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Population Dynamics Based on Birth Intervals and Parity Progression*

G. FEENEY†

The Chinese population policy of 'later–longer–fewer' enjoins women to marry later, to wait longer before having a first or a next child, and to have fewer children altogether.[1] It is intuitively clear that specific stipulations along these lines will determine the course of population growth. If we can specify how many children each woman can have and when she will have them, the numbers of births in every future period are evidently determined. But how specifically do birth-interval distributions – which are the formal demographic embodiment of the 'later–longer' – and parity progression ratios – which represent the 'fewer' – translate into population growth?

The parity progression model developed in this paper enables us to calculate the future population that will result from the operation of a given set of parity progression ratios and birth-interval distributions on an initial series of births distributed by order, just as the conventional method of population projection allows us to calculate the future population that will result from the operation of given age-schedules of fertility and mortality on an initial age distribution.

Parity progression ratios and birth interval[2] distributions are here defined for parity

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[1] For general discussions of China's population policies see C. H. Tuan, 'China's population in perspective', pp. 69–83 in H. Brown (ed.), *China Among the Nations of the Pacific* (Westview Press, Boulder, Colorado, 1982); and P. C. Chen, 'Population and birth planning in the People's Republic of China', *Population Reports*, series J, no. 25, January–February 1982 (Population Information Program, The Johns Hopkins University, Baltimore, Maryland).

[2] This is in contrast to the conventional approach to birth-interval analysis, in which birth-interval distributions are defined for birth or marriage cohorts. See, for example, Mindel C. Sheps, 'An analysis of reproductive patterns in an American isolate', *Population Studies* **19** (1965), pp. 65–80; Chapter 9 of Mindel C. Sheps and Jane A. Menken, *Mathematical Models of Conception and Birth* (University of Chicago Press, Chicago, 1973); and J. A. Ross and Shanta Madhavan, 'A Gompertz model for birth interval analysis', *Population Studies* **35** (1981), pp. 439–454. Parity cohorts are by no means new, however. They were explicitly introduced by L. Henry, *Fertility of Marriages: A New Method of Measurement*, Population Studies Translation Series, no. 3 (United Nations, 1980; French edition published 1953), and have been noted by W. K. Poole, 'Fertility measures based on birth interval data', *Theoretical Population Biology* **4** (1973), pp. 357–387, N. B. Ryder, 'Fertility measurement through cross-sectional surveys', *Social Forces* **54** (1975), pp. 7–35, and G. J. Wunsch and M. G. Termote, *Introduction to Demographic Analysis* (Plenum Press, New York, 1978), Section 5.4. For an empirical analysis in which parity cohorts are used see R. R. Rindfuss, L. L. Bumpass, J. A. Palmore and Dae Woo Han, 'The transformation of Korean child-spacing practices', *Population Studies* **36** (1982), pp. 87–104, and L. Bumpass, R. R. Rindfuss, J. A. Palmore, Mercedes Concepcion and B. M. Choi, 'Intermediate variables and educational differentials: fertility in Korea and the Philippines', *Demography* **19** (1982), pp. 241–260.

cohorts – groups of women who have a birth of a given order during a given period. We regard the proportion of women in a birth cohort who become mothers as a parity progression ratio of order zero and the distribution of mothers in the cohort by age at first birth as a birth interval distribution. It is convenient to combine the parity progression ratio and the birth-interval distribution for each birth order into a single schedule, which we refer to as the *parity progression schedule*.

Once parity progression dynamics have been formulated in this way it is natural to ask what would happen in a closed population in which parity progression schedules remain constant over a long period. In asking this question we expect results analogous to those of Lotka's stable population theory, and they are, indeed, forthcoming. We prove convergence to a stable growth rate and birth-order distribution and give formulae for calculating these quantities from the parity progression schedules. The result is an alternative to Lotka's theory, in which age has been replaced by parity and interval since previous birth.

We next compare parity progression dynamics with those of Lotka empirically for a population with low fertility (the United States) and with high fertility (Costa Rica). The two are remarkably similar, suggesting that the choice between them may be made on the basis of available data and the aims of analysis. As might be expected, the differences between the two dynamics are more pronounced for the population with high fertility.

The parity progression model embodies an approach to the measurement of fertility which is based on parity progression schedules, just as Lotka's model is based on age-specific birth rates. Parity progression schedules which incorporate parity progression rates and birth-interval distributions are arguably the most natural approach to the measurement of fertility. When people think about having children, they think in terms of whether or not and when to have a first or a subsequent birth. The aggregate results of these individual decisions are directly represented in parity progression ratios and birth-interval distributions. Age-specific birth rates, by contrast, are highly abstract even to the demographer. They are familiar, but do not relate in any obvious way to the process of childbearing. The analysis of birth-interval distributions and parity progression ratios poses what have appeared to be difficult technical problems, however, and the relation between these measures and more conventional tools of demographic analysis has been imperfectly understood.

The parity progression model makes two contributions in this context. First, we find that when birth-interval analysis is suitably formulated, the technical problems of censoring and selection turn out to have rather simple solutions by invoking standard formal demographic concepts. The essential tool is the parity cohort, which plays, for populations of women of a given parity, the role played by the birth cohort in populations at large. These matters are discussed in the penultimate section of the paper. Secondly, the parity progression model provides a framework for integrating micro-level birth-interval analyses into macro-level studies of fertility and population growth trends. There has been a tendency in birth-interval analysis to look at what happens within birth intervals. The parity progression model shows how to look in the other direction, the way in which birth intervals affect aggregate fertility and trends in population growth.

