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ARE MORTALITY PROJECTIONS ALWAYS MORE PESSIMISTIC WHEN DISAGGREGATED BY CAUSE OF DEATH?

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It is often observed that mortality projections are more pessimistic when disaggregated by cause of death. This article explores the generality and strength of this relationship under a variety of forecasting models. First, a simple measure of the pessimism inherent in cause-based mortality forecasts is derived. Second, it is shown that the pessimism of cause-based forecasts can be approximated using only data on the distribution of deaths by cause in two previous time periods. Third, using Japanese mortality data during 1951–1990, the analysis demonstrates that the pessimism of cause-based forecasts can be attributed mainly to observed trends in mortality due to cancer and heart disease, with smaller contribution due to trends in stroke (women only), pneumonia/bronchitis, accidents, and suicide. The last point requires the important qualification, however, that observed trends in cancer and heart disease may be severely biased due to changes in diagnostic practice.

KEY WORDS: Mortality projection, forecasting, causes of death, disaggregation.
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1. INTRODUCTION

It is often observed that mortality projections are more pessimistic when disaggregated by cause of death. The reason that this should be true is intuitively simple: if projections are made separately for individual causes of death and then combined, the resulting projection of all-cause mortality comes to be dominated by those causes of death that are decreasing the most slowly or that are increasing. Thus, the results of an empirical mortality forecast may depend critically on the choice of which mortality series to extrapolate (total or cause-specific). In a recent set of mortality projections for Japan, the divergence in forecasted e_0 was about 5 years in 2020, and around 10 years in 2050, with the lower values of e_0 always associated with the cause-based forecast (Wilmoth 1993b).

Is it true that mortality forecasts are always more pessimistic when based on separate projections of cause-specific death rates? This article seeks to answer that question and to develop methods for quantifying the degree of pessimism associated with cause-based forecasts. At present, I focus on mortality forecasting methods that fall under the general rubric of "proportional rate of change models." These include the common extrapolatory methods (see Pollard 1987 for a review) and the more elegant technique proposed recently by Lee and Carter (1992). All empirical examples here are based on Japanese mortality data during the period 1951–1990. Total and cause-specific death rates for Japan were calculated using published vital statistics and census data.

In this article, I explore the general conditions under which proportional rate of change models yield more pessimistic mortality forecasts when disaggregated by cause of death. In a simple but plausible case, it is possible to show that the cause-based forecasts are always more pessimistic. One of the main findings of the article is that we can approximate the pessimism implicit in cause-specific mortality forecasts by means of a simple formula based on changes in the distribution of deaths by cause. This approximation formula also helps to provide an intuitive understanding of the situations in which cause-based forecasts should deviate most noticeably from aggregate forecasts. Further calculations serve to identify the causes of death that account for the pessimism of cause-specific forecasts.

The purpose of this discussion is not to defend the appropriateness or accuracy of one projection method or another. My current goal is more modest: to develop methods for understanding the origin and magnitude of the pessimism implicit in mortality forecasts based on cause-specific trends using familiar projection techniques. In the Japanese example considered here, this article documents that cancer mortality trends are the most important source of the pessimism inherent in cause-based forecasts, raising the level of projected death rates over a broad age range but especially at older ages. Other causes that are important contributors to this pessimism include heart disease at the oldest ages, along with suicide and accidents at adolescent and young adult ages.

2. GENERAL MATHEMATICAL RESULTS

In this section, I develop a general mathematical framework for understanding the differences between forecasts of total and all-cause mortality. The analysis is limited to the most common class of projection techniques, namely, proportional rate of change models. It is difficult to justify this restriction without reference to empirical examples. The appeal of proportional rate of change models for forecasting mortality is easily demonstrated, however, if one examines age-specific mortality trends in a variety of industrialized countries. For example, Figure 1 shows the trend in the age-adjusted death rate for Japanese women and men during 1951–1990.[1] The near-linearity of this composite index of mortality, when plotted in a logarithmic scale, is evidence of the appropriateness of projection models that assume a constant proportional rate of change in age-specific death rates, at least for total mortality. More detailed graphs of mortality trends for individual age groups support a similar conclusion, as seen for Japanese women and men in Figures 2(a) and 2(b).

Cause-specific trends are not, of course, as regular as total mortality trends. Figures 3(a) and 3(b) show age-adjusted mortality trends by cause for nine categories

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[1]The age-adjusted total mortality rate is the weighted average of the age-specific death rates for each year, where the weights are held constant across different years. The weights in this case were set equal to the estimated mid-year population in each age group in 1980.

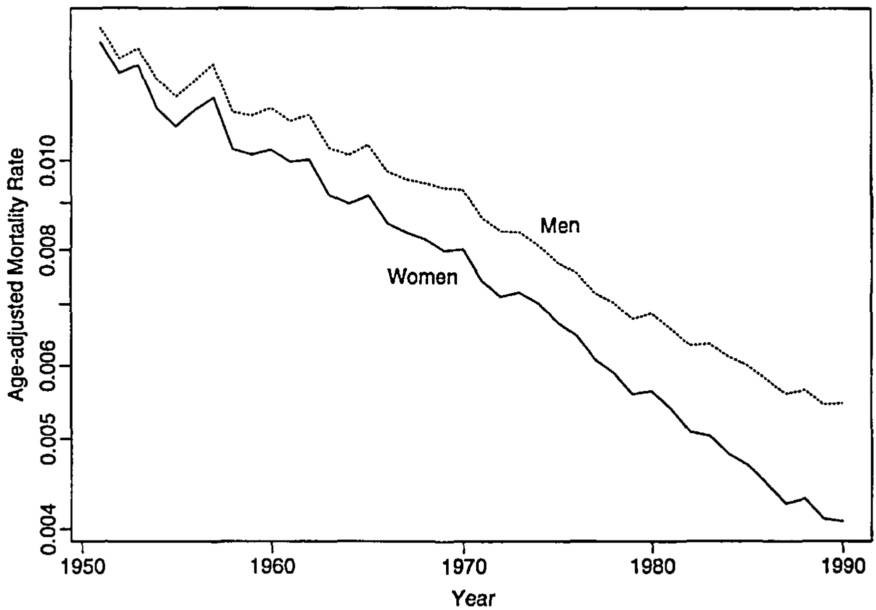

FIGURE 1. Age-adjusted total mortality rate, Japanese women and men, 1951–1990 (semi-logarithmic scale).

of causes (see Appendix A for a list of these causes along with their ICD-9 codes) for Japanese women and men during 1951–1990. This graph includes a line (labeled '0') corresponding to total mortality; to facilitate comparison to the cause-specific trends, this line plots the level of the age-adjusted total mortality rate (in Figure 1) divided by 9. Although cause-specific trends (plotted in a logarithmic scale) are not as consistently well approximated by a straight line as total mortality trends, it is also true that a log-linear model is not far from reasonable in the majority of cases. In other words, a proportional rate of change model may be a plausible means of projecting total *or* cause-specific mortality trends. The trend in stroke mortality (#4) is the least linear of all if we consider the entire period form 1951 to 1990, but note that this trend becomes almost linear during the last two decades of the observation period. Therefore, when we consider actual projection techniques in the next section, we will include methods for deriving the linear trend that give additional weight to more recent observations.

It is on the basis of these empirical observations, and in recognition of common practice among mortality forecasters, that I have chosen to focus the discussion in this article on proportional rate of change models. The general form of these models in the case of total mortality can be written

$$\hat{m}_{xt} = m_{xt_0}(1 - r_x)^{t-t_0}, \tag{1}$$

where $\hat{m}_{xt}$ is the projected total mortality rate at age x and t, t_0 is the last year of the observation period (thus $t > t_0$), m_{xt_0} are observed age-specific total mortality rates in year t_0, and r_x is the assumed annual rate of mortality decline at age x (held

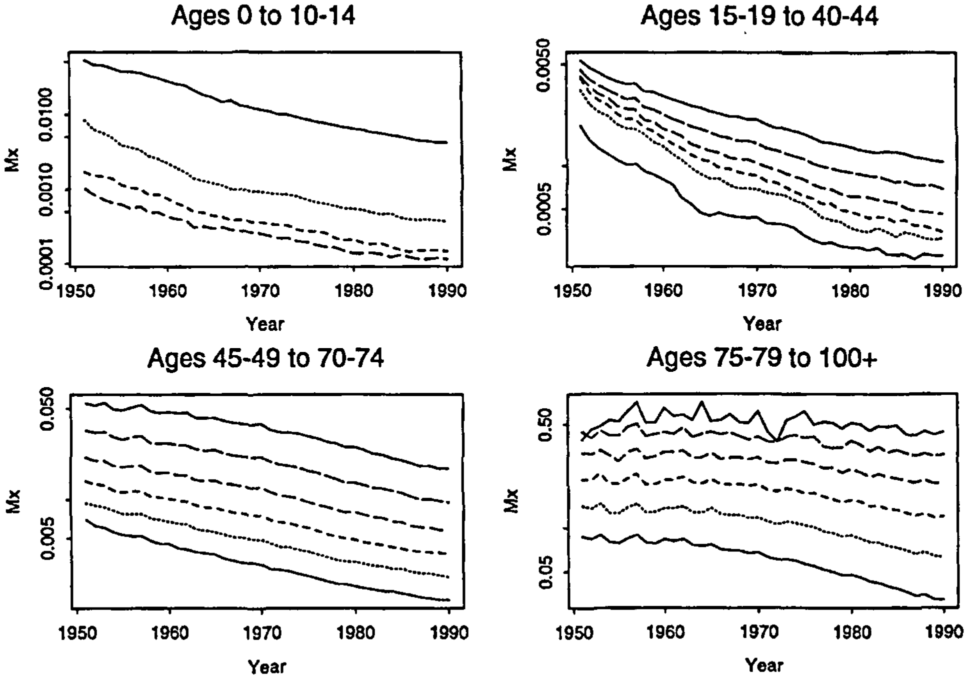

FIGURE 2(a). Trends in mortality rates by age, Japanese women, 1951–1990 (semi-logarithmic scale).

