Age Patterns of Marriage*

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From the earliest studies of death records in Graunt's *Observations Made Upon the Bills of Mortality* demographers have tried to find regularities in the age patterns of death rates. Makeham, Gompertz and others looked for mathematical formulae to represent what they hoped were universal laws of mortality; recently regularities in the age patterns of human mortality have been embodied in model life tables. These tables, incorporating empirical associations among mortality rates at different ages, are now widely used in research, in making population projections, and so on.

Attempts to find laws of fertility and to construct model schedules of fertility have been less frequent and less successful. Still less attention has been given to the age pattern of marriage. Yet, as will be shown in this paper, there are precisely defined age patterns of nuptiality, readily approximated by a simple mathematical expression, that are followed very closely indeed in populations under widely different social conditions – for example, western European populations before, during, and after the revolution in nuptiality that occurred since just before World War II.

The existence of common age patterns of marriage was discovered as a by-product of research on the historic decline of marital fertility in Europe now being conducted at the Office of Population Research. In the course of this research we needed to estimate the distribution of women by age and marital status in instances where the relevant cross tabulations of census data were non-existent or insufficiently detailed. We were thus drawn to examine age patterns of the marital status distribution of women in different populations in the hope of finding regularities that might be useful in preparing estimates.

UNIFORMITY IN THE AGE PATTERN OF PROPORTIONS EVER-MARRIED

An examination of the rising curve of proportions of women ever-married by single years of age in a number of European populations in the late nineteenth and early twentieth centuries (when within each population age at marriage and proportions ultimately marrying were generally stable) suggested that in these populations the curves were essentially the same in structure, differing only in the age at which marriage began, the rate at which marriage increased, and the ultimate proportion ever-married – differing, in other words, in origin, horizontal scale, and vertical scale, but with the same functional form.

Fig. 1 shows a pair of curves (for Netherlands, 1859, and Germany, 1910) selected from eight schedules of proportions ever-married calculated for populations in Western Europe before World War I. (The others examined were Sweden, 1890, Denmark, 1890 and 1911, Norway, 1876, and Belgium, 1846 and 1910.) In Fig. 2 the two curves in Fig. 1 are re-drawn with a common starting point, with a vertical scale for each adjusted by a multiple that makes the proportion ever-married at advanced ages equal to 1·0, and with a horizontal scale chosen so that the average rate of rise for the two curves is about the same. (See the Appendix for the method of determining origin

* The ideas summarized in this paper have emerged over a period of several years in conversations between the author and others at the Office of Population Research, especially Ivan Lakos and Etienne van de Walle. Professor Lakos, Chairman of the Department of Economics at Antioch College, devoted the academic year of 1968–69 to research and study in demography at Princeton. Among his projects was the estimation of age-and-marital-status distributions from incomplete data, and it was in connection with his work that the existence of a standard pattern of nuptiality was first noted. Dr. van de Walle's estimations of age-and-marital-status distributions in nineteenth-century France helped develop some of the techniques described below.

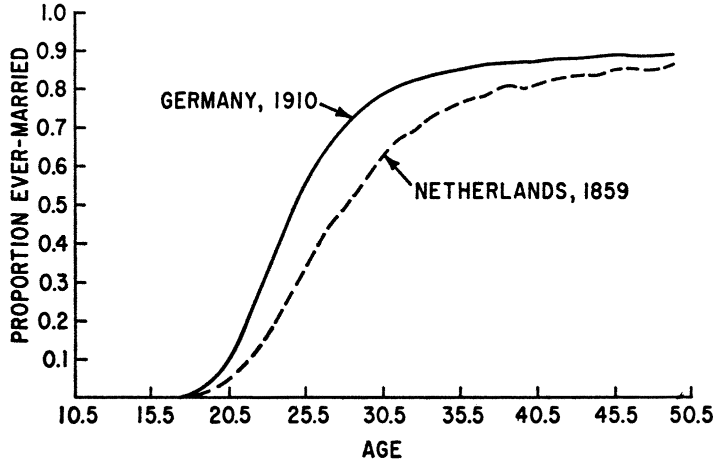

FIGURE 1. *Proportion ever-married, Netherlands, 1859, and Germany, 1910*

and scales.) Note that although origin and scale have been altered for each schedule, the points plotted in Fig. 2 are a graph of exactly the same individual proportions ever-married plotted in Fig. 1. The two curves in Fig. 2 are certainly very much alike. The data on the other six pre-World War I western European populations form a cluster of virtually identical first-marriage frequencies when drawn from a common origin and with appropriately chosen horizontal and vertical scales.