THE PARITY PROGRESSION MODEL

The essential idea of the parity progression model is that total $(i+1)$th births in a closed population during any short period may be expressed as the sum of the $(i+1)$th births contributed by the cohorts of women who had had an ith birth previously. Thus we write

$$B_{i+1}(t) = \int B_i(t-a)\,\psi_i(a)\,\mathrm{d}a \quad (i = 0, 1, 2, \ldots). \tag{1}$$

where $B_i(t)\,dt$ denotes the number births of order i between time t and time $t+dt$ and $\psi_i(a)\,da$ denotes the proportion who have an $(i+1)$th birth between a and $a+da$ years, after their ith birth among all women who had an ith birth previously. We refer to $\psi_i(a)$ as *parity progression schedules.*

Equation (1) is analogous to Lotka's integral equation,[3] with total births disaggregated by order, the parity progression schedules $\psi_i(a)$ replacing the net maternity function $\phi(a)$, and length of interval since the ith birth replacing age. The equation has a meaning for $i = 0$ if $B_0(t)\,dt$ is taken to denote total births between time t and time $t+dt$ and $\psi_0(a)\,da$ the proportion of all persons born during any given period who have a first birth between age a and age $a+da$. Thus $\psi_0(a)$ is a net maternity function for first births. Note, however, that since the model is defined for the total and not the female population the denominators in the expression for $\psi_0(a)$ are total and not female births.

In this way the distribution of births by order at time t is expressed in terms of the distributions of births by order at previous times and the parity progression schedules $\psi_i(a)$, just as in Lotka's model total births at time t are expressed in terms of total births at previous times and the net maternity schedule $\phi(a)$. We refer to this model as the *parity progression model.*[4]

The parity progression schedules incorporate both the parity progression ratios and the birth interval distributions. The parity progression ratio of order i is $\int_0^{30} \psi_i(a)\,da$ and the corresponding birth interval distribution is $\psi_i(a)$ divided by this integral.

Lotka's model is conventionally applied to female births only. The parity progession model can evidently be applied only to the total population, both because birth order is usually defined in terms of numbers of births independent of sex, and because distributions of intervals between successive births independent of sex are a more useful index of fertility.

It is useful to have a discrete as well as a continuous version of the parity progression model. For this purpose let $B_i(y)$ denote the number of births of order i in year y and let $\psi_i(a)$ denote the proportion who have an $(i+1)$st birth a years later among all women who have had an ith birth during a given year, and where y and a assume only integral values. Then we may write

$$B_{i+1}(y) = \sum_{a>0} B_i(y-a)\,\psi_i(a) \quad (i = 0, 1, 2, \ldots). \tag{2}$$

This equation expresses $(i+1)$th births in year y in terms of ith births in year y and preceding years and the parity progression schedules, now in discrete form, $\psi_i(a)$.[5]

Equations (1) and (2) both define cohort formulations of the parity progression model. The model may also be formulated in period terms, with the initial birth-order series replaced by a table of women distributed by parity and length of interval since previous birth (in completed years) and the parity progression schedules transformed into fertility rates specific for parity and birth interval. This period formulation of the model bears the same relation to the cohort formulation that Leslie's matrix formulation of stable

[3] See, for example Chapter 5, and in particular Formula (5.1.2), p. 99, and the following text, in N. Keyfitz, *Introduction to the Mathematics of Population* (Addison-Wesley, Reading, Massachusetts, 1968).

[4] This model has been presented in two earlier versions of this paper, 'Population projection based on time series of births distributed by order', presented to the 1981 Annual Meetings of the Population Association of America, Washington, D.C., and 'Population dynamics based on birth intervals and parity progression', Paper contributed to the 1981 General Conference of the International Union for the Scientific Study of Population, Manila.

[5] On the assumption that births of each order are uniformly distributed during calendar years, the discrete parity progression schedule value $\psi_i(a)$ equals the integral between a and $a+1$ of the corresponding continuous schedule. Normal seasonality of births does not significantly violate this assumption. Similar statistics are discussed in Section 5.4, p. 168, Wunsch and Termote, *op. cit.*, in footnote 2.

population theory (or, equivalently, conventional component population projection) bears to Lotka's integral equation.[6]

POPULATION PROJECTION BASED ON THE PARITY PROGRESSION MODEL

The parity progression model allows us to calculate the future population that will result from the operation of a given set of parity progression schedules on an initial time series of births distributed by order. Given a closed population for which numbers of births by order are available (from vital registration, for example) for a series of years ending in year $y-1$, and given a set of parity progression schedules $\psi_i(a)$, we may calculate first births in year y from Equation (2), then second births, and so on. Total births may then be calculated by summing births of all orders. This will yield a projected distribution of births by order for year y. The same process may now be repeated to obtain projected births by order for year $y+1$, and so on for any desired number of years into the future.

Note that since a woman may have both her *ith* and her $(i+1)$th birth in the same year, either because of twinning or because of an exceptionally short birth interval, the calculation of $(i+1)$th births in year y involves ith births in the same year as well as ith births in preceding years. Thus, in applying Equation (2), first births in year y must be calculated first, then second births, and so on.

For some purposes it will be more useful to cast the projection in period terms. The initial series of births of order i is replaced by a distribution of women of parity i by completed years since their last birth. These initial distributions may be obtained from survey data, or constructed from time series of births distributed by order and corresponding birth interval distributions. In the latter case the process is analogous to constructing an age distribution from a time series of births and corresponding life tables. Distributions of women of each parity by completed years since their last birth are then projected forward in the same manner as age distributions in conventional component projection, using parity progression ratios, analogously to survivorship ratios, derived from the parity progression functions.

STABILITY THEORY FOR THE PARITY PROGRESSION MODEL

Following the idea of Lotka's stable population theory, we ask what will happen in a population in which parity progression schedules remain constant over a long period of time. What will happen if the projection procedure of the last section is carried forward for, say, a century, assuming an arbitrary initial distribution of births by order and constant values of $\psi_i(a)$? We shall see that analogues of all the familiar results from stable population theory may be derived.