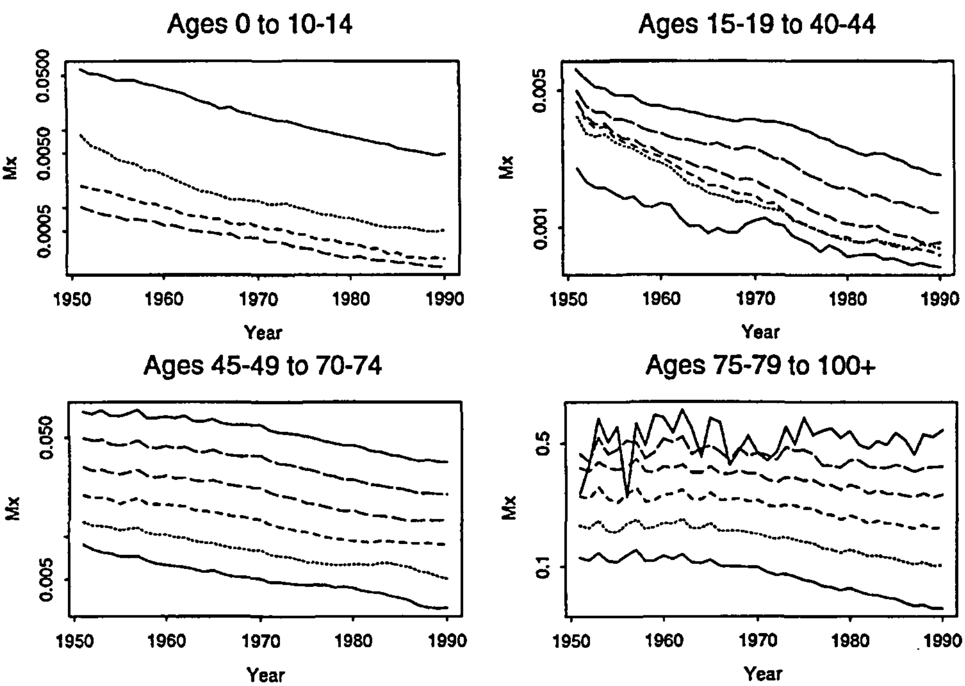

FIGURE 2(b). Trends in mortality rates by age, Japanese men, 1951–1990 (semi-logarithmic scale).

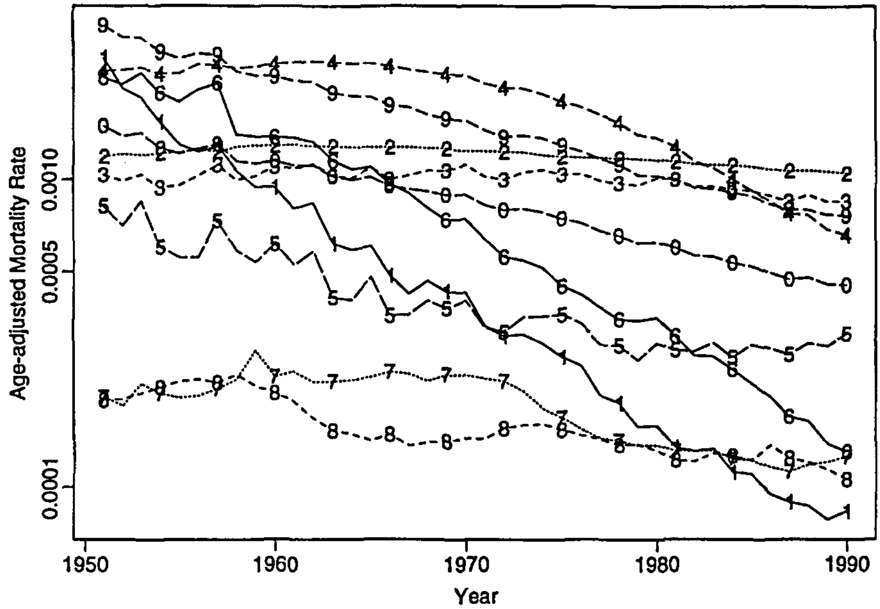

FIGURE 3(a). Age-adjusted total mortality rates by cause, Japanese women, 1951–1990 (0 = all-cause/9, 1 = infectious, 2 = cancer, 3 = heart, 4 = stroke, 5 = pneumonia/bronchitis, 6 = senility, 7 = accidents, 8 = suicide, 9 = other).

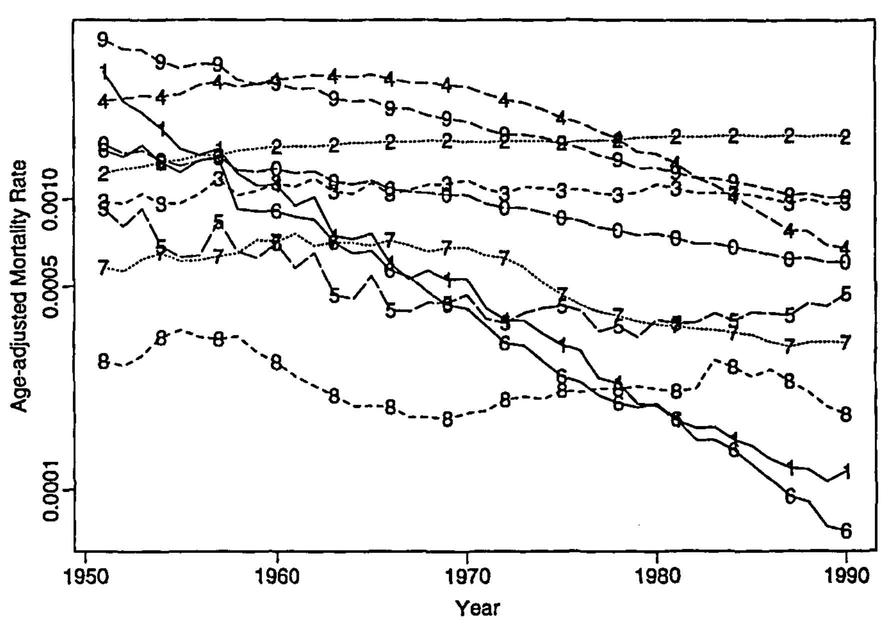

FIGURE 3(b). Age-adjusted total mortality rates by cause, Japanese men, 1951–1990 (0 = all-cause/9, 1 = infectious, 2 = cancer, 3 = heart, 4 = stroke, 5 = pneumonia/bronchitis, 6 = senility, 7 = accidents, 8 = suicide, 9 = other).

constant over the projection period). To simplify some of the formulas later on, it is also useful to introduce the notation $s_x = 1 - r_x$, and thus

$$\hat{m}_{xt} = m_{xt_0} s_x^{t-t_0}, \tag{2}$$

where s_x is the age-specific ratio of projected mortality rates in year t divided by those in year $t-1$. We can thus think of s_x as a multiplier that moves the projection of m_{xt} forward by one year of time.

Similar notation can be used to describe cause-specific mortality forecasts. Thus, for cause i

$$\begin{aligned}\hat{m}_{xt}^{(i)} &= m_{xt_0}^{(i)}(1 - r_x^{(i)})^{t-t_0} \\ &= m_{xt_0}^{(i)}(s_x^{(i)})^{t-t_0}.\end{aligned} \tag{3}$$

The all-cause mortality forecast, therefore, can be written

$$\begin{aligned}\hat{m}_{xt}^{(\cdot)} &= \sum_i \hat{m}_{xt}^{(i)} \\ &= \sum_i m_{xt_0}^{(i)}(1 - r_x^{(i)})^{t-t_0}.\end{aligned} \tag{4}$$

Note the important distinction made here between *total* and *all-cause* mortality projections. The former is the result of projecting total mortality rates, while the latter is the result of projecting cause-specific mortality rates and adding them together. The central purpose of this article, then, is to determine when and why $\hat{m}_{xt}$ (total mortality forecast) differs from $\hat{m}_{xt}^{(\cdot)}$ (all-cause mortality forecast) in the context of proportional rate of change models.

If we let $p_x^{(i)}$ be the proportion of deaths at age x that are due to cause i in the last year of the observation period, t_0, then the following equations demonstrate that a simple relationship exists between the forecasts of total and all-cause mortality:

$$\begin{aligned}\hat{m}_{xt}^{(\cdot)} &= \sum_i m_{xt_0}^{(i)}(1 - r_x^{(i)})^{t-t_0} \\ &= \sum_i m_{xt_0} p_x^{(i)}(1 - r_x^{(i)})^{t-t_0} \\ &= \sum_i \frac{\hat{m}_{xt}}{(1 - r_x)^{t-t_0}} p_x^{(i)}(1 - r_x^{(i)})^{t-t_0} \\ &= \hat{m}_{xt} \sum_i p_x^{(i)} \left(\frac{1 - r_x^{(i)}}{1 - r_x} \right)^{t-t_0} \\ &= \hat{m}_{xt} \sum_i p_x^{(i)} \left(\frac{s_x^{(i)}}{s_x} \right)^{t-t_0}.\end{aligned} \tag{5}$$

Thus, the ratio of the all-cause forecast to the total mortality forecast at age x can be written

$$\frac{\hat{m}_{xt}^{(\cdot)}}{\hat{m}_{xt}} = \sum_i p_x^{(i)} \left(\frac{s_x^{(i)}}{s_x} \right)^{t-t_0} . \tag{6}$$

This ratio is the weighted average of the exponentiated ratios of the cause-specific forecast multipliers, $s_x^{(i)}$, to the total mortality multiplier, s_x, where the weights equal the proportion of deaths due to cause i in the last observation period, t_0. I will call the quantity in equation (6) the "m-ratio" and will use it as a measure of the pessimism of cause-based forecasts.[2]

