity $C(\partial g/\partial x_i)$ must be added to $(\partial y/\partial x_i)$ above.

The mean parity M is given by $M = \Sigma_i\, i p_i$, where p_i is the proportion of women at parity i, with the side condition that $\Sigma_i\, p_i = 1$ (or $\Sigma_i\, p_i - 1 = 0$). The sensitivity of M to changes in a specific parity i is therefore given not by $\partial M/\partial p_i = i$, but rather by $\partial M/\partial p_i = i + C$, where C is some constant (not a function of i) required by the side condition. This is an exact relationship, rather than an approximation, because equation (2) is linear.

To specify C, it is reasonable to require as a boundary condition that if the mean M happens to be exactly at the specific integer i, then a change in p_i will not in itself induce any change in M. This requirement implies that the constant C must be given by $C = -M$. That is, $\partial M/\partial p_i = i - M$. We are then led to this formula for the change between two successive cohorts: $\Delta M = \Sigma_i\, (i - M)\, \Delta p_i$. To avoid ambiguity over whether M on the right side comes from the first or second cohort, we will use

$$\Delta M = \sum_i (i - M^*)\, \Delta p_i, \tag{5}$$

where M is the difference $M_2 - M_1$ and M^* is the arithmetic average of M_1 and M_2. This choice of M^* is arbitrary. Results would differ slightly if it were replaced by the weighted average of M_1 and M_2 or the geometric average, and so on. The ith term on the right side, $(i - M^*)\, \Delta p_i$, will be interpreted as the change in M that is due to the change in p_i.

If a specific p_i increases by Δp_i, then one or more of the other proportions must decline because the proportions always add to unity. The quantity $(i - M^*)$ must be regarded as a partial effect, in the same sense as a regression coefficient in multiple regression when the predictor variables are intercorrelated.

Expressing Change in Terms of Parity Progression Ratios

Another behavioral model for family formation assumes that women or couples arrive at their completed parity through a sequence of decisions or behaviors at each parity (Udry, 1983). Under this model, it is sensible to describe fertility and its changes in terms of the proportions who progress across each of the successive parities. In response to this perspective, the mean will now be expressed as a function of parity progression ratios and the delta procedure will be applied to that function. Let P_i be the parity progression ratio for transitions out of parity 0, defined in terms of the proportions p_i by $P_i = (\Sigma_{j=i+1}^{\infty} p_j)/(\Sigma_{j=i+1}^{\infty} p_j)$. From the reverse relationship $p_i = (1 - P_i) \Pi_{j=0}^{i-1} P_j$ (and $p_0 = 1 - P_0$) and the definition of the mean, $M = \Sigma_i \, ip_i$, it can be shown that

$$M = P_0 + P_0P_1 + P_0P_1P_2 + \cdots = \sum_{i=0}^{\infty} \prod_{j=0}^{i} P_j. \tag{6}$$

This formula appears to have been first used by Ryder (1980:49). Again employing the chain rule for differentiation, we now have $dM = \Sigma_i \, (\partial M/\partial P_i) \, dP_i$ or the approximation

$$\Delta M = \sum_i (\partial M/\partial P_i) \, \Delta P_i. \tag{7}$$

In contrast with the decomposition of proportions, there is no side condition, because the parity progression ratios can vary completely independently of one another. The partial derivative $\partial M/\partial P_i$ is simply the coefficient of P_i (after grouping and simplifying) in equation (6):

$$\partial M/\partial P_i = \left[\sum_{l=1}^{\infty} \prod_{j=0}^{l} P_j \right] \Big/ P_i. \tag{8}$$

This quantity is calculated by using the first and second cohorts' parity progression ratios and then averaging, to improve the approximation. This averaging will be indicated by attaching an asterisk. Then $(\partial M/\partial P_i)^* \, \Delta P_i$ is interpreted as the marginal change in the mean that is due to change in the ith parity progression ratio—that is, as the change in M that would have occurred if *only* the ith parity progression ratio had changed. In general, the sum of these marginal changes will differ slightly from the observed total change because equation (7) is an approximation.

Synthesizing Several Parity Distributions: The Relational Logit

The decomposition of change according to the specific proportions or parity progression ratios is useful for comparing distributions with one another but will not bring out any of their common features. An approach that is better suited for identifying similarities of distributions and continuities in change will now be developed.