Since second births during the current period may be expressed in terms of first births during the current and previous periods, and since these first births may in turn be expressed in terms of total births in previous periods, second births in the current period may evidently be expressed directly in terms of total births in previous periods. Thus, from Equation (2) we obtain

$$\begin{aligned} B_2(y) &= B_1(y)\,\psi_1(0)+B_1(y-1)\,\psi_1(1)+\dots \\ &= [B_0(y)\,\psi_0(0)+B_0(y-1)\,\psi_0(1)+\dots]\,\psi_1(0) \\ &\quad +[B_0(y-1)\,\psi_0(0)+B_0(y-2)\,\psi_0(1)+\dots]\,\psi_1(1) \\ &\quad +[B_0(y-2)\,\psi_0(0)+B_0(y-3)\,\psi_0(1)+\dots]\,\psi_2(2)+\dots \end{aligned}$$

[6] See, for example, Chapter 2, pp. 27–32, of Keyfitz, *op. cit.*, in footnote 3.

or, more compactly

$$B_2(y) = \sum_{a \geqslant 0} B_0(y-a)\,\phi_2(a),$$

where

$$\phi_2(a) = \sum_{j=0}^{a} \psi_0(a-j)\,\psi_1(j).$$

More generally we find that

$$B_i(y) = \sum_{a \geqslant 0} B_0(y-a)\,\phi_i(a), \tag{3}$$

where $\phi_1(a) = \psi_0(a)$ and

$$\phi_{i+1}(a) = \sum_{j=0}^{a} \phi_i(a-j)\,\psi_i(j). \tag{4}$$

Since total births are the sum of births of all orders, we may write

$$B(y) = \sum_{a \geqslant 0} B(y-a)\,\phi(a), \tag{5}$$

where the zero subscript on B is dropped and $\phi(a)$ denotes the sum over i of the $\phi_i(a)$.

We have thus shown that in the long run, after parity progression schedules have been constant for a sufficient number of years, parity progression dynamics are equivalent to those in Lotka's system with the net maternity function defined by Equation (4). The number of years required for equivalence is determined by how many years we have to go back to go exhaust the birth cohorts that contribute births to the current period – approximately 50 years.

The proof of convergence to stability for the parity progression model is now reduced to the proof for Lotka's model. Given any initial series of births by order and any set of parity progression schedules, we may execute the dynamics defined by Equation (2) until Lotka's equation (5) applies. Convergence will then occur subject to the usual conditions on the net maternity function.

The simplest condition for convergence, and one that is sufficient for demographic purposes, is that the net maternity function $\phi(a)$ should have at least two consecutive positive values.[7] In view of the way in which $\phi(a)$ is calculated from the parity progression schedules $\psi_i(a)$, it suffices that any one of these parity progression schedules should have at least two consecutive positive values. This will, of course, always be true in demographic applications.

To calculate the stable growth rate r corresponding to a given set of parity progression schedules $\psi_i(a)$ we first calculate the corresponding values of $\phi(a)$ in (5) and then solve the familiar characteristic equation

$$1 = \sum_{a \geqslant 0} \phi(a)\,e^{-ra} \tag{6}$$

by the usual methods.[8]

To obtain a formula for the stable birth-order distribution, we substitute $B_i e^{ry}$ for $B_i(y)$ in Equation (2), extract the factor $B_i e^{ry}$ from under the summation, and transfer it to the left side of the equation, giving

$$\frac{B_{i+1}}{B_i} = \sum_{a \geqslant 0} e^{-ra}\psi_i(a) \quad (\mathrm{i} = 0, 1, 2, \ldots). \tag{7}$$

[7] W. B. Arthur, 'Why a population converges to stability', *American Mathematical Monthly*, **88** (1981), pp. 557–563.

[8] See Keyfitz, *op. cit.*, in footnote 3, chapter 5, section 3, pp. 108–114. I have found the method of function iteration very effective. Calculation of $\phi(a)$ from $\psi_i(a)$ values is, of course, very tedious and is best done by computer.

Denoting the sum on the right by $\Psi_i(r)$ and taking cumulative products gives the stable birth-order distribution,

$$\frac{B_i}{B_0} = \prod_{S=0}^{i-1} \Psi_S(r) \quad (i = 1, 2, \ldots). \tag{8}$$

Note that since the value of $\Psi_0(r)$ here is a constant factor in terms that must sum to unity, it is not necessary to know its value for numerical calculation. To obtain the stable birth-order distribution we may put $\Psi_0(r)$ equal to any value (unity is most convenient), calculate the values (8), and then normalize, i.e. divide each value by the sum of all the values.

It is interesting to note that when $r = 0$, the case of a stationary population, $\Psi_i(r)$ reduces to the sum of the $\psi_i(a)$ values, i.e. to the parity progression ratio. The stationary proportion of ith births is thus the product of the first i parity progression ratios, and this gives the proportion of persons in any birth cohort who have at least i children. Thus, in a stationary population, the proportion of ith births during any period equals the proportion of persons in any birth cohort who have i or more children.

Preston has pointed out that the summation on the right in Equation (7) may be approximated by $a_i e^{-rM(i)}$, where a_i denotes the parity progression ratio and $M(i)$ denotes the mean birth interval. The approximation is extremely accurate, and the equation provides a useful statement of the relationship between parity progression ratios, birth-order distributions and the growth rate in a stable population. Preston has proposed using it to estimate fertility from incomplete statistics on birth order.[9]

NUMERICAL COMPARISON WITH LOTKA'S MODEL

It is intuitively clear that the parity progression model and Lotka's model define different dynamics in the short run. Imagine an hypothetical population with several recent relatively large birth cohorts. Under Lotka's dynamics, the impact of these extra births will not become apparent for about 15 years, the time it takes to enter reproductive age. Under parity progression dynamics, however, the impact will be felt immediately, as additional births of each order generate additional births of the next higher order. At the same time, the parity progression model incorporates the delay between a woman's own birth and the birth of her first child, and we, therefore, expect generational waves similar to those observed in Lotka's model. We thus have reasons for believing both that the growth trends produced by the two models will be similar and that they will be different. The purpose of the numerical comparison is to give a quantitative indication of the difference between the two models.

A comparison for the United States is shown in Table 1. The input for the calculation of Lotka's model consists of annual births during 1909–70 and 1970 births by single year of age of mother.[10] Net maternity values $\phi(a)$ were obtained by dividing the average number of births in 1970 to women aged $a-1$ and a by total births in year $1970-a$ and multiplying the resulting values by a constant factor chosen so that projected births for 1970 equal actual births for that year.[11] Projected births for 1971–2071 were then calculated from Equation (5).