At first glance, there seems to be no reason to conclude that the sum in equation (6) will be greater than one for all values of t, hence that the all-cause forecast of m_{xt} will always be greater (thus, more pessimistic) than the total mortality forecast. Nevertheless, if $s_x^{(i)} > s_x$ for at least one cause i, then it is obvious that this sum tends toward infinity as t gets very large, and thus the all-cause mortality forecast must *eventually* exceed the total mortality forecast in this case. Clearly, the all-cause and total mortality forecasts will be equal for all $t > t_0$ if $s_x^{(i)} = s_x$ for all causes i. Furthermore, if $s_x^{(i)} \neq s_x$ for at least one i, it is possible to show (see Appendix B) that the m-ratio will exceed unity for all values of $t > t_0$ if and only if the following inequality holds true:

$$\sum_i p_x^{(i)} \ln(s_x^{(i)}/s_x) \geq 0. \tag{7}$$

It is easy to check whether the inequality in equation (7) is correct for a particular projection method. For all six forecasting techniques considered in the following section, which are applied separately to male and female mortality, I have verified that this inequality holds true in all age groups. In one simple case, it is possible to demonstrate (see Appendix B) that this inequality is always true. Thus, let s_x and $s_x^{(i)}$ be defined as follows:

$$s_x = \left(\frac{m_{xt_0}}{m_{x\tau}} \right)^{1/(t_0-\tau)} , \tag{8}$$

and

$$s_x^{(i)} = \left(\frac{m_{xt_0}^{(i)}}{m_{x\tau}^{(i)}} \right)^{1/(t_0-\tau)} , \tag{9}$$

where τ and t_0 refer to the first and last years, respectively, of the observation period (as above, t_0 is the starting point for the forecast of future mortality). We call this model the "first-to-last" proportional change method, since it derives the

[2]Equations (5) and (6) assume, implicitly, that the observed mortality level in the final year, t_0, is used as the baseline for projection. Alternatively, projections could be based on some predicted value of age- and cause-specific mortality in this year, thus $\hat{m}_{xt_0}$ and $\hat{m}_{xt_0}^{(i)}$. In this case, it is possible to substitute $p_x^{*(i)}$ for $p_x^{(i)}$ in these equations, where $p_x^{*(i)} = \hat{m}_{xt_0}^{(i)}/\hat{m}_{xt_0}$. The interpretation is similar although not identical to the previous case: since it is not necessarily true that $\sum_i p_x^{*(i)} = 1$, equation (6) may no longer be a proper weighted average.

proportional rate of change during the forecast period simply by calculating the (geometric) average annual change from the first to the last year of past observations. Note that the annual rate of mortality decline assumed by this forecast method is $r_x = 1 - s_x$ for total mortality, and $r_x^{(i)} = 1 - s_x^{(i)}$ for cause i. For this particular method, it is shown in Appendix B that the all-cause mortality forecast is always greater than the total mortality forecast for any $t > t_0$.

At present, I have not succeeded in demonstrating that the inequality in equation (7) is true in general for more complicated forecast methods. Nevertheless, since other plausible proportional change models seem likely to produce results that are similar to the projections of the first-to-last method, it is not surprising that the inequality was found to hold true in practice for the other methods considered here. The usefulness of focusing our attention on the first-to-last method, however, goes beyond our ability to derive precise mathematical results. The first-to-last method can also be manipulated in a way that offers an informative interpretation of the mechanics of the pessimism implicit in cause-based mortality forecasts.

Note that the ratio of the $s_x^{(i)}$ to s_x forecast multipliers in the first-to-last method can be re-arranged as follows:

$$\frac{s_x^{(i)}}{s_x} = \left(\frac{m_{xt_0}^{(i)} / m_{x\tau}^{(i)}}{m_{xt_0} / m_{x\tau}} \right)^{1/(t_0-\tau)}$$

$$= \left(\frac{m_{xt_0}^{(i)} / m_{xt_0}}{m_{x\tau}^{(i)} / m_{x\tau}} \right)^{1/(t_0-\tau)}$$

$$= \left(\frac{p_x^{(i)}}{q_x^{(i)}} \right)^{1/(t_0-\tau)}, \qquad (10)$$

where $q_x^{(i)}$ and $p_x^{(i)}$ are the proportion of deaths due to cause i (at age x) in years τ and t_0, respectively. Thus, combining equations (6) and (10), the ratio of the all-cause to total mortality forecasts in this case becomes

$$\frac{\hat{m}_{xt}^{(\cdot)}}{\hat{m}_{xt}} = \sum_i p_x^{(i)} \left(\frac{s_x^{(i)}}{s_x} \right)^{t-t_0}$$

$$= \sum_i p_x^{(i)} \left(\frac{p_x^{(i)}}{q_x^{(i)}} \right)^{(t-t_0)/(t_0-\tau)} \qquad (11)$$

Therefore, the pessimism of cause-based mortality forecasts in this particular case can be interpreted in terms of the changing distribution of deaths by cause. Clearly, if this distribution is unchanging (thus if $p_x^{(i)} = q_x^{(i)}$ for every i), then the ratio of all-cause to total mortality equals one for any time horizon t. On the other hand, any change in the distribution of deaths by cause leads to differences in the two forecasts.

TABLE 1
Emergent vs. Receding Causes: 4 Hypothetical Examples

	Hypothetical Distribution of Causes (in percent)					M-Ratios 2010	M-Ratios 2040
Emergent (strong)	1951:	1	30	30	39	1.65	9.92
	1990:	20	25	25	30		
Emergent (weak)	1951:	10	30	30	30	1.08	1.4
	1990:	25	25	25	25		
Receding (strong)	1951:	20	25	25	30	1.11	1.3
	1990:	1	30	30	39		
Receding (weak)	1951:	25	25	25	25	1.05	1.17
	1990:	10	30	30	30		

Notes: Each example supposes a distribution of deaths by cause in 1951 and 1990. The m-ratios in 2010 and 2040 are ratios of the forecasts of all-cause to total mortality based on the first-to-last projection model, which assumes a continuation of historic average proportional rates of change in age- and cause-specific death rates (see text for further explanation).

It is useful to distinguish between the effects of "emergent" and "receding" causes of death. An emergent/receding cause is one whose proportion of the total number of deaths (across all ages or in a particular age group) rises/declines substantially over the observation period. Note that it is possible to observe an emergent cause without receding causes, if the "substantial" increase in the emergent cause results from small decreases in a number of causes; similarly, a receding cause could be matched by a number of slowing expanding causes. Although this definition remains rather vague (since I have given no precise definition of what is meant by a "substantial" rise or decline in a given cause), it is still useful for building intuition about the role of the changing distribution of deaths by cause in the pessimism of cause-based forecasts.

Consider the simple case of four causes of death whose distributions are changing over time. Table 1 presents four scenarios of emergent and receding causes, together with the pessimism of the cause-based forecast suggested by the ratio in equation (11) for time horizons of $t = 20$ and 50 years. The four cases illustrate two instances each of emergent and receding causes. Note that, for changes of similar magnitude in both directions, the emergent causes are associated with much larger m-ratios than the receding causes. From inspection of equation (11), this result can be explained by the fact that the ratio $p_x^{(i)}/q_x^{(i)}$ quickly becomes large as t increases for an emergent cause, but decreases rapidly for a receding cause. The lesson to be retained from this simple illustration is that we should expect a relatively large divergence between all-cause and total mortality forecasts when there is one cause or several causes of death that have increased substantially during the observation period. The presence of a cause that recedes or even disappears over time, on the other hand, will have a much smaller effect on the pessimism of the all-cause forecast.

It is true, of course, that this illustration has been based on formulas derived for a particular case, referred to here as the first-to-last projection method. It seems plausible, however, that similar results may be derived or observed for any proportional change projection method. Again, although I have not succeeded in deriving precise analytical results for other projection methods, I show in the following section

TABLE 2
Pessimism (M-Ratios) of Cause-Based Mortality Forecasts by Two "Approximation" Methods

	Women				Men			
	2010		2040		2010		2040	
	First-to-last	First-to-second	First-to-last	First-to-second	First-to-last	First-to-second	First-to-last	First-to-second
0	1.49	1.77	6.27	7.48	1.49	1.70	6.80	5.97
1-4	1.91	2.90	6.95	14.01	1.73	2.37	4.98	8.30
5-9	1.94	2.96	8.40	17.05	1.60	2.09	5.10	8.19
10-14	1.84	2.24	7.64	9.44	1.44	1.74	3.56	5.41
15-19	2.01	2.17	7.87	7.59	1.57	1.55	3.57	2.96
20-24	1.97	1.96	7.59	5.98	1.47	1.37	2.98	2.22
25-29	1.87	2.16	5.99	6.78	1.56	1.63	3.38	3.42
30-34	1.78	2.14	5.16	6.28	1.62	1.85	3.82	4.65
35-39	1.55	1.84	3.52	4.47	1.49	1.73	3.12	4.22
40-44	1.37	1.50	2.51	2.88	1.38	1.58	2.55	3.64
45-49	1.28	1.34	2.11	2.21	1.27	1.41	2.03	2.67
50-54	1.25	1.31	2.01	2.13	1.22	1.36	1.85	2.33
55-59	1.25	1.32	2.03	2.13	1.23	1.37	1.90	2.28
60-64	1.25	1.30	2.04	2.08	1.27	1.36	2.11	2.28
65-69	1.28	1.33	2.21	2.21	1.31	1.35	2.32	2.29
70-74	1.34	1.40	2.50	2.48	1.36	1.40	2.61	2.53
75-79	1.44	1.51	3.00	2.97	1.44	1.52	3.11	3.13
80-84	1.59	1.64	4.05	3.65	1.63	1.69	4.47	4.23
85-89	1.71	1.76	4.99	4.36	1.73	1.86	5.35	5.55
90-94	1.79	1.81	5.91	4.82	1.83	1.94	6.61	6.26
95-99	1.84	1.84	7.28	5.08	1.65	1.90	4.78	5.52
100+	1.39	1.78	3.33	5.26	1.82	1.54	9.37	3.24

Notes: Baseline = 1951–1990, Horizon = 2010 and 2040. M-ratios are ratios (all-cause: total mortality) of forecasted age-specific death rates. The two sets of forecasts used here were derived using the first-to-last and first-to-second projection models, which assume a continuation of historic average proportional rates of change in age- and cause-specific death rates (see text for further explanation).

how equation (11) may be used in a general fashion to approximate the pessimism of cause-based mortality forecasts. Thus, the intuition that it affords (connecting the pessimism of cause-based forecasts to changes in the distribution of deaths by cause) seems to be generally useful. The formula itself, because it is so simple, can also be manipulated quite easily in order to partition the pessimism of cause-based forecasts among individual causes of death.