Define d_i to be the proportion of the cohort that has experienced parity i. Thus $d_0 = 1$, since all women have been childless at some point; $d_1 = P_0$, since this is the proportion of women who have gone on to have one child; and in general

$$d_i = \prod_{j=0}^{i-1} P_j \qquad \text{for } i > 0. \tag{9}$$

This proportion, the parity attainment proportion, is a direct analog of the survival function l_x in a life table (with radix $l_0 = 1$) except that l_x is a continuous function of age x, whereas d_i is only defined for integral numbers of children i. By establishing an isomorphism with the life table, one has the possibility of applying to the parity distribution a collection of analytical tools originally established for the study of mortality. Efforts in this direction have been made by Chiang and van den Berg (1982), Lutz and Feichtinger (1983), and Feeney (1983). Life-table methodology has already been applied with success to the study of birth intervals, another aspect of the childbearing process (e.g., see Bumpass, Rindfuss, and Janosik, 1978; Pebley, 1981; Rindfuss et al., 1982).

Our application of the relational logit to the parity attainment proportions of several cohorts is based on successful applications of this technique to the survival functions from sets of life tables. The logit or log-odds was first applied to the life-table survival function by Brass (1974, 1975; Brass et al., 1968) and has also been used by Stoto (1982).[2] The logit of a proportion such as l_x or d_i has several desirable properties not shared by the proportion itself. Of these, two of the most important are (1) that the logit has a full range from $-\infty$ to $+\infty$, rather than simply from 0 to 1, and better differentiates proportions near 0 and 1 and (2) that the logit is completely symmetric with respect to the criterion category, so a relabeling of the two categories is equivalent to simply multiplying the logit by -1. The logit is not defined when the proportion is exactly 0 or 1; since $d_0 = 1$, the logit will only be calculated for parities 1 and above.

Given several cohorts or distributions, the logit of the parity attainment proportions d_{ic} for parity i and cohort c can be described by a relational logit transformation,

$$\text{logit } d_{ic} = a_c + b_c (\text{logit } d_i^*), \qquad \text{for } i > 0. \tag{10}$$

Here the proportions d_i^* refer to a synthetic or summary distribution that is modified in each cohort's experience by parameters a_c and b_c. The model asserts that under this transformation, all of the distributions in a set can be fitted with these two cohort-specific parameters and the synthetic pattern. We regard this model primarily as a device for integrating and comparing distributions.

Figures 1 and 2 illustrate the roles of a_c and b_c in altering the level and dispersion of fertility. With an arbitrary choice of attainment proportions d_i^*, Figure 1 shows the effect on d_i of variations of a_c about zero when $b_c = 1$. An increase in a_c will increase all of the parity attainment proportions, raising the mean of the distribution. Figure 2 shows the effect on d_i of variations of b_c about 1 when $a_c = 0$. An increase in b_c will increase the early proportions but decrease the later ones, thereby changing the dispersion of the distribution.

Data to Be Analyzed

The three procedures will now be applied to the completed fertility of white women in the United States during the 20th century. The data consist of the completed parity

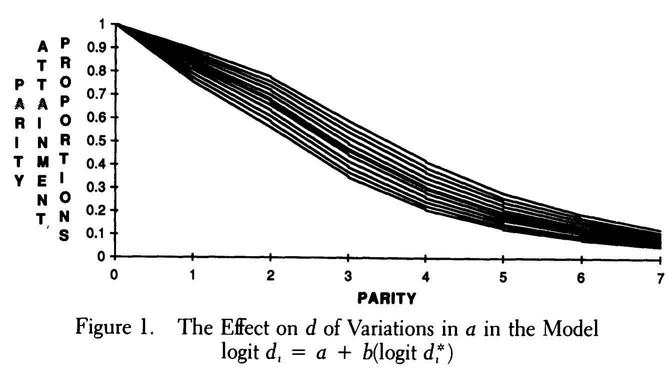

Figure 1. The Effect on d of Variations in a in the Model logit $d_i = a + b(\text{logit } d_i^*)$