[9] Personal communication dated 2 June 1981.

[10] Annual births for 1909–70 from Table B 1–4, p. 49 of U.S. Bureau of the Census, *Historical Statistics of the United States: Colonial Times to 1970*, Part 1 (Washington, D.C.: Government Printing Office, 1975); numbers of births for 1909–58 adjusted for under-registration. 1970 births by single year of age from table 1–55, p. 173 of U.S. National Center for Health Statistics, *Vital Statistics of the United States: 1970*, vol. 1, *Natality* (Washington, D.C.: Government Printing Office).

[11] $\phi(a)$ is defined as the number of births in year $y+a$ to women born in year y, divided by the number of persons (not women) born in year y. Taking $y+a$ to be the current year, this is equivalent to the number

The input for the parity progression calculation consists of annual numbers of births for 1909–70, annual numbers of births by order for 1960–70, 1970 first births by single year of age of mother, and birth interval data from the United States 1970 National Fertility Survey (NFS).[12] Values of $\psi_0(a)$ were calculated in the same manner as $\phi(a)$ in Lotka's model.[13] Values of $\psi_1(a)$ were calculated directly from the NFS data.[14] Values of $\psi_i(a)$ for $i > 1$ were calculated from registered births by order on the assumption that $\psi_i(a) = k_i \psi_1(a)$, a crude assumption empirically, but adequate for present purposes.[15] Projected births for 1971–2171 were then calculated from Equation (2), and the stable growth rate obtained by solving the characteristic equation (6), having first calculated $\phi(a)$ from formula (4).[16]

To examine convergence to stability we use the concept of the 'growth corrected' birth sequence recently introduced by Arthur.[17] The growth-corrected birth sequence is defined as $B(y)\,e^{-ry}$, where $B(y)$ denotes the projected birth sequence and r denotes the stable growth rate. Since $B(y)$ approaches $B(y)\,e^{ry}$ asymptotically as y increases, the growth-corrected birth sequence converges to a constant value. The fluctuations that occur during convergence are easier to see in the growth corrected sequence because they have been isolated from the underlying exponential growth.[18]

The results of the comparison are shown in Table 1, with the results for Lokta's model in the upper panel of the table and the parity progression results in the lower panel. The first four columns of the table show the peaks and troughs of the growth-corrected projected births. We thus see that for Lotka's model these numbers of births rise from 3,731,000 in 1970 to 4,360,000 in 1983, then decline to 3,939,000 in 1995, and so on. The

of births occurring during the current year to women who reached exact age a during the year, divided by the number of persons born a years ago. The number of births in the current year to women who reached exact age a during the year may be approximated by the average number of births during the year to women aged $a-1$ and a in completed years at the time of birth. (To clarify this, draw a Lexis diagram.) It was not considered worthwhile to take account of migration in this calculation. The resulting values of 10,000 $\psi(\alpha)$ are, beginning with $a = 15$: 51, 126, 243, 395, 552, 675, 777, 824, 873, 764, 733, 831, 770, 645, 546, 497, 435, 379, 312, 270, 240, 199, 163, 145, 114, 88, 63, 44, 29, 16, 9, 5. This gives a net reproduction rate of 1.1813 and a mean age at childbirth of 26.04.

[12] Annual births as in footnote 10. Births by order (1, 2, ..., 7, 8+; not stated values pro-rated) and 1970 first births by single year of age of mother from annual vital statistics publications. The 1970 National Fertility Survey is described in C. F. Westoff and N. B. Ryder, *The Contraceptive Revolution* (Princeton, New Jersey: Princeton University Press, 1977), chapter 1.

[13] The values (multiplied by 10,000) are, beginning at age 15: 47, 114, 209, 314, 408, 447, 443, 406, 371, 280, 227, 220, 171, 121, 85, 65, 49, 38, 29, 24, 20, 15, 11, 9, 6, 4, 4. The mean age at first birth is 22.4 years, the proportion of persons who become mothers 0.4137. Assuming a sex ratio of birth of 1.06, the proportion of women who become mothers is 2.06 times this value, or about 85 per cent.

[14] By dividing second births in 1970 to women in the survey, who have lived for a complete years after their first birth at the beginning of the year by first births during year $1970 - a$, $a = 0, 1, \ldots$ To smooth out random variation, numerators and dominators were aggregated for the years 1966–70, yielding values of 10,000 $\psi,(a)$, of 4, 163, 244, 170, 96, 61, 28, 9, 15, 10, and 8, with a mean of 2.93 years. Second births occurring more than ten years after the first, about one per cent of all births according to the survey, were ignored. These values of $\psi(a)$ may be checked for consistency with vital registration data by applying them to registered first births in 1970, 1969,... to yield projected second births in 1970, using Equation (2) with $i = 1$. This calculation yields 1,027,000 first births, as compared with 1,025,000 registered first births.

[15] The implied parity progression ratios, beginning with progression from first to second birth, are 0.8063, 0.6236, 0.5251, 0.4704, 0.4427, 0.4384 and 1.1224. The last value is, of course, impossible and reflects treating eighth- and higher-order births as though they were eighth births. In view of the relatively small proportion of eighth- and higher-order births (about two per cent) in 1970, it was not considered worth while to elaborate the calculation to eliminate this inconsistency.

[16] The values of 10,000 $\phi(a)$ beginning at age 15, are: 47, 122, 241, 391, 553, 679, 767, 813, 841, 793, 755, 740, 674, 601, 529, 466, 405, 351, 300, 258, 221, 186, 157, 132, 110, 91, 76, 60, 49, 40, 32, 26, 20, 16, 13, 10, 8, 6, 5, 4, 3, 2, 2, 1, 1, 1. The mean age of childbearing is 26.3 and the net reproduction rate is 1.1598.

[17] See section 3 of Arthur, *loc. cit.*, in footnote 7.