3. PESSIMISM OF DISAGGREGATED FORECASTS

The examples in this section are all based on trends in Japanese mortality statistics for the years 1951–1990. These data are used for illustrative purposes only. While it may be true that some features of mortality change in Japan are shared by other industrialized countries over this time period, there are no doubt important differences. For this reason, no claim is made here about the general applicability of these results.

Approximations

Two approximations of the pessimism of cause-based mortality forecasts for women and men in Japan are shown in Table 2 at time horizons of 20 and 50 years (thus corresponding to years 2010 and 2040). I refer to these calculations as "approximations," since actual forecasting techniques are likely to be based on more elaborate methods, although in principle both approximations could also be considered actual forecasts.

Both approximation methods are based on the first-to-last forecasting technique described in the previous section (see equations 8–11) applied separately to 5-year age groups. Due to random fluctuations in annual mortality data, however, it is advisable to compare the change in the distribution of deaths by cause for periods of more than one year. Thus, equation (11) can be modified as follows:

$$\frac{\hat{m}_{xt}^{(\cdot)}}{\hat{m}_{xt}} = \sum_i p_x^{(i)} \left(\frac{p_x^{(i)}}{q_x^{(i)}} \right)^{(t-t_2)/(t_2-t_1)}, \tag{12}$$

where $q_x^{(i)}$ and $p_x^{(i)}$ describe the distribution of deaths by cause in the first and second time periods, respectively, and where t_1 and t_2 are the midpoints of those two periods.

In empirical applications here, the first-to-last method is based on the first and last 3-year time periods, as shown in Table 2. These m-ratios are exactly the ratios of all-cause to total mortality that would result from a forecast based on the first-to-last proportional change method using the first and last 3-year time periods as the basis for the forecast. Because the first and last 3-year periods may still be atypical with respect to the long-term trend, however, it is sensible to make a similar calculation based on broader time periods. Thus, the second method considered here for approximating the pessimism of cause-based forecasts is based on the first and second halves of the entire observation period (thus, comparing the distribution of deaths by cause in 1951–1970 to the distribution in 1971–1990), again using equation (12). This calculation will be referred to as the "first-to-second" approximation method.

In the following paragraphs, m-ratios for both sets of approximations are compared to the m-ratios that result from a variety of actual cause-based forecasts. Although both approximation techniques perform reasonably well, the approximation based on the two halves of the observation period yields estimates that are somewhat more consistent with the range of projection techniques considered here than the first-to-last method.

Projections

As noted earlier, all projection techniques considered here fall under the general rubric of "proportional rate of change models". Based on empirical observations, this class of models seems to be a reasonable choice, although within this class of techniques there remain numerous decisions about how to derive a set of mortality forecasts. Chief among these is the decision about whether to make separate projections by cause of death, which is the topic of this article. Other choices relate to the manner of deriving the proportional rate of change for each age group. One technique has already been described and is known as the "first-to-last" method. It

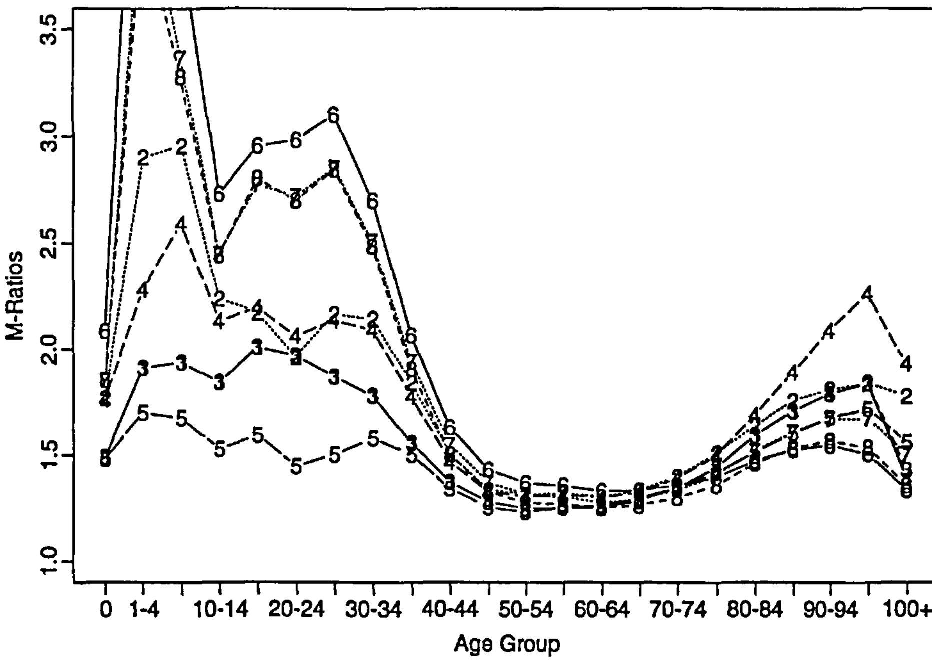

FIGURE 4(a). M-ratios, approximations vs. projections, Japanese women, baseline = 1951–1990, horizon = 2010 (1 = approx 1, 2 = approx 2, 3 = project 1a, 4 = project 1b, 5 = project 1c, 6 = project 2a, 7 = project 2b, 8 = project 2c).

is applied separately to each age group to derive a proportional rate of change for the forecast based on the historic average rate of change between the first and last years (or 3-year time periods).

The forecasting techniques considered here can be divided into two groups: 1) methods that are applied separately to each age group; and 2) methods derived from the Lee-Carter procedure, which uses a parametric model to condense age-specific changes into a single time trend. Within each of these two classes of techniques, I examine three sets of total and all-cause mortality forecasts, in which the proportional rate of change (or, equivalently, the trend in the logarithm of the age-specific death rates) is derived by three methods: a) the average change from the first to the last year (or 3-year intervals) of the observation period; b) the slope of an ordinary least squares (OLS) regression line fit to the logarithm of the age-specific mortality rates (or to the time index in the Lee-Carter model); and c) same as b) but using weighted least squares (WLS), where the weights are chosen to give greater emphasis to more recent trends. These six projection techniques are described in more detail in Appendix C.

For each of these six sets of actual mortality forecasts, it is possible to compare the projected levels of all-cause and total mortality for each age group. The ratio of these two levels, known as the m-ratio, is plotted in Figure 4(a) and 4(b) (Japanese women and men, respectively) for the six projection techniques and the two approximation methods at a time horizon of 20 years (thus for the year 2010). Lines 1

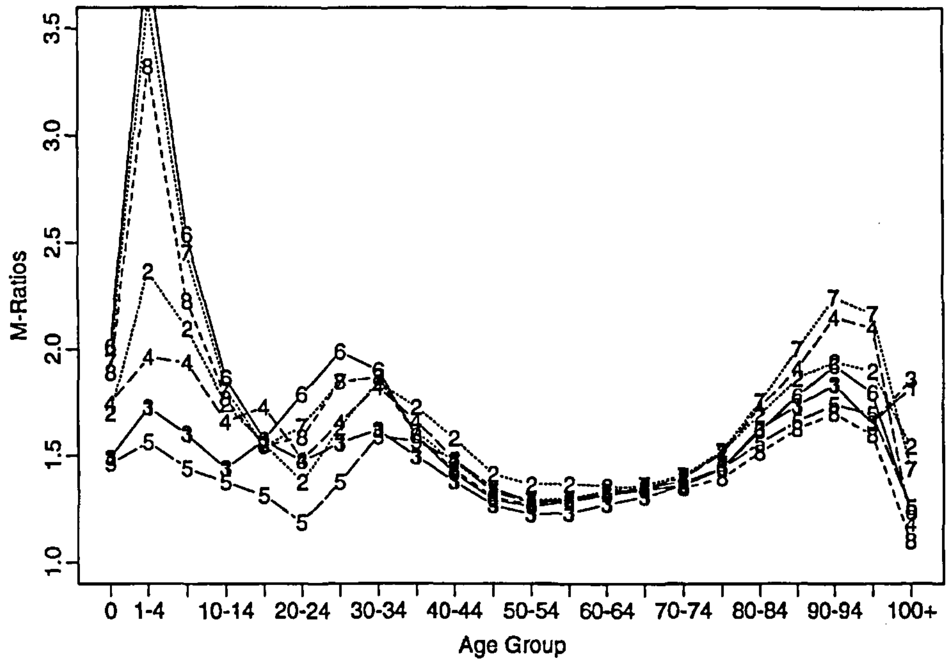

FIGURE 4(b). M-ratios, approximations vs. projections, Japanese men, baseline = 1951–1990, horizon = 2010 (1 = approx 1, 2 = approx 2, 3 = project 1a, 4 = project 1b, 5 = project 1c, 6 = project 2a, 7 = project 2b, 8 = project 2c).

and 2 trace the m-ratios implied by the two approximation techniques. Lines 3, 4, and 5 show the m-ratios for the projection methods based on individual age-specific trends. Lines 6, 7, and 8 give the m-ratios based on the three applications of the Lee-Carter technique. Note that lines 1 and 3 on this graph are identical, since the first approximation method is the same as projection method 1a (the first-to-last method).

The most important result to emerge from Figure 4 is that the age profiles of the pessimism of cause-based forecasts have important similarities for a variety of projection techniques (at least within the class of proportional rate of change models). Through the age range from 40 to 80 years, there is strikingly little disparity between the m-ratios yielded by various methods. Nonetheless, there is a rather large divergence in the m-ratios at ages below 40, especially for women, but these differences are probably unimportant: although the relative differences between the methods may be large, as measured by the m-ratios, the absolute differences are quite small, since mortality at these ages is comparatively low. Furthermore, it is shown in the next section that the contribution of these ages to the pessimism of the all-cause forecast of life expectancy at birth, e_0, is a small fraction of the total difference.