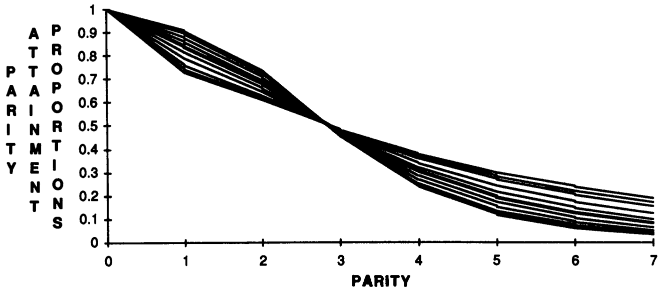

Figure 2. The Effect on d of Variations in b in the Model logit $d_i = a + b(\text{logit } d_i^*)$

distributions of white American women who completed their fertility between 1920 and 1980. Specifically, using Heuser (1976:237) for the years 1920–1970 and an updating to 1980 (see Bogue, 1985:283, for a convenient summary), we are able to monitor these distributions, as of exact age 45, for women who were aged 45–49 on January 1 in 1920, 1925, . . . , 1980—a total of 13 five-year cohorts. The birth years of these cohorts were 1871–1875, 1876–1880, . . . , 1931–1935. These distributions will be identified by the labels 1873, 1878, . . . , 1933, which give the midpoint years of birth of the five-year cohorts. Table 1 presents the 13 basic distributions in tabular form, including their means and variances.[3]

Other researchers, particularly Ryder (1969, 1980, 1986), Masnick (1981), Cutright and Shorter (1979), and Vaupel and Goodwin (1987), have examined family building in the United States during the same interval, using the same data. Al-Osh (1986) used time series methods to generate plausible forecasts of future U.S. fertility with these data. Research by Lee (1974, 1980), Bouvier (1980), Smith (1981), and others is related as well. The analysis is limited to white women to improve the homogeneity of the data; the principal differences between white and nonwhite fertility in the United States have been described by Cutright and Shorter (1979), Masnick (1981), and Evans (1986).

The analysis will use only a minute fraction of the available data on these cohorts of women. We will make no reference to changing patterns of marriage, access to contraception, infant and childhood mortality, female employment, known trends in sterility, and other known correlates of fertility.[4] These are the factors through which changes in the parity distribution came about.

Decomposition by Proportions

The numerical results of the partitioning into parity-specific components of change are shown in Table 2. The first column gives the net change that is decomposed in subsequent columns. All calculations are based on more than the two decimal places that appear in the table. Rounding error accounts for any discrepancies between the table entries and quantities derived from them.

The pattern in Table 2 can be summarized easily. Up to 1908, with only 2 exceptions out of 56 components of change, all parities participated in the concentration toward the mean or had no impact at all. Specifically, mean completed parity fell by 1.41 children between the 1873 and 1908 cohorts. By adding components in Table 2, we can ascertain what proportion of this decline can be attributed to changes in the proportion of women at specific parities or groups of parities. The net contributions of parities 0 and 3 were negligible. Changes in parity 1 accounted for -0.20, or 14 percent of the decline; changes in parity 2 for -0.10, or 7 percent; changes in parities 4–6 for -0.22, or 15 percent; and changes

Table 1. Means, Variances, and Proportions of Women (p) at Each Completed Parity Within Cohorts of White Women in the United States

	Completed parity									
Cohort	0	1	2	3	4	5	6	7+	Mean	Variance
1873	0.2098	0.1121	0.1280	0.1131	0.0956	0.0773	0.0668	0.1973	3.6553	10.1740
1878	0.2183	0.1219	0.1362	0.1170	0.0950	0.0740	0.0642	0.1734	3.4377	9.5674
1883	0.2177	0.1322	0.1479	0.1226	0.0960	0.0743	0.0579	0.1514	3.2486	8.7876
1888	0.2091	0.1444	0.1603	0.1325	0.1002	0.0704	0.0548	0.1283	3.0892	7.9912
1893	0.1918	0.1675	0.1846	0.1414	0.0980	0.0643	0.0478	0.1046	2.8880	6.9461
1898	0.1932	0.1921	0.2059	0.1415	0.0890	0.0552	0.0411	0.0820	2.6291	6.0195
1903	0.1980	0.2195	0.2183	0.1358	0.0823	0.0494	0.0336	0.0631	2.3975	5.2310
1908	0.2079	0.2181	0.2332	0.1389	0.0796	0.0446	0.0280	0.0497	2.2474	4.5857
1913	0.1774	0.2050	0.2541	0.1590	0.0880	0.0460	0.0274	0.0431	2.3122	4.1237
1918	0.1346	0.1759	0.2687	0.1852	0.1065	0.0553	0.0303	0.0435	2.5259	3.9079
1923	0.0942	0.1507	0.2645	0.2088	0.1280	0.0679	0.0368	0.0491	2.7931	3.8594
1928	0.0942	0.1142	0.2402	0.2223	0.1488	0.0812	0.0445	0.0547	2.9858	3.9803
1933	0.0785	0.0968	0.2311	0.2365	0.1659	0.0905	0.0482	0.0525	3.1010	3.6618