[18] The analysis given here is similar in spirit to but technically simpler than that given in Chapter 3, 'Convergence of a population to the stable form', in A. J. Coale, *The Growth and Structure of Human Populations* (Princeton, New Jersey: Princeton University Press, 1972).

Table 1. *Numerical comparison of parity progression and Lotka dynamics for the United States, 1970*

Cycle years		Growth corrected births		Peak-trough difference		Contraction factor	Length of cycle
Peak	Trough	Peak	Trough	Number	%		
Lotka model (Initial births = 3,731; final births = 4,101; stable growth rate = 0.62%)							
1,983	1,995	4,360	3,939	421	10.1	—	26
2,009	2,022	4,196	4,042	154	3.7	0.37	26
2,035	2,048	4,135	4,080	55	1.3	0.36	26
2,061	2,074	4,113	4,093	20	0.5	0.36	—
Parity progression model (Initial births = 3,731; final births = 4,080; stable growth rate = 0.57%)							
1,982	1,994	4,303	3,950	353	8.6	—	24
2,006	2,019	4,153	4,037	116	2.8	0.33	26
2,032	2,046	4,105	4,066	39	1.0	0.34	25
2,057	2,068	4,088	4,076	12	0.3	0.31	—

Note: see text for explanation.

next two columns show the difference between the highest and lowest values, a measure of the amplitude of the cycles, both in absolute terms and relatively to their average. The next-to-last column shows the ratio of the differences between the highest and lowest values in successive cycles, a measure (in combination with the cycle length) of the speed of convergence. The last column shows the length of the cycles measured from peak to peak.

The results of the two models differ very little.The cycles in the parity progression model are slightly shorter, with slightly smaller amplitudes and a slightly smaller contraction factor, all indicating a slightly more rapid convergence to stability. The parity progression model yields also a slightly lower stable growth rate and a slightly lower ultimate value of growth-corrected births. But these are very fine differences. The overall picture provided by the two models is essentially the same.

This similarity between Lotka's model and the parity progression model in this case is not surprising, for the models are the same if all births are first births, and when fertility is low first births account for a large proportion of total births. This observation leads us to suspect that the difference between the two models will be greater for populations of high fertility, however, and we, therefore, proceed to another example.

In Table 2 we show a comparison for Costa Rica in 1963. Essentially the same procedure was followed here as in the previous example. Some estimations and interpolations were required to complete the input vital registration data, however, and as no information on birth intervals was available, statistics for Colombia were used.[19]

[19] Data on births by order are given in the United Nations *Demographic Yearbooks*, as follows: 1959, Table 15, p. 319, for 1953–4; 1965, Table 16, p. 440, for 1955–63. These tables terminate with the open birth-order group 10+ —much too low for so high a fertility population. Numbers in the 10+ group were distributed among the categories 10, 11, ..., 14, 15+ according to the observed distribution in 1964, 2,084, 1,538, 1,185, 797, 532, and 802, respectively (Costa Rica, Departmento Estadisticos Sociales, *Estadistica Vital–1964*, Table 16, p. 28). Total births for the years before 1953 were obtained by a crude extrapolation of births between 1953 and 1963, with births in year $1953-y$ estimated as $37{,}000 \times e^{-0.0359Y}$. Total and first births for 1963, available only by five-year groups, were interpolated to single years by using the Gompertz relational model described in W. Brass, 'The use of the Gompertz relational model to estimate fertility', *International Population Conference: Manila, 1981*, vol. 3 (International Union for the Scientific Study of Population), pp. 345–362. The procedure consists of cumulating, normalizing and transforming the births in five-year age groups to give values $Y(x) = -\log_e[\log_e B(x)/B(50)]$, for $x = 20, 25, \ldots, 45$, where $B(x)$ denotes cumulative births by exact age x. Values of $y(x)$ for $x = 16, 17, \ldots, 49$ are then obtained by linear interpolation against the standard values

Table 2. *Numerical comparison of parity progression and Lotka dynamics for Costa Rica, 1963*

Cycle years		Growth-corrected births		Peak-trough difference		Contraction factor	Length of cycle
Trough	Peak	Trough	Peak	Number	%		
Lotka model							
(Initial births = 63,638; final births = 60,460; stable growth rate = 3.66%)							
1,973	1,985	58,335	61,806	3,471	5.8	—	26
1,999	2,012	59,750	60,859	1,109	1.8	0.32	26
2,025	2,038	60,236	60,853	617	1.0	0.31	27
2,052	2,065	60,391	60,499	108	0.2	0.31	—
Parity progression model							
(Initial births = 63,638; final births = 60,050; stable growth rate = 3.40%)							
1,975	1,987	58,344	60,775	2,431	4.1	—	25
2,000	2,013	59,777	60,190	413	0.7	0.17	26
2,026	2,038	60,012	60,084	72	0.1	0.17	26
2,052	2,064	60,047	60,059	12	0.0	0.17	—

Note: see text for explanation.

Regarded as a description of the demography of Costa Rica, the results could no doubt be improved upon, but refinements are not necessary to provide a reasonable example of a population of high fertility for which to compare Lotka's and parity progression models.

The second comparison differs from the first in one striking particular only – the much more rapid speed of convergence indicated for the parity progression model by the contraction factors. The explanation for this divergence lies in the distinction made in the parity progression model between first births and those of higher orders. In Lotka's model, current total births fluctuate in response to fluctuations in total births in previous years. The fluctuation is governed by the net maternity function and consists of cycles with period approximately equal to the mean of this function. In the parity progression model, first births in the current period fluctuate in response to fluctuations in total births in previous years, and this introduces cycles similar to those of Lotka's model, but with a slightly shorter period. But births of second and higher orders fluctuate in response to fluctuations in births of the next lower order. Fluctuations in the first births are thus smoothed out as they are transmitted to births of higher order, with the result that fluctuations in total births are attenuated.

IMPLICATIONS FOR BIRTH-INTERVAL ANALYSIS

The birth-interval distributions and parity progression ratios that occur in the parity progression model are defined for parity cohorts, i.e. groups of women who have a birth of a given order during a given period. This is in contrast to the conventional approach, in which they are defined for birth or marriage cohorts. In this section we point to some advantages of parity cohorts for birth-interval analysis, particularly in connection with censoring and selection problems.