More important than the variability of the m-ratios at younger ages is their variability at the highest ages, since these ages are major contributors to differences in forecasts of life expectancy or other aggregate measures of mortality. The exact source of the divergence in the m-ratios above 80 years is unknown. There does not appear to be any clear pattern whereby one method yields consistently higher or lower results at these ages. In contrast, the m-ratios at younger ages appear to follow a consistent pattern: the pessimism of the all-cause forecast is consistently highest for the Lee-Carter methods, lowest for the age-specific extrapolation methods, and intermediate for the first-to-second approximation method. At ages above 40, it is difficult to argue that one method produces results that are consistently in the middle of the pack. Nonetheless, I will focus on the first-to-second approximation in the section that follows for several reasons: its simplicity facilitates the decompositions that I wish to apply, it is clearly the intermediate method at younger ages, and it is never far from the average level at older ages.

Decompositions

Thanks to the simplicity of equations (11) or (12), it is possible to decompose the approximate m-ratios presented in Table 2 and plotted in Figure 4. The first decomposition described here partitions the m-ratios by cause of death and thus attributes the pessimism of cause-based forecasts to specific causes. This decomposition is very simple to compute but gives results separately for each age group. A more elaborate decomposition uses a technique proposed by Pollard (1982, 1990) to partition this pessimism by age and cause simultaneously and to attribute differences in forecasts of e_0 to specific cause categories or age groups. These decompositions are presented only for the approximation based on the first and second halves of the observation period, since we have seen that it is broadly representative of a range of actual projection techniques.

The first decomposition partitions the quantity, m-ratio minus one, among the various causes. Clearly, if the m-ratio equals one, then the all-cause and total mortality forecasts are identical. Therefore, it is excess of the m-ratio above one that represents the pessimism of the all-cause forecast. Based on equation (6), this quantity can be written as follows:

$$\frac{\hat{m}_{xt}^{(\cdot)}}{\hat{m}_{xt}} - 1 = \sum_i p_x^{(i)} \left(\frac{s_x^{(i)}}{s_x} \right)^{t-t_0} - 1$$

$$= \sum_i p_x^{(i)} \left(\left(\frac{s_x^{(i)}}{s_x} \right)^{t-t_0} - 1 \right). \qquad (13)$$

Therefore, the excess of the m-ratio above one in this case can be easily decomposed as the sum of cause-specific elements. For first-to-last type forecasts (see equation 12), this equation can be written simply in terms of the proportional distribution of deaths by cause in two time periods:

$$\frac{\hat{m}_{xt}^{(\cdot)}}{\hat{m}_{xt}} - 1 = \sum_i p_x^{(i)} \left(\left(\frac{p_x^{(i)}}{q_x^{(i)}} \right)^{(t-t_2)/(t_2-t_1)} - 1 \right). \qquad (14)$$

Therefore, the appeal of the "approximation" methods considered here is, again, their simplicity, since they permit a decomposition of the pessimism of cause-based forecasts given only a knowledge of the distribution of deaths by cause in two time periods.

Tables 3 and 4 presents the results of this decomposition calculated using only the change in the distribution of deaths by cause from 1951–1970 to 1971–1990 (known as the second approximation method, or the first-to-second forecast technique), assuming a forecast horizon of 20 years (thus, the year 2010). This table is very simple to calculate using equation (14). The most striking result is the overwhelming contribution of trends in cancer mortality towards the pessimism of cause-based mortality forecasts in almost every age group. Other causes that are important in this regard include heart disease at older ages, stroke (to a lesser extent) at the highest ages, accidents during childhood thru late adolescence and early adulthood, and suicide from adolescence until around age 50. Causes of death that have been declining more rapidly than average contribute negatively in this calculation, but their cumulative effect is much smaller than the positive effects of the causes that have been declining less rapidly than average (or even increasing over time).

The above decomposition is very simple to carry out, since it can be computed using only the two observed distributions of deaths by cause. It partitions the pessimism of the cause-based forecast separately for each age group, but it does not yield information about the relative importance of each age group towards differences in aggregate measures of mortality, such as life expectancy at birth, e_0. It is possible to assess the contribution of age and cause groups simultaneously using a decomposition proposed by Pollard (1982, 1990), although it requires significantly more calculations, including complete forecasts of all-cause and total mortality rates.

Using Pollard's formula, life expectancies derived from forecasted all-cause and total mortality rates can be decomposed as follows:

$$\begin{aligned} e_0^{\text{Tot}} - e_0^{\text{All}} &= w_0 \sum_i ({}_1m_0^{(i)\text{All}} - {}_1m_0^{(i)\text{Tot}}) \\ &+ 4w_2 \sum_i ({}_1m_1^{(i)\text{All}} - {}_1m_1^{(i)\text{Tot}}) \\ &+ 5w_{7.5} \sum_i ({}_5m_5^{(i)\text{All}} - {}_5m_5^{(i)\text{Tot}}) \\ &+ 5w_{12.5} \sum_i ({}_5m_{10}^{(i)\text{All}} - {}_5m_{10}^{(i)\text{Tot}}) + \cdots . \end{aligned} \tag{15}$$

where the weights, w_x, are defined by

$$w_x = \tfrac{1}{2}({}_xp_0^{\text{Tot}} e_x^{\text{All}} + {}_xp_0^{\text{All}} e_x^{\text{Tot}}) \tag{16}$$

(${}_xp_0^{\text{Tot}}$ and ${}_xp_0^{\text{All}}$ represent the life table probabilities of survival from age 0 to x based on the total and all-cause mortality forecasts, respectively). Thus, the difference between the total and all-cause projections of e_0 can be partitioned among the

TABLE 3(a)
Simple Decomposition of M-Ratios, Japanese Women

	Infectious	Cancer	Heart	Stroke	Pneu/Bron	Senility	Accidents	Suicide	Other	M-Ratio
0	-0.04	0.03	0.18	0.11	-0.06	--	0.07	--	0.47	1.77
1-4	-0.08	0.71	0.16	0.04	-0.05	--	0.58	--	0.54	2.90
5-9	-0.05	1.31	0.01	0.08	-0.03	--	0.47	0.01	0.15	2.96
10-14	-0.04	1.01	-0.03	0.04	-0.01	--	0.06	0.21	0.00	2.24
15-19	-0.03	0.42	-0.02	0.02	-0.01	--	0.84	-0.01	-0.04	2.17
20-24	-0.03	0.47	-0.01	0.03	-0.01	--	0.43	0.12	-0.04	1.96
25-29	-0.04	0.72	-0.01	0.05	-0.01	--	0.12	0.38	-0.06	2.16
30-34	-0.03	0.77	-0.01	0.05	-0.01	--	0.07	0.38	-0.08	2.14
35-39	-0.03	0.58	-0.02	0.06	-0.01	--	0.04	0.30	-0.08	1.84
40-44	-0.03	0.37	-0.01	0.02	-0.01	--	0.03	0.20	-0.07	1.50
45-49	-0.03	0.31	0.00	-0.03	0.00	0.00	0.02	0.13	-0.06	1.34
50-54	-0.02	0.32	0.01	-0.07	0.00	0.00	0.02	0.10	-0.04	1.31
55-59	-0.02	0.36	0.03	-0.08	0.00	0.00	0.02	0.05	-0.04	1.32
60-64	-0.02	0.35	0.05	-0.09	0.00	0.00	0.01	0.03	-0.03	1.30
65-69	-0.02	0.35	0.09	-0.09	0.00	0.00	0.01	0.02	-0.03	1.33
70-74	-0.02	0.34	0.15	-0.07	0.01	-0.01	0.01	0.01	-0.03	1.40
75-79	-0.02	0.32	0.24	-0.02	0.02	-0.03	0.01	0.01	-0.02	1.51
80-84	-0.02	0.29	0.33	0.05	0.04	-0.07	0.01	0.00	0.00	1.64
85-89	-0.02	0.25	0.42	0.13	0.06	-0.11	0.01	0.00	0.02	1.76
90-94	-0.02	0.17	0.51	0.18	0.07	-0.15	0.01	0.00	0.04	1.81
95-99	-0.02	0.10	0.58	0.18	0.10	-0.17	0.02	0.00	0.05	1.84
100+	-0.02	0.03	0.71	0.08	0.10	-0.18	0.02	0.00	0.04	1.78

Notes: Baseline = 1951–1990, Horizon = 2010, Forecast Method = first-to-second. The m-ratios for each age group are decomposed into components representing the (approximate) contribution of each cause category to the pessimism of cause-based forecasts. For each age group, the sum of the cause components in this table equals the m-ratio minus one.

TABLE 3(b)
Simple Decomposition of M-Ratios, Japanese Men

	Infectious	Cancer	Heart	Stroke	Pneu/Bron	Senility	Accidents	Suicide	Other	M-Ratio
0	-0.04	0.02	0.15	0.09	-0.06	--	0.11	--	0.42	1.70
1-4	-0.07	0.44	0.14	0.03	-0.04	--	0.62	--	0.25	2.37
5-9	-0.03	0.80	0.01	0.03	-0.02	--	0.29	0.01	0.01	2.09
10-14	-0.03	0.55	-0.01	0.02	-0.01	--	0.02	0.23	-0.03	1.74
15-19	-0.01	0.07	-0.01	0.00	-0.01	--	0.62	-0.04	-0.07	1.55
20-24	-0.02	0.13	0.04	0.01	0.00	--	0.23	0.02	-0.05	1.37
25-29	-0.02	0.26	0.11	0.03	0.00	--	0.03	0.27	-0.05	1.63
30-34	-0.02	0.26	0.14	0.05	0.00	--	-0.03	0.50	-0.05	1.85
35-39	-0.03	0.20	0.12	0.06	0.00	--	-0.04	0.45	-0.04	1.73
40-44	-0.03	0.15	0.10	0.03	0.00	--	-0.02	0.37	-0.02	1.58
45-49	-0.03	0.17	0.10	-0.02	0.00	0.00	-0.01	0.23	-0.02	1.41
50-54	-0.03	0.26	0.09	-0.06	0.00	0.00	0.00	0.13	-0.02	1.36
55-59	-0.02	0.37	0.08	-0.08	0.00	0.00	0.01	0.04	-0.03	1.37
60-64	-0.02	0.40	0.09	-0.09	0.00	0.00	0.01	0.01	-0.04	1.36
65-69	-0.02	0.39	0.09	-0.09	0.01	0.00	0.01	0.00	-0.04	1.35
70-74	-0.02	0.41	0.10	-0.08	0.03	-0.01	0.01	0.00	-0.04	1.40
75-79	-0.02	0.44	0.14	-0.05	0.05	-0.02	0.01	0.00	-0.04	1.52
80-84	-0.02	0.48	0.21	0.00	0.08	-0.05	0.01	0.00	-0.02	1.69
85-89	-0.02	0.48	0.31	0.05	0.12	-0.08	0.00	0.00	0.00	1.86
90-94	-0.02	0.37	0.44	0.11	0.15	-0.13	0.00	0.00	0.02	1.94
95-99	-0.02	0.20	0.49	0.10	0.25	-0.16	0.00	0.00	0.03	1.90
100+	-0.03	0.03	0.41	0.06	0.17	-0.15	0.00	0.00	0.05	1.54

Notes: See notes for Table 3(a).