Notes: Cohorts are identified by the middle year of the birth cohort of women, and parity is defined as of age 45.

Table 2. Change in Mean Completed Parity due to Changes in the Proportion at Specific Parities (p)

		Parity							
Cohort	Net change	0	1	2	3	4	5	6	7+
1873–1878	−0.22	−0.03	−0.02	−0.01	0.00	0.00	0.00	−0.01	−0.14
1878–1883	−0.19	0.00	−0.02	−0.02	0.00	0.00	0.00	−0.02	−0.13
1883–1888	−0.16	0.03	−0.03	−0.01	0.00	0.00	−0.01	−0.01	−0.13
1888–1893	−0.20	0.05	−0.05	−0.02	0.00	0.00	−0.01	−0.02	−0.15
1893–1898	−0.26	0.00	−0.04	−0.02	0.00	−0.01	−0.02	−0.02	−0.14
1898–1903	−0.23	−0.01	−0.04	−0.01	0.00	−0.01	−0.01	−0.03	−0.12
1903–1908	−0.15	−0.02	0.00	0.00	0.00	0.00	−0.01	−0.02	−0.09
1908–1913	0.06	0.07	0.02	−0.01	0.01	0.01	0.00	0.00	−0.05
1913–1918	0.21	0.10	0.04	−0.01	0.02	0.03	0.02	0.01	0.00
1918–1923	0.27	0.11	0.04	0.00	0.01	0.03	0.03	0.02	0.03
1923–1928	0.19	0.00	0.07	0.02	0.00	0.02	0.03	0.02	0.03
1928–1933	0.12	0.05	0.04	0.01	0.00	0.02	0.02	0.01	−0.02
1873–1908	−1.41	0.01	−0.20	−0.10	−0.01	−0.02	−0.07	−0.12	−0.90
1908–1933	0.85	0.33	0.21	0.02	0.04	0.11	0.10	0.07	−0.02

in parities 7 and above for −0.90, or 64 percent. Avoidance of the highest parities was clearly the main factor in the decline.

Working with nearly the same interval (from 1868 to 1910), Cutright and Shorter (1979) concluded that "changes in the distribution of women to 0 and 1 parities explain nearly a third [31%] of the difference in completed fertility between the two cohorts. The remaining difference is due to the larger numbers of children born to mothers with at least two children to c. 1868, in contrast to c. 1910" (p. 206). They reached this conclusion with the direct standardization procedure described earlier, simply distinguishing between parities 0–1 and 2+.

After 1908 (i.e., after the cohort that was aged 45–49 in 1955), the Baby Boom is shown in the form of a steadily increasing mean. Mean completed parity rose from 2.25 to 3.10 children, an increase of 0.85. As Bean (1983) remarked, the Baby Boom was less significant in cohort than in period terms; the period TFR (for whites) increased from less than 2.2 in the mid-1930s to more than 3.6 in 1957 (Bogue, 1985:255). For the five successive cohorts that had ever-increasing mean completed parities, the most important source of change was reductions in parities 0 and 1. If the five cohorts are combined, 63 percent of the increase in the mean can be attributed to parities 0 and 1, 20 percent to parities 2–4, and 17 percent to increase in parities 5 and above.

We also differ substantially with Cutright and Shorter regarding the partitioning of this increase in mean fertility. They calculated that 91 percent of the increase in white fertility from 1910 to 1934 was due to the proportions at parities 0 and 1, compared with our allocation of 63 percent of the increase from 1908 to 1933. For the intervals of decline and increase, they attached considerably more importance to the proportions at 0 and 1 than we do.