The significance of the parity cohort concept is perhaps best indicated by observing that such cohorts are analogous to birth cohorts. Given any population, e.g. the

and the cumulation–normalization–transformation process reversed to give values of $B(x)$ for single years (note, however, that $B(15) = 0$ and $B(50) = 1$ are given). Differencing these then gives births by single years. Values of $\psi(a)$ were estimated from the Colombia data given in Table 3.1, p. 46, in Rodriguez and Hobcraft, *loc. cit.*, in footnote 24.

population of the United States, the sub-population of women of parity i may be regarded as a population in its own right. Familiar formal demographic concepts are then seen to have analogues in that population. Women enter the population by having an ith birth, and this is analogous to birth in an ordinary population. They leave the population by having an $(i+1)$th birth, and this is analogous to death.[20] Parity cohorts, are thus analogous to birth cohorts, and interval since the ith birth is analogous to age. Such familiar formal demographic concepts as the demographic equation, life tables, stationary populations and Lexis diagrams may all be applied to the population of women of parity i.

We may thus consider the analysis of parity-specific birth intervals exactly as though it were an analysis of mortality, with the transition from ith to $(i+1)$th birth regarded as formally identical to the transition from birth to death. The practical implications of this idea for birth-interval analysis are reasonably straightforward. The essential idea is to take the parity cohort as defining the basic groups of women (or, equivalently, of birth intervals) to which life-table calculations are applied. What is less obvious is that this approach to birth-interval analysis effectively solves problems of censoring and selection in so far as these can be solved.

We take 'birth-interval distribution' to mean the proportions of women progressing to a next birth among *all* women in a specified group who have had a given birth, rather than only the women who go on to have a next birth.[21] Although at present most data available for birth-interval analysis come from fertility surveys, we do not assume any particular method of data collection. Vital registration systems in which the date of the previous birth is recorded on birth certificates, and prospective surveys, are also possible sources, and it is useful to distinguish problems inherent in the phenomena from problems specific to particular types of data.

Censoring problems may be illustrated with a simple example.[22] Consider two birth-interval distributions that could be calculated from fertility surveys: the distributions

[20] Women may also leave by death proper, hence parity populations are subject to attrition from two causes: progression to next birth and death. The application of multiple decrement theory to this situation is developed in G. Feeney and J. A. Ross, 'Analyzing open birth interval distributions', Paper presented to the 1982 annual meetings of the Population Association of America, San Diego.

[21] There is, remarkably, no standard terminology. The phrase 'birth interval distribution', for example, appears in none of the following sources on demographic terminology (page numbers indicate where an entry would appear): Demographic Dictionary Committee of the International Union for the Scientific Study of Population (IUSSP), *Multilingual Demographic Dictionary*, English section, Population Studies, no. 29 (United Nations Department of Economic and Social Affairs, 1958), p. 37; C. Lucas and M. Osburn, *Population/Family Planning Thesaurus: An Alphabetical and Hierarchical Display of Terms Drawn from Population-Related Literature in the Social Sciences*, 2nd ed. (Carolina Population Center Library, University of North Carolina at Chapel Hill, 1978), p. 24; Committee for International Cooperation in National Research in Demography (CICRED), *Population Multilingual Thesaurus*, English edition prepared by J. Viet, 1979, p. 85; R. Pressat, *Dictionnaire de Démographie* (Paris: Presses Universitaires de France, 1979), p. 98.; and *Popline*; *Thesaurus* (published by the Population Information Program, Johns Hopkins University, and the Library/Information Program, Center for Population and Family Health, Columbia University, and authored by these organizations and *Population Index*, Office of Population Research, Princeton University), p. 13. The significance of this terminological lacuna may be more than usually important. There has been a tendency in empirical work to segregate closed birth-interval distributions from the corresponding parity progression ratios, thus directing attention away from women who do not progress to another birth. This is regrettable, for non-progression is as interesting and important an aspect of fertility as the timing of progression for those women who do progress. In the analysis of data on incomplete birth histories, as for example from any retrospective fertility survey, the segregation of closed birth intervals is a serious technical blunder. See Rindfuss, Palmore and Bumpass, *loc. cit.*, in footnote 29.

[22] Censoring refers in this context to data sets in which all women are censored at a single point in time, as in a retrospective fertility survey. This must be distinguished from the censorship that may occur in prospectively gathered data, in which different women may be lost to observation at different times. The terms 'singly censored' and 'progressively censored' have been used to designate data subject to these two types of censoring. See E. T. Lee, *Statistical Methods for Survival Data Analysis*, (Belmont, California: Lifetime Learning Publications, 1980), pp. 2–3, who cites A. C. Cohen, 'Maximum likelihood estimation in the Weibull distribution based on complete and censored samples', *Technometrics* 7 (1965), pp. 579–588.

of intervals between first and second births for women aged 15–19 and 45–49 respectively at the time of the survey. Suppose further that the mean interval for the younger women is shorter. This cannot be interpreted as an effect of age ('younger women give birth at shorter intervals'), or time ('intervals have been getting shorter'), because it may reflect nothing more than that the younger women have had less time to have a second birth.

The essential problem here is failure to control for time of first birth. Women who had their first birth x years ago have only been exposed to the risk of a second birth for x years; their experience at higher durations, while having one child only, is 'censored'. Unless x is sufficiently large to lead us to assume that there will be no more second births, we must consider *all* women who had a first birth x years ago, not only those who had a second birth by the time of the survey, and we must not attempt to calculate any statistic that depends on the incidence of second birth at intervals exceeding x years from the first birth. This rules out, in particular, the calculation of a mean birth interval for these women, though, of course, a truncated mean for intervals of less than x years duration may be calculated.