TABLE 4(a)
Simple Decomposition of M-Ratios (In Percent), Japanese Women

	Infectious	Cancer	Heart	Stroke	Pneu/Bron	Senility	Accidents	Suicide	Other	All
0	-5	4	24	15	-8	--	9	--	61	100
1-4	-4	37	9	2	-3	--	31	--	28	100
5-9	-2	67	1	4	-1	--	24	1	7	100
10-14	-3	81	-2	3	-1	--	5	17	0	100
15-19	-3	36	-2	2	-1	--	72	-1	-3	100
20-24	-4	49	-1	3	-1	--	45	13	-4	100
25-29	-3	61	-1	4	0	--	11	33	-5	100
30-34	-3	67	-1	4	-1	--	6	34	-7	100
35-39	-4	69	-2	7	-1	--	5	35	-10	100
40-44	-6	74	-2	4	-1	--	6	39	-14	100
45-49	-8	92	-1	-9	-1	0	7	37	-17	100
50-54	-7	103	3	-21	-1	0	7	31	-13	100
55-59	-7	114	8	-25	-1	0	5	17	-12	100
60-64	-7	116	18	-29	-1	0	4	9	-10	100
65-69	-6	105	28	-27	0	-1	4	6	-9	100
70-74	-5	86	39	-18	1	-3	3	3	-7	100
75-79	-4	64	47	-4	4	-6	2	2	-4	100
80-84	-3	46	52	8	6	-10	1	1	0	100
85-89	-3	33	56	17	8	-14	1	0	3	100
90-94	-3	21	63	22	9	-18	1	0	5	100
95-99	-3	12	70	22	12	-21	2	0	6	100
100+	-3	4	91	10	13	-23	2	0	5	100

Notes: Baseline = 1951–1990, Horizon = 2010, Forecast Method = first-to-second. The numbers in this table show the (approximate) percent contribution by cause category towards the pessimism of cause-based forecasts. For each age group, the percent contribution of a cause category equals the absolute contribution in Table 3 divided by the quantity, m-ratio minus one.

TABLE 4(b)
Simple Decomposition of M-Ratios (In Percent), Japanese Men

	Infectious	Cancer	Heart	Stroke	Pneu/Bron	Senility	Accidents	Suicide	Other	All
0	-5	3	22	13	-8	--	15	--	60	100
1-4	-5	32	10	2	-3	--	45	--	18	100
5-9	-3	73	1	3	-2	--	27	1	1	100
10-14	-4	74	-1	2	-1	--	3	32	-5	100
15-19	-2	13	-1	0	-2	--	112	-6	-14	100
20-24	-4	36	12	2	-1	--	62	6	-13	100
25-29	-3	41	18	5	-1	--	5	43	-8	100
30-34	-3	30	16	6	0	--	-3	59	-6	100
35-39	-4	28	17	8	-1	--	-5	61	-5	100
40-44	-5	26	18	5	-1	--	-4	64	-4	100
45-49	-7	42	23	-6	-1	0	-3	55	-4	100
50-54	-7	71	24	-18	-1	0	0	35	-5	100
55-59	-6	103	22	-22	-1	0	2	11	-8	100
60-64	-6	111	24	-25	0	0	3	3	-10	100
65-69	-6	113	25	-26	3	-1	4	1	-12	100
70-74	-5	102	26	-20	6	-2	3	1	-10	100
75-79	-4	86	28	-10	10	-4	2	0	-7	100
80-84	-3	70	31	0	12	-7	1	0	-3	100
85-89	-3	55	36	6	14	-10	0	0	0	100
90-94	-2	39	47	11	15	-14	0	0	2	100
95-99	-3	22	55	11	28	-17	1	0	3	100
100+	-5	6	75	12	31	-28	-1	0	10	100

Notes: See notes for Table 4(a).

TABLE 5(a)
Pollard-Type Decomposition of Δe_0, Japanese Women

	Infectious	Cancer	Heart	Stroke	Pneu/Bron	Senility	Accidents	Suicide	Other	Total
0	0.00	0.00	0.01	0.01	0.00	0.00	0.01	0.00	0.04	0.06
1-4	0.00	0.01	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.03
5-9	0.00	0.01	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.02
10-14	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
15-19	0.00	0.01	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.02
20-24	0.00	0.01	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.02
25-29	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.03
30-34	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.04
35-39	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.05
40-44	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.02	-0.01	0.05
45-49	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.02	-0.01	0.05
50-54	0.00	0.06	0.00	-0.01	0.00	0.00	0.00	0.02	-0.01	0.06
55-59	-0.01	0.09	0.01	-0.02	0.00	0.00	0.00	0.01	-0.01	0.08
60-64	-0.01	0.11	0.02	-0.03	0.00	0.00	0.00	0.01	-0.01	0.10
65-69	-0.01	0.16	0.04	-0.04	0.00	0.00	0.01	0.01	-0.01	0.15
70-74	-0.01	0.22	0.10	-0.05	0.00	-0.01	0.01	0.01	-0.02	0.25
75-79	-0.02	0.29	0.22	-0.02	0.02	-0.03	0.01	0.01	-0.02	0.46
80-84	-0.03	0.35	0.39	0.06	0.04	-0.08	0.01	0.01	0.00	0.75
85-89	-0.03	0.28	0.48	0.14	0.07	-0.12	0.01	0.00	0.03	0.86
90-94	-0.02	0.13	0.39	0.14	0.05	-0.11	0.01	0.00	0.03	0.61
95-99	-0.01	0.03	0.18	0.06	0.03	-0.05	0.00	0.00	0.02	0.26
100+	0.00	0.00	0.08	0.01	0.01	-0.02	0.00	0.00	0.00	0.08
Total	-0.15	1.94	1.92	0.26	0.22	-0.42	0.12	0.15	0.03	4.07

Notes: Baseline = 1951–1990, Horizon = 2010, Forecast Method = first-to-second. The numbers in this table show the (approximate) contribution of each age and cause category towards the difference in e_0 based on forecasts of total and all-cause mortality.

various age and cause groups, after making a full set of projections.[3] The numbers in Table 5 were calculated based on this formula. The simple first-to-second forecast was used for these calculations so that the results would be fully comparable to those in Tables 3 and 4.

These calculations verify the assertion, made earlier, that the effects of the pessimism at younger ages in the cause-based forecast are negligible if expressed in terms of aggregate measures of mortality. Although accidents are a major contributor to the m-ratios from childhood through young adulthood, this cause contributes very little to the overall difference in e_0. The trend in suicide during the middle adult ages is somewhat more important than accidents overall, especially for men, but its effect is still small compared to trends in the major degenerative diseases.

It is possible to relate the calculations in Table 5 to those in Tables 3 and 4 by manipulating equation (15). Note that the difference in the projected values of age-specific all-cause and total death rates can be written

$$\sum_i ({}_nm_x^{(i)\text{All}} - {}_nm_x^{(i)\text{Tot}}) = {}_n\hat{m}_x^{(\cdot)} - {}_n\hat{m}_x = {}_n\hat{m}_x \left(\frac{{}_n\hat{m}_x^{(\cdot)}}{{}_n\hat{m}_x} - 1 \right), \qquad (17)$$

[3]In equation (15), forecasts of cause-specific mortality rates based on the *total* mortality forecast are found by assuming that the distribution of deaths by cause remains constant and equals the distribution in the second half of the observation period, 1971–1990. Thus, implicitly, the total mortality forecast constrains the rate of change for each cause to equal the rate of change for total mortality within each age group.

TABLE 5(b)
Pollard-Type Decomposition of Δe_0, Japanese Men

	Infectious	Cancer	Heart	Stroke	Pneu/Bron	Senility	Accidents	Suicide	Other	Total
0	0.00	0.00	0.02	0.01	-0.01	0.00	0.01	0.00	0.04	0.07
1-4	0.00	0.01	0.00	0.00	0.00	0.00	0.02	0.00	0.01	0.04
5-9	0.00	0.02	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.02
10-14	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
15-19	0.00	0.01	0.00	0.00	0.00	0.00	0.07	0.00	-0.01	0.07
20-24	0.00	0.01	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.03
25-29	0.00	0.02	0.01	0.00	0.00	0.00	0.00	0.02	0.00	0.04
30-34	0.00	0.02	0.01	0.00	0.00	0.00	0.00	0.04	0.00	0.07
35-39	0.00	0.03	0.02	0.01	0.00	0.00	0.00	0.06	0.00	0.10
40-44	-0.01	0.04	0.02	0.01	0.00	0.00	-0.01	0.09	0.00	0.14
45-49	-0.01	0.06	0.03	-0.01	0.00	0.00	0.00	0.08	-0.01	0.14
50-54	-0.01	0.11	0.04	-0.03	0.00	0.00	0.00	0.05	-0.01	0.16
55-59	-0.01	0.19	0.04	-0.04	0.00	0.00	0.00	0.02	-0.01	0.19
60-64	-0.01	0.24	0.05	-0.06	0.00	0.00	0.01	0.01	-0.02	0.22
65-69	-0.02	0.32	0.07	-0.07	0.01	0.00	0.01	0.00	-0.03	0.28
70-74	-0.02	0.41	0.10	-0.08	0.03	-0.01	0.01	0.00	-0.04	0.40
75-79	-0.02	0.51	0.16	-0.06	0.06	-0.03	0.01	0.00	-0.04	0.60
80-84	-0.02	0.53	0.23	0.00	0.09	-0.05	0.01	0.00	-0.02	0.76
85-89	-0.02	0.36	0.24	0.04	0.09	-0.06	0.00	0.00	0.00	0.66
90-94	-0.01	0.14	0.16	0.04	0.05	-0.05	0.00	0.00	0.01	0.35
95-99	0.00	0.03	0.07	0.01	0.03	-0.02	0.00	0.00	0.00	0.12
100+	0.00	0.00	0.02	0.00	0.01	-0.01	0.00	0.00	0.00	0.02
Total	-0.18	3.07	1.30	-0.22	0.35	-0.23	0.17	0.38	-0.16	4.48