Decomposition by Parity Progression Ratios

We shall now partition change in mean parity according to changes in specific parity progression ratios (P_i' s). The impact of a fixed change in any P_i (e.g., a change of 0.01) will have maximum effect if it occurs at $i = 0$ and a progressively diminishing effect for higher parities because progressively fewer women will ever attain each higher parity and thereby have the potential to pass beyond it. The results of this decomposition are given in Table 3. The first column repeats the net change in the mean, exactly as in the first column of Table 2, and the subsequent columns allocate that change across specific parities. The components of change are equal to the changes in parity progression ratios, weighted by the sensitivity of the mean to each change.

Except for two positive contributions from P_0, the interval from 1873 to 1908 is marked by negative contributions to change from all ratios. Even among the subsequent cohorts, for which the ratios for low parities show sharp increases, the contributions to change from parities above 3 continue to be zero or negative.

More specifically, the decline can be allocated across four parity progression ratios or groups of ratios that shared approximately equal responsibility for the change from 1873 to 1908. Decline in movement out of parity 1 accounted for 25 percent of the reduction in the mean; decline in movement out of parity 2 accounted for another 31 percent; decline in movement out of parity 3 accounted for 21 percent; and decline in the other ratios accounted for another 22 percent.

From 1908 to 1933, the bulk of the increase of 0.85 in mean completed parity can be attributed to changes in P_0 and P_1. If the parity progression ratios for parities 1 and above had not changed at all across these cohorts and only the recorded change in P_0 had occurred, then the mean would have increased by 0.38, or 45 percent of the total increase. The observed changes in P_1 would have served to increase the mean by 0.37, or 43 percent of

Table 3. Change in the Mean due to Change in Specific Parity Progression Ratios (P)

Cohort	Net change	Parity 0	1	2	3	4	5	6	Residual
1873–1878	−0.22	−0.04	−0.05	−0.05	−0.04	−0.02	−0.01	−0.01	−0.01
1878–1883	−0.19	0.00	−0.04	−0.05	−0.04	−0.03	−0.02	0.00	−0.01
1883–1888	−0.16	0.03	−0.04	−0.05	−0.05	−0.04	−0.01	−0.01	0.00
1888–1893	−0.20	0.06	−0.07	−0.08	−0.06	−0.03	−0.01	−0.01	−0.01
1893–1898	−0.26	0.00	−0.08	−0.09	−0.05	−0.02	−0.01	−0.01	0.00
1898–1903	−0.23	−0.01	−0.08	−0.07	−0.03	−0.02	−0.01	0.00	0.00
1903–1908	−0.15	−0.03	0.00	−0.05	−0.03	−0.02	−0.01	0.00	0.00
1908–1913	0.06	0.09	0.05	−0.01	−0.03	−0.02	−0.01	−0.01	0.00
1913–1918	0.21	0.12	0.09	0.03	0.00	−0.01	−0.01	0.00	−0.01
1918–1923	0.27	0.12	0.08	0.07	0.02	0.00	0.00	0.00	−0.01
1923–1928	0.19	0.00	0.09	0.08	0.03	0.00	0.00	0.00	−0.01
1928–1933	0.12	0.05	0.05	0.04	0.01	−0.01	−0.01	−0.01	−0.01
1873–1908	−1.41	0.02	−0.35	−0.44	−0.29	−0.17	−0.09	−0.05	−0.03
1908–1933	0.85	0.38	0.37	0.22	0.02	−0.04	−0.04	−0.03	−0.03

the total. Adding these two percentages, 88 percent of the increase in the mean was due to increased transitions out of parities 0 and 1. Changes in P_2 and P_3 would have increased the mean by 0.24, or 28 percent of the total. Changes in the higher parities actually served to counteract the effects of changes in the low-order ratios.

Continuities: The Relational Logit Model

When the relational logit is applied[5] to the U.S. data in Table 1, the fitted d_i^* vector may be regarded as a synthesis or summary of the parity attainment proportions across the full interval of data. From this vector, it is possible to compute a set of synthetic parity progression ratios using $p_i^* = d_i^* - d_{i+1}^*$ for $i \geq 0$ ($P_i^* = 0$ and $p_i^* = d_i^*$ for the highest parity). Table 4 gives the estimated coefficients a_c and b_c associated with each cohort c and the logit d_i^* associated with each i. Table 5 then gives the fitted or synthetic vectors of proportions, parity attainment proportions, and parity progression ratios for the entire interval. It is necessary to supply two constraints to the parameters for them to be identifiable. We have required that the arithmetic average of the a_c's be 0 and that of the b_c's be 1 across all cohorts.[6]