Observe the analogy with mortality.[23] We may calculate a life table formally for any group of deceased persons whatever, but the result makes sense only if the group is suitably specified. To calculate a distribution of intervals between first and second births (or successive births of higher orders) for women currently aged 15–19 is like calculating a life table for persons who were born within the past five years and who are now dead. We know this to be nonsense. The expectation of life at birth, for example, must necessarily be less than five years in this case. We can calculate useful mortality statistics from information on persons born during the past five years, but only by considering those who survive as well as those who die, and only for the incidence of mortality during the first five years of life.

The analogy with mortality shows that what is at first sight a new and potentially difficult problem is in fact an old and thoroughly understood one. If we want to calculate a distribution of intervals between first and second births for women currently aged 15–19 we must disaggregate them according to time elapsed since first birth, i.e. by parity cohort. For the cohort of women who had their first birth x years ago, or from any earlier cohort, we can calculate parity progression ratios for single years of intervals up to x years. If we wish to obtain a single summary birth-interval distribution, it must be a synthetic one obtained by merging the information for several parity cohorts, as information for different birth cohorts is merged in the construction of a period life table.[24] It will still be impossible to obtain any information on the incidence of progression to second birth beyond the longest interval since first birth represented in the data, however. If we assume no birth to take place to women below age fifteen, for example, women currently aged 15–19 cannot provide any information on the incidence of progression to second birth for intervals exceeding five years from the first birth.

Several kinds of selection problems have been identified in birth-interval analysis. One is concerned with the ascertainment of birth intervals. Thus, James notes that 'studies such as that of Whitelaw which ascertain a representative sample of pregnancies *ipso facto* secure a biased sample of women' because women with relatively high fecundity will tend to have more pregnancies and hence a greater chance of being represented in the sample.[25]

This problem does not arise with the use of parity cohorts because there is a one–one

[23] This seems to have been first pointed out by Henry, *op. cit.*, in footnote 2, pp. 10–14.

[24] This was clearly recognized by Henry, *op. cit.*, in footnote 2, p. 13. For a detailed explanation see G. Rodriguez and J. N. Hobcraft, *Illustrative Analysis: Life Table Analysis of Birth Intervals in Colombia*, (World Fertility Survey, 1980), and J. Hobcraft and G. Rodriguez, 'Methodological issues in life table analysis of birth histories', Paper presented to the Seminar on the Analysis of Maternity Histories, London, April 1980.

[25] W. H. James, 'Estimates of fecundability', *Population Studies* **17** (1963), pp. 57–65. The reference is to M. J. Whitelaw, 'Statistical evaluation of female fertility', *Fertility and Sterility* **11** (1960), pp. 428–436.

correspondence between women having an *i*th birth during a given period and *i*th births during this period. Thus statistics on, say, progression to second birth for United States women who had a first birth in 1965 may be collected either retrospectively in a survey in (say) 1970 or prospectively by means of a vital registration system in which births in 1965 and subsequent years are classified by order of birth and time of (or time elapsed since) last birth. The retrospective data will, of course, exclude intervals for women in the parity cohort who died before the survey, but the distinction between 'selecting women' and 'selecting births' disappears with the introduction of parity cohorts, and with it the necessity for Wolfers's[26] distinction between 'mean birth interval (women)' and 'mean birth interval (births)'.

A second selection problem arises from the exclusion of intervals beginning after the latest point in time for which data are available.[27] Consider, for example, the comparison of intervals between first and second births for women aged 20–24 and 45–49 respectively at the time of a fertility survey, and suppose that censoring has been controlled by the life-table techniques indicated above. It remains true that the distributions for both groups of women necessarily exclude birth intervals beginning after the time of the survey. This means that the distribution for the women aged 20–24 excludes all birth intervals beginning at ages over 25, whereas the distribution for the 45–49-year-old women excludes only a portion of intervals beginning at ages over 45. Differences between the two distributions may thus be due only to this lack of comparability of the two groups with respect to age at beginning of interval.

The essential problem here is the implicit stipulation that we should be looking at the set of all birth intervals begun by women in a birth cohort. We may define selection problems generally as problems that arise when (*a*) for one reason or another we are forced to look at a sub-group of the group we should look at, and when (*b*) the way in which the members of this sub-group are chosen, or 'selected', threatens or ensures that the aggregate characteristics of the sub-group will be different from those of the group as a whole. Consideration of selection problems thus presupposes an understanding of what group should be looked at.[28] The accepted standard in the case of both mortality and (age-specific) fertility is the birth cohort, and it is natural that this standard should have been carried over into birth-interval analysis.

The parity cohort suggests an alternative standard. Instead of looking at all birth intervals of a given order begun by women in a given birth cohort, we may look at all intervals of a given order begun during a given period of time, i.e. at the intervals corresponding to a parity cohort. Parity cohorts evidently provide a reasonable standard, and they have the advantage of eliminating by definition the problem of exclusion of birth intervals which will be begun in the future.

To illustrate the use of parity cohorts in this context, suppose we are interested in the effects of age at first birth on the subsequent birth interval. Then we may consider the cohort of women who had a first birth during some past period, five to ten years preceding a survey, say, and calculate values of the parity progression function for sub-groups of this cohort defined according to age of woman at first birth. Alternatively, suppose we

[26] D. Wolfers, 'A method of analysis for contemporary birth interval data', *Contributed Papers*, Sydney IUSSP Conference, 1967, pp. 289–299, and 'Determinants of birth intervals and their means', *Population Studies* **22** (1968), pp. 253–262.

[27] Mindel C. Sheps, Jane A. Menken, Jeanne C. Ridley and J. W. Lingner, 'Birth intervals: artifact and reality', *Contributed Papers*, Sydney IUSSP Conference, 1967, pp. 857–868, and 'Truncation effect in birth interval data', *Journal of the American Statistical Association* **65** (1970), pp. 678–693.

[28] Sheps and Menken refer to this group specification as the 'sampling frame'. See Mindel C. Sheps and Jane A. Menken, 'Distribution of birth intervals according to the sampling frame', *Theoretical Population Biology* **3** (1972), pp. 1–26.

are interested in trends in birth intervals and parity progression. Then we may consider cohorts of women who had an *i*th birth during a series of periods and calculate, say, the proportion of women whose next birth occurred within five years.