Notes: See notes for Table 5(a).

where ${}_n\hat{m}_x^{(\cdot)}$ and ${}_n\hat{m}_x$ are the all-cause and total mortality forecasts, respectively, at some unspecified time t (equivalent to the quantities defined previously, $\hat{m}_{xt}^{(\cdot)}$ and $\hat{m}_{xt}$, but with notation to emphasize the width of the age interval, n, rather than the time index, t). In the case of the first-to-second projection method, this equation equals

$$= {}_n\hat{m}_x \left(\sum_i {}_np_x^{(i)} \left(\frac{{}_np_x^{(i)}}{{}_nq_x^{(i)}} \right)^{(t-t_2)/(t_2-t_1)} - 1 \right)$$

$$= {}_n\hat{m}_x \sum_i {}_np_x^{(i)} \left(\left(\frac{{}_np_x^{(i)}}{{}_nq_x^{(i)}} \right)^{(t-t_2)/(t_2-t_1)} - 1 \right). \tag{18}$$

The portion inside the summand now consists of the cause-specific contributions to the pessimism of the m-ratio for this age group (as derived in equation 14 and shown in Table 3). If we label this value ${}_nC_x^{(i)}$, then Pollard's decomposition (equation 15) can be re-written

$$e_0^{\text{Tot}} - e_0^{\text{All}} = (w_0)({}_1\hat{m}_0) \sum_i {}_1C_0^{(i)} + 4(w_2)({}_4\hat{m}_1) \sum_i {}_4C_1^{(i)}$$

$$+ 5(w_{7.5})({}_5\hat{m}_5) \sum_i {}_5C_5^{(i)} + 5(w_{12.5})({}_5\hat{m}_{10}) \sum_i {}_5C_{10}^{(i)} + \cdots. \tag{19}$$

Thus, the values in Table 5 equal the values in Table 3 multiplied by the width of the age interval, the Pollard weights w_x, and the projected total age-specific mortality rate. Although the weights w_x generally decline with age, the projected total mortality rates increase rapidly with age. Thus, the values of ${}_nC_x^{(i)}$ from the older age groups play a much more significant role in the decomposition of e_0 than those at younger ages.

Table 5 confirms the primary importance of cancer mortality trends, followed (closely for women, but at some distance for men) by heart disease in accounting for the pessimism of cause-based forecasts. Observed trends in stroke mortality have declined faster than average below age 80, but slower than average among the oldest old. The combined effect of these trends is a positive contribution to the pessimism of cause-based forecasts for women, but a negative contribution for men (it may be that, for men, the trend in cancer is so unfavorable that, in comparison, the trend in stroke is more favorable than the average trend). Pneumonia/bronchitis, accidents, and suicide are relatively smaller components of the differences in forecasted e_0.

The most important causes contributing negatively to the pessimism of cause-based forecasts for both men and women are infectious diseases and senility. Both categories of causes have seen quite dramatic declines in mortality rates during 1951–1990. The rapid decline in infectious diseases should come as no surprise. The dramatic decline in mortality attributed to senility, however, is somewhat more troublesome and suggests the strong influence of changes in diagnostic practice. In 1951–1955, for example, 42 percent of all deaths aged 80 and above in Japan were attributed to senility, while only 8 percent of all deaths in this age range were so classified in 1986–1990. Undoubtedly, a portion of the deaths that previously were classified as senile deaths are now assigned to one of the major categories of degenerative disease, such as cancer, heart disease, or stroke. If true, the importance of this gradual change in diagnostic procedures operates more through its effect on trends in degenerative diseases (which, in varying degrees, may be thought of as emergent causes) than through its effect on trends in senility itself (a receding cause). Thus, the net effect of such a shift in diagnostic categories would be to increase (erroneously) the pessimism of cause-based forecasts.

4. CONCLUSIONS

The major findings of this article can be summarized as follows:

1. The pessimism of cause-based forecasts can be summarized by the m-ratio, which is the ratio (all-cause:total mortality) of forecasted age-specific death rates. In the case of a simple projection model that assumes an exact continuation of historic average annual proportional rates of mortality change, it can be shown that the all-cause projection is always more pessimistic than the total mortality projection. Empirically, other proportional change projection methods seem to behave in a similar manner, although precise analytical results are not available at this time.
2. It is possible to calculate the approximate contribution of individual causes of death to the pessimism of cause-based forecasts for a single age group using only data on the distribution of deaths by cause in two previous time periods. More

elaborate calculations permit a full decomposition of the difference in forecasted e_0 by age and cause.

3. Using Japanese mortality during 1951–1990 as an example, the pessimism of cause-based forecasts can be attributed mainly to observed trends in mortality due to cancer and heart disease, with smaller contributions due to trends in stroke (women only), pneumonia/bronchitis, accidents, and suicide.

The third conclusion requires the important qualification, noted earlier, that the observed unfavorable trends in cancer and heart disease may be severely affected by changes in diagnostic practice, reflected in the dramatic decline in deaths attributed to senility. *Science* magazine reported in 1990 that there is enormous disagreement about the proper interpretation of observed cancer mortality trends among experts in the field (*Science* 1990). Given that trends in cancer, along with heart disease, are the driving force behind the pessimism of cause-based forecasts, it is important to keep in mind at least the possibility of a major bias introduced by changes in diagnostic procedures. On the other hand, the relatively less favorable trends in accidents and suicide at younger ages (compared to overall mortality trends at those ages) might be considered more reliable (i.e., less subject to changes in diagnostic or coding practices). In the absence of more conclusive evidence than is available at this time, I would certainly feel more comfortable in taking account of differential trends in accidents and suicide when projecting mortality at ages below 50, than in treating observed trends in cancer and other degenerative diseases as the literal truth when forecasting mortality at older ages.[4]

The important general point that emerges from this discussion is that the pessimism inherent in cause-based mortality forecasts may or may not be justified in particular cases. It is perhaps best justified in the case of external causes of death (accidents, suicide, and homicide), where trends are likely to be relatively well documented over time. It is probably least well justified in the case of chronic degenerative diseases, where trends are doubtless contaminated by spurious changes in coding practices or diagnostic standards. In general, whenever there is a change in cause of death coding practices or diagnostic procedures, mortality forecasts that are disaggregated by cause are *always* unduly biased toward pessimism if historic proportional rates of change are merely projected into the future.

Even if mortality trends by cause of death are historically well recorded, however, there remain important arguments against disaggregated forecasts that deserve mention here (although it is not the purpose of this article to develop this line of discussion to the fullest extent possible). First, it may be inappropriate to forecast causes of death separately, as though these trends operate independently from one another, if in fact the various trends are interrelated. For example, when one major cause of death declines substantially or effectively disappears, the societal effort that

[4]One strategy for improving the plausibility of cause-based forecasts at older ages might be to distribute some portion of the deaths attributed to senility (or other ill-specified residual categories) to more descriptive classifications. In practice, this adjustment might or might not increase the accuracy of the cause-specific trends. In the Japanese case, I suspect strongly that it would yield an improvement in accuracy, although this conjecture is difficult or even impossible to verify. In any event, such an adjustment would automatically attenuate the pessimism of the cause-based forecast.

had been focussed on combating that disease may be re-directed to other causes of death. Second, adverse trends for a particular cause may be met with a conscious societal response. If accident rates go up, for example, the media and other influential observers may begin to talk more about accidents, and new measures to counter their rise may be implemented. Therefore, given the possibilities of interdependencies among cause-specific trends and feedback mechanisms of various kinds, the pessimism inherent in cause-based mortality forecasts may be overstated for reasons more substantial than the inaccuracies of recorded trends.

One final point regards the effect of incorporating expert opinion into cause-based forecasts on the relative pessimism of those projections. As noted by Alho (1991) and Alho and Spencer (1990), expert opinion is often used to identify an ultimate proportional rate of mortality decline for individual causes. In the mortality projections of the Social Security Administration, for example, historic average annual rates of mortality change are forced to converge over a 25-year forecast period to a set of ultimate cause-specific rates of decline, chosen based on expert opinion (Bell et al. 1992). Alho and Spencer note that these ultimate rates tend to be less variable than observed cause-specific rates of mortality decline. Thus, the use of expert opinion in the form of ultimate cause-specific rates of decline can serve to attenuate the pessimism inherent in cause-based forecasts. A full exploration of this topic in relation to the methods and findings of this article may be an important next step in understanding whether or not it is advisable to disaggregate by cause of death when making a mortality forecast.

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APPENDIX A

This appendix describes the cause of death categories used in this article. The codes given here correspond to the 9th revision of the International Classification of Diseases.

The nine cause of death categories employed in the article are as follows:

1. Infectious diseases (000–139, 487, 535, 555, 556, 558, 562, 590, 460–466, 680–686, 690–698)
2. Malignant neoplasms, or Cancer (140–208)
3. Heart diseases (393–398, 410–429)
4. Cerebrovascular disease, or Stroke (430–438)
5. Pneumonia and bronchitis (466.0, 480–486, 490, 491)
6. Senility without mention of psychosis (797)
7. Accidents (E800–E949)
8. Suicide (E950–E959)
9. Other remaining causes of death

APPENDIX B

This appendix gives proofs of two mathematical results referred to in the main text of this article. Let the m-ratio be defined as in equation (6):

$$\frac{\hat{m}_{xt}^{(\cdot)}}{\hat{m}_{xt}} = \sum_i p_x^{(i)} \left(\frac{s_x^{(i)}}{s_x} \right)^{t-t_0}. \qquad (6)$$

Recall that s_x and $s_x^{(i)}$ are forecast multipliers, which move the projection of m_{xt} forward from one year to the next, and $p_x^{(i)}$ is the proportion of deaths due to cause i in the baseline year of the forecast, t_0.