It is more a matter of judgment than of statistical test whether the model provides an acceptable summary of Table 1, because that table is based on population rather than sample data. To indicate the quality of fit, we have computed the index of dissimilarity for each of the 13 cohorts. It ranges from a low of 1.01 percent for 1923 to a high of 4.25 percent for 1933, with an average value of 2.22 percent. That is, an exact correspondence between the observed and fitted parity distributions could be obtained by shifting an average of only 2.22 percent of the women in each cohort. In our judgment, this is a good fit.

Although the model was stated in terms of d_i^*, it is notable that the parity progression ratios P_i^* computed from the fitted d_i^* are essentially two-valued. Table 5 suggests that the synthetic distribution could be constructed completely by using a probability of approximately 0.82 for transitions out of parities 0 and 1 and a probability of approximately 0.67 for all subsequent transitions. Ryder (1969) hypothesized that such a two-valued set of ratios may be characteristic of settings of high fertility control. None of the observed cohorts shows this two-valued pattern as clearly as the synthetic distribution.

Table 4. Parameter Estimates for the Relational Logit Model Applied to Completed Parity Distributions of Birth Cohorts of White Women in the United States, 1873–1933

Cohort	*a*	*b*	Parity	logit(d^*)
1873	0.2668	0.6562	0	0.0000
1878	0.1685	0.6821	1	1.6308
1883	0.0955	0.7257	2	0.7380
1888	0.0402	0.7875	3	−0.1350
1893	−0.0338	0.8826	4	−0.8358
1898	−0.1836	0.9575	5	−1.4350
1903	−0.3432	1.0278	6	−1.9507
1908	−0.4579	1.0780	7+	−2.4533
1913	−0.3716	1.1599		
1918	−0.1368	1.2299		
1923	0.1532	1.2955		
1928	0.3235	1.2367		
1933	0.4791	1.2805		

Notes: The estimates are parameterized such that the mean value of the *a* parameters is 0 and the mean value of the *b* parameters is 1.

Table 5. The Synthetic Proportions (p) at Specific Completed Parities, the Proportions (d) at or Above Specific Parities, and the Parity Progression Ratios (P) Implied by the Fitted Vector d^*

Parity	p	d	P
0	0.1637	1.0000	0.8363
1	0.1597	0.8363	0.8090
2	0.2103	0.6766	0.6892
3	0.1639	0.4663	0.6486
4	0.1101	0.3024	0.6359
5	0.0678	0.1923	0.6473
6	0.0453	0.1245	0.6362
7+	0.0792	0.0792	0.0000

If one accepts equation (10) as an adequate representation of the completed parity distribution across 13 cohorts, then successive changes in that distribution have been reduced to changes in only two coefficients, a_c and b_c. These coefficients show only one departure from an extraordinary continuity: a single change of direction in the a_c parameters, a restatement of the increase in the mean during the Baby Boom.

Discussion

We have described procedures for allocating the change in mean fertility into changes in the outcome itself—that is, in the proportions of women who terminate at specific parities or who ever pass through specific parities. The decompositions rest on two equivalent definitions of the mean, one in terms of the proportions at each parity and the other in terms of the parity progression ratios. The first definition of the mean distinguishes women according to the end result of family formation. If completed parity is the result of a planned and controlled process, then this decomposition will be more appropriate. The second approach distinguishes women according to whether they made specific transitions, regardless of how much farther they subsequently proceeded. This decomposition thus focuses on the process rather than the end result and is more appropriate when decisions are sequential or when control is less effective. It is rarely possible to characterize an entire population's family formation as wholly by targets or wholly sequential, so the two decompositions are best regarded as complementary rather than as mutually exclusive.

A conclusion reached through the relational logit transformation is that a single model can link the parity distributions of successive cohorts of U.S. women. Despite large shifts and contractions in the distributions, there is a continuity that is not immediately obvious in either the proportions or the parity progression ratios. Rather than formal distributions or models that have their basis in statistical theory—for example, the Poisson, the negative binomial, and the hypergeometric (see Pittenger, 1973, for efforts to use formal distributions)—we used a relational model and an empirical standard. This integrative formulation built on the analogy between family building and progression through the familiar life table.