A third selection effect is peculiar to parity cohort data obtained from retrospective surveys.[29] Given a fertility survey including (as is more or less standard) women aged 15–50, consider the comparison of birth intervals and parity progression for two groups of women, those who had a first birth during the fifth year preceding the survey, and those who had a first birth during the twenty-fifth year preceding to the survey. We should look at all intervals begun by women who had a first birth during either of these two years, but because the survey excludes women over 50 years old, we will in fact be looking only at the sub-group of women who were below that age at the time of the survey. This means that instead of looking at the set of all intervals begun during the fifth year before the survey, we consider only intervals begun by women under 45 years of age at the beginning of that year; and that instead of looking at all intervals begun during the twenty-fifth year before the survey, we will be looking only at intervals begun by women aged less than 25 at the beginning of that year. As before, this incomparability confounds interpretation. Suppose we find higher parity progression ratios and shorter mean intervals for the older cohort. Does this signify a trend towards lower fertility? Or does the difference merely reflect the disproportionate exclusion of birth intervals begun at older ages for the older cohort?

A natural approach to this problem is to restrict the birth intervals considered to those begun below some given age.[30] Thus, for example, if we consider only birth intervals begun at ages below 40 we may go back ten years without encountering incomparabilities. Or, if we consider only birth intervals initiated at ages less than 30, we may go back 20 years. The data will represent only a sub-set of the experience we are interested in, but a bias that changes over time and may thus introduce spurious trends, will have been eliminated. While this approach works reasonably well at low parities, the procedure tends to break down at high parities simply because the number of excluded intervals increases.

Observe the duality between these last two selection effects In both cases the selection operates on age of women who begin a birth interval. In the first case, however, the intervals excluded are those begun after the survey by the women included in the survey, whereas in the second case they are those that were begun during the 35 years before the survey by women who had reached age 50 by the time of the survey.

The selection effects due to the exclusion of older women in fertility surveys could, in principle, be eliminated simply by including these women. (The practicality of obtaining reliable birth history information from older women is, of course, another matter.) This issue has received little attention in the literature, perhaps because eliminating the upper age limit would only bring another selection effect, that due to mortality, into play. Even in a retrospective survey in which complete and accurate birth histories are obtained for all women in a population, the fertility experience of women who died before the survey, is necessarily excluded. The effects of this selection will evidently be qualitatively similar to those due to survey age-limit selection. Among the women who begin a birth interval a given number of years before the survey, older women will be more likely to have died by the time of the survey. There will thus be a selection for intervals begun at relatively young ages, and the strength of this selection will increase with the remoteness from the survey.

[29] R. R. Rindfuss, J. A. Palmore and L. L. Bumpass, 'Selectivity and the analysis of birth intervals', *Asian and Pacific Census Forum* **8**, pp. 5–10, 15–16.

[30] This is suggested by Rindfuss, Palmore and Bumpass, *loc. cit.*, in footnote 29.

CONCLUSION

The parity progression model provides a complete, formal alternative to conventional age-based approaches to the study of fertility and population growth. Birth interval distributions and parity progression ratios, combined to form what we have termed parity progression schedules, are the fundamental measures of fertility, and the model provides a method of population projection and a stable population theory. It seems likely that these tools will prove a useful complement to existing methods and may in some respects supplant them. Three particular areas may be noted: trend analysis, determinants analysis, and family planning programmes.

In the analysis of population trends, age-specific birth rates suffer from a serious defect – they may rise and fall with changes in the timing of births independently of changes in the level of fertility. This is well known, but there has been a tendency to regard it as something that must be controlled for in the interpretation of the rates. It may also be regarded as indicating a serious deficiency in age-specific rates that ought to be rectified by resort to alternative fertility measures. Parity progression schedules, which combine birth-interval distributions and parity progression ratios, are obvious candidates. The principal source of changes in the timing of fertility has been a change in age at first marriage and age at first birth. These will, of course, be reflected in the zero-order parity progression schedule which is essentially the age distribution of women who have a first birth, but they will have relatively little effect on parity progression schedules at higher orders. Thus, the schedules of progression from first to second birth and the schedules for higher-order transitions are measures of marital fertility that are essentially independent of changes in age at first birth. They may be said to control for timing effects, just as age-specific rates control for age distribution effects.

The parity progression model would seem to be particularly well suited to the analysis of the demographic impact of family-planning programmes. The focus on parity instead of age is natural, both because the number of children a women has is more important than the age at which she has them, and because the immediate behavioural purpose of contraception is to defer or prevent a next birth. In so far as the goal of family-planning programmes is to reduce fertility, they must do so by bringing down the values of the parity progression schedules. Setting targets for ultimate parity progression for each parity thus provides a natural planning tool for operations and a standard for evaluation of impact. This is precisely the course the Chinese have adopted, both in the 'later–longer–fewer' policy and in the one-child family programme. Though this is only one small element of the programme, it is an essential element, and one that, unlike much of the rest of the Chinese approach to population policy, may be reasonably easily emulated in other countries.

With respect to the analysis of the determinants of fertility, birth interval and parity progression measures appear to offer advantages in two areas. The most obvious is in analysis of the 'intermediate variables', such as length of breastfeeding and contraceptive use, through which any variable (education, rural–urban residence, and so on) must operate if it is to influence fertility. The intermediate variables are by definition variables that intervene between the occurrence of one birth and the possible occurrence of the next, and parity progression fertility measures are thus the natural ones to adopt in measuring their effects. A second area is the study of differentials by characteristics that undergo substantial change as women reproduce. Women's labour-force participation and migration provide two obvious examples. The interpretation of conventional age-specific fertility measures for women with specific labour force or migration characteristics is complicated both by the rapidity with which these characteristics may change with age and by the probable existence of causal effects in both directions. Parity progression measures would appear to be preferable on both counts. The number of children a women has and their ages are probably more relevant to her participation in the labour force than her age, for example, within the relevant age limits. The general approach would be to disaggregate parity cohorts by characteristics and then to trace the progression to next birth simultaneously with changes in characteristics.