The first result states that, if $s_x^{(i)} \neq s_x$ for at least one cause i, the m-ratio in equation (6) will be greater than one for all values of $t > t_0$ if and only if the inequality in equation (7) holds true:

$$\sum_i p_x^{(i)} \ln(s_x^{(i)}/s_x) \geq 0. \qquad (7)$$

PROOF. Without loss of generality, simplify the notation in equation (6) by dropping the age subscript and setting $t_0 = 0$. Thus, let

$$M_t = \sum_i p_i \left(\frac{s_i}{s}\right)^t, \tag{B.1}$$

where $\sum_i p_i = 1$, $0 < s < \infty$, $0 < s_i < \infty$, and $s_i \neq s$ for at least one i. We must show that $M_t > 1$ for all $t > 0$ if and only if

$$\sum_i p_i \ln(s_i/s) \geq 0. \tag{B.2}$$

Compute the first and second partial derivatives of M_t with respect to t, and note that the second derivative is always positive:

$$\frac{\partial}{\partial t} M_t = \sum_i p_i \left(\frac{s_i}{s}\right)^t \ln(s_i/s) \tag{B.3}$$

$$\frac{\partial^2}{\partial t^2} M_t = \sum_i p_i \left(\frac{s_i}{s}\right)^t (\ln(s_i/s))^2 > 0. \tag{B.4}$$

Let S be the first partial derivative of M_t evaluated at $t = 0$:

$$S = \left.\frac{\partial}{\partial t} M_t\right|_{t=0} = \sum_i p_i \ln(s_i/s). \tag{B.5}$$

Suppose that $S \geq 0$. Since $(\partial^2/\partial t^2)M_t > 0$, then $(\partial/\partial t)M_t > 0$, and therefore $M_t > M_0 = 1$ for all $t > 0$. Alternatively, suppose $S < 0$. Then, M_t must fall below one for some values of t (although it will eventually rise above one since its second derivative is positive). □

The second result states that the m-ratio will always be greater than one if the forecast multipliers, s_x and $s_x^{(i)}$, are derived as follows:

$$s_x = \left(\frac{m_{xt_0}}{m_{x\tau}}\right)^{1/(t_0-\tau)}, \tag{8}$$

and

$$s_x^{(i)} = \left(\frac{m_{xt_0}^{(i)}}{m_{x\tau}^{(i)}}\right)^{1/(t_0-\tau)}, \tag{9}$$

where τ and t_0 refer to the first and last years, respectively, of the observation period. When the forecast multipliers are so defined, equations (10) and (11) demonstrate that the m-ratio can be written

$$\frac{\hat{m}_{xt}^{(\cdot)}}{\hat{m}_{xt}} = \sum_i p_x^{(i)} \left(\frac{p_x^{(i)}}{q_x^{(i)}}\right)^{(t-t_0)/(t_0-\tau)}, \tag{11}$$

where $q_x^{(i)}$ and $p_x^{(i)}$ are the proportion of deaths due to cause i (at age x) in years τ and t_0, respectively. If the two distributions, $q_x^{(i)}$ and $p_x^{(i)}$, are different, then this sum is always greater than one for $t > 0$.

PROOF. Without loss of generality, simplify the notation in equation (11) by dropping the age subscript and re-scaling the time index. Thus, let

$$M_t = \sum_i p_i \left(\frac{p_i}{q_i}\right)^t, \tag{B.6}$$

where $0 < p_i < 1$ and $0 < q_i < 1$ for all i, and $\sum_i p_i = \sum_i q_i = 1$. We will show that $M_t \geq 1$, with equality if and only if $p_i = q_i$ for all $i = 1,2,\ldots,n$.

1) First, show that a local minimum for M_t exists when $p_i = q_i$ for all i and that this minimum equals one. Note that M_t can be re-written

$$M_t = \sum_{i=1}^{n-1} \frac{p_i^{(t+1)}}{q_i^t} + \frac{(1 - p_1 - \cdots - p_{n-1})^{(t+1)}}{(1 - q_1 - \cdots - q_{n-1})^t}. \tag{B.7}$$

For a given set of q_i's, any local extremum must satisfy

$$\frac{\partial}{\partial p_a} M_t = (t+1)\left(\frac{p_a}{q_a}\right)^t - (t+1)\left(\frac{1 - p_1 - \cdots - p_{n-1}}{1 - q_1 - \cdots - q_{n-1}}\right)^t = 0, \tag{B.8}$$

eq. for all $a = 1,\ldots,n-1$. Thus,

$$\frac{p_a}{q_a} = \frac{1 - p_1 - \cdots - p_{n-1}}{1 - q_1 - \cdots - q_{n-1}} = \frac{p_n}{q_n}, \tag{B.9}$$

for all $a = 1,\ldots,n-1$. Clearly, if $p_i = q_i$ for all i, then (B.9) is satisfied. To show that this extremum is a local minimum, note that

$$\frac{\partial^2}{\partial p_a^2} M_t = t(t+1)\left(\frac{p_a}{q_a}\right)^t - 1 + t(t+1)\left(\frac{1 - p_1 - \cdots - p_n - 1}{1 - q_1 - \cdots - q_n - 1}\right)^t - 1 > 0. \tag{B.10}$$

Obviously, when $p_i = q_i$ for all i, then $M_t = 1$.

2) Next, show (by contradiction) that this minimum is unique. Other minima or maxima would need to satisfy (B.9). For a given set of q_i's, suppose there were some $p_i \neq q_i$ that satisfied (B.9). Thus, for some i, we would have

$$\frac{p_i}{q_i} = r \neq 1. \tag{B.11}$$

But according to (B.9), in order for the sum M_t to be at a local minimum or maximum, this ratio must be the same for all i. Thus, $p_i = r \cdot q_i$ for all i, and

$$\sum_i p_i = r \cdot \sum_i q_i = r \neq 1, \tag{B.12}$$

which contradicts the stated conditions. Thus, the minimum that occurs when $p_i = q_i$ is the unique extremum for the sum, M_t. □

APPENDIX C

This appendix provides details about the six projection methods employed in this article. Tabulations of deaths and estimates (by the author) of the exposure-to-risk were available by age, sex, year, and cause of death. Age groups were 0, 1–4, 5–9, 10–14,...,95–99, and 100+. Years were 1951, 1952,...,1990. Cause of death categories were those described in Appendix A. All projections were made separately for male and female mortality. The "all-cause" mortality forecast always refers to the sum of the forecasts of the individual causes; the "total" mortality forecast refers to the projection of aggregate mortality rates.

Three of the forecasts consisted of separate projections of trends in age-specific death rates, for both total and cause-specific mortality:

1a. Method 1a is the first-to-last projection technique described in the main text (see equations 8–12). Age-specific death rates for the first and last 3-year periods were calculated by dividing total deaths in each interval by the estimated exposure-to-risk. The historic average annual proportional rate of change for each age group was calculated as follows:

$$r_x = 1 - \left(\frac{m_{xt_2}}{m_{xt_1}} \right)^{1/(t_2 - t_1)} \tag{C.1}$$

for total mortality;

$$r_x^{(i)} = 1 - \left(\frac{m_{xt_2}^{(i)}}{m_{xt_1}^{(i)}} \right)^{1/(t_2 - t_1)} \tag{C.2}$$

for mortality due to cause i. These rates of change are held constant over the forecast interval. It is important to remember that t_1 and t_2 are the mid-points of the first and last 3-year intervals in the baseline period, and that all projections of future mortality should use t_2 (not the last year of observed mortality) as their starting point.

1b. Method 1b fits ordinary least squares (OLS) regression lines to the time trend of the log-death rates for each age group. The forecasted value of (total or cause-specific) mortality is then found by making a simple extrapolation of this linear trend and computing the anti-log.

1c. Method 1c is identical to 1b, except that it uses weighted least squares (WLS) rather than OLS. The weights in this example were assumed to be 1, 2,...,40. Weights that increase more steeply were found to diminish the difference between total and all-cause mortality forecasts in most cases. An inspection of Figure 4 verifies that the m-ratios for the WLS projections are usually, but not always, below those of the comparable OLS and first-to-last projections (i.e., compare line 5 to lines 3 and 4, or line 8 to lines 6 and 7). It is unclear why this result was observed and whether it should be true in general.

The remaining three forecasts were adaptations of the procedure proposed by Lee and Carter (1992). The Lee-Carter model is written as follows:

$$f_{xt} = \ln(m_{xt}) = a_x + b_x k_t + e_{xt}, \tag{C.3}$$

where m_{xt} are death rates by age and time, a_x is the "average" age schedule of log-mortality, b_x is an age profile of mortality change, k_t is a time index, and e_{xt} is an error term. This model was fit separately to total and cause-specific mortality rates using a WLS procedure described in Wilmoth (1993a). Mortality forecasts were found by projecting the time index, k_t, then computing mortality rates based on forecasted values of k_t and the original fitted values of a_x and b_x. The three methods of projecting k_t were as follows:

2a. Method 2a follows the original recommendation of Lee and Carter by setting the slope of projected k_t equal to the average difference between the first and last years of the observation period. (Unlike Method 1a, I did not average over the first and last 3 years of the observation period: annual mortality fluctuations are less important in this case, since k_t is already the composite of trends across the age range.) Projected values of k_t begin at the fitted value of k_t for the final year of the observation period, with the change in each subsequent year equal to the average change. Thus,

$$\hat{k}_t = \hat{k}_{t_0} + (t - t_0)\left(\frac{\hat{k}_{t_0} - \hat{k}_\tau}{t_0 - \tau}\right), \tag{C.4}$$

where $t > t_0$, and τ and t_0 are the first and last years of the observation period.

2b. Method 2b extrapolates the fitted values of k_t by fitting an OLS regression line.

2c. Method 2c extrapolates the fitted values of k_t by fitting a WLS regression line, with weights (in this example) equal to $1, 2, \ldots, 40$.