An earlier analysis (Pullum, 1980) took a more conventional data base (although similarly limited to fertility outcomes), namely the array of age-specific fertility rates from 1920 to 1970. The goal of that analysis was specifically to distinguish the roles of age, period, and cohort in accounting for variations in age-specific rates. That analysis found that age was by far the most important source of variation, followed by period-related influences, and finally by cohort-related influences. The complementary analysis given here suppresses short-term period variation and better reveals the continuity from one cohort to the next. The cohort data imply that period-related factors will influence the *timing* of childbearing but do not dominate or drive the process by which a final completed parity is achieved.

The methods presented here can be applied to successive cross-sections, such as the distributions of children ever born in decennial censuses, and to the parity distributions at successive stages in the life course of a specific cohort. In those applications, some or all of the women appearing in one distribution will also appear in another one. The methods can also be applied to cohorts or cross-sections that are differentiated by covariates such as type of place of residence, in an effort to identify the sources of differences in means. It is thus possible to describe the roles of different parities in accounting for subgroup differentials in fertility—in much the same way that differentials can be attributed to different levels of the proximate determinants with Bongaarts's decomposition (1978).

In work not presented here, we have applied the relational logit model to sets of parity distributions from Asia and Latin America to generate synthetic distributions that characterize those regions of the world. It is similarly possible to synthesize parity distributions arising internationally in settings of low control, fertility transition, and high control to characterize changes in the distribution beyond the decline in its mean, with the same objectives as David et al. (1988).

Because the chain rule of differentiation can be extended to higher order differentials, it is possible to develop formulas to decompose change in the second moment and in the variance of the parity distribution (as well as higher moments). Since a change that increases the mean will generally serve, however, to increase the second moment and also the variance, little appears to be gained from taking such a step; most of the information is contained in the decomposition of the mean.

Notes

[1] There are other problems with the Cutright and Shorter (1979) decomposition, particularly involving the treatment of an interaction term. The first author will provide a more detailed evaluation on request.

[2] The conventional form of the logit transformation in the statistical literature, and the one used here, is

$$\text{logit } l_x = \log[l_x/(1 - l_x)].$$

Following Brass (1975), however, demographers have often defined the logit of the survival function by

$$\text{logit } l_x = \tfrac{1}{2} \log[(1 - l_x)/l_x].$$

This definition can be obtained from the other form after multiplication by $-\tfrac{1}{2}$.

[3] The published parity distributions end with an open-ended category (7+). By using the mean completed parity, we are able to infer the mean parity within the open-ended category. For the calculation of the variance and standard deviation, it was arbitrarily assumed that all women in the open-ended category were concentrated at the value thus computed.

[4] Although it would be desirable to articulate changes in fertility with changes in the proximate determinants, the picture would not necessarily be clarified by doing so. For example, changes in nuptiality should have their greatest impact on P_0, the rate of parity progression out of the childless state. Yet during the interval of decline in the mean, P_0 showed virtually no change, despite increases in marital exposure. During the period of increase in the mean, contraception was becoming progressively more widely known and available. These examples illustrate that trends in the most important proximate determinants were often contrary to trends in completed fertility.

A factor that could potentially account for changes in fertility is the underlying fecundity of each cohort, which would be affected in turn by the health of women and their partners. Most of the evidence of primary or secondary sterility has pertained to the black population (e.g., see Farley, 1970;

McFalls, 1973; Tolnay, 1981). White life expectancy steadily increased and other health indicators improved during the entire interval of analysis, with major gains through the decades of high childlessness and long before the cohorts of Baby Boom mothers. The long-term trend in this variable also does not parallel the long-term trends in fertility.

[5] The parameters of the relational logit model were estimated by an iterative procedure equivalent to that used in a different context by Stoto (1982). On request, the first author will provide details and a copy of the computer program.

[6] One must be cautious in the interpretation of specific values of a_c and b_c (as well as d_i^*) because of the underidentification problem. The *ratio* of successive b_c's is invariant with respect to alternative parameterizations; the monotonic increase of about 7 percent that continued into the Baby Boom is thus not a consequence of how one deals with underidentification. No equally simple function of the a_c's is invariant, but the concave pattern of these estimates is observed under any plausible parameterizations.

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