The startling conformity of these linearly transformed schedules of proportions ever-married suggests the existence of some sort of law governing the ages at which first marriages occur. However, the data examined relating to this point are from populations subject to what Hajnal calls the European pattern of marriage – a pattern of marriage that occurs late relative to most human experience, a pattern in which the proportions remaining permanently single are high.[1]

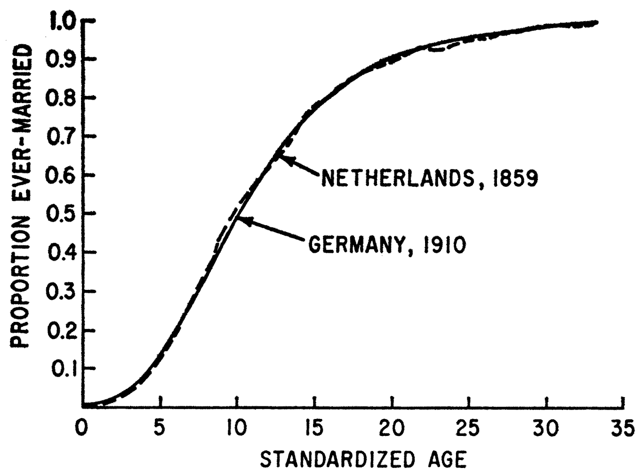

FIGURE 2. *Proportions ever-married, adjusted scale and origin, Netherlands, 1859, and Germany, 1910*

[1] J. Hajnal, 'European marriage patterns in perspective', in D. V. Glass and D. E. C. Eversley (Eds.), *Population in History* (London, 1964).

A natural question is whether the common structure (subject to choice of scale and origin) found in western Europe also characterizes proportions ever-married in populations where first-marriage occurs earlier and is more nearly universal. The requisite schedules of proportions ever-married by single years of age are not easily found, because conformity to a standard pattern is logically characteristic of the cumulative first-marriage experience of a cohort of women as they move through life, rather than of a cross-section – a population in which cumulative first-marriage experience is recorded at a given moment of time for different women at each age. However, when nuptiality customs are stable, the marital status distribution by age of a population resembles that of any of the different cohorts constituting the population. Therefore, to determine whether the pattern of proportions ever-married by age found in turn-of-the-century west European populations also characterizes populations with earlier and more universal marriage, we needed to find data on marital status either for cohorts, or for cross-sections when nuptiality had been stable, or nearly so. Three additional schedules of proportions ever-married by age are shown (together with the two already examined) in Fig. 3. The U.S. schedule for 1930 was chosen because proportions ever-married by age in the United States had changed only slightly since 1890 (the first census to record marital status); the Hungarian schedule for 1960 was chosen because in Hungary a pattern of early marriage had been followed since the late nineteenth century at least, (nevertheless an imperfect choice because there had been disturbances in nuptiality during World War II and in the late 1950's); the Taiwanese schedule was chosen because of the extraordinary accuracy of the censuses in Taiwan, and the very early marriage in the Taiwanese population. There were moderate changes in nuptiality from 1915 to 1935, as shown in Table 1. Data on marital status by single years of age were available to us only for the censuses of 1915, 1920 and 1935. If the three cross-sections are plotted separately, each shows moderate deviations from the other two over a range of ages, reflecting the typical difference between cohorts and cross-sections when nuptiality is not constant. The closest approach to cohort nuptiality is obtained by taking the average of the proportion ever-married in the three censuses.

TABLE 1. *Per cent single among females in Taiwan, five-year age intervals, 1915–1940*

	1915	1920	1930	1935	1940
15–19	65·292	67·218	67·387	71·881	70·472
20–24	12·615	13·368	13·709	17·048	15·595
25–29	3·355	3·064	3·866	4·080	4·072
30–34	1·465	1·515	2·003	2·302	1·657
35–39	0·883	1·000	1·194	1·590	1·188
40–44	0·605	0·723	0·756	1·094	0·841
45–49	0·487	0·584	0·637	0·735	0·617
50–54	0·298	0·358	0·525	0·764	0·432
55–59	0·204	0·221	0·592	1·003	0·508

SOURCE: *Report of the Seventh Population Census of Taiwan, 1940*, Table 14, pp. 58–59. Bureau of Accounting and Statistics, Provincial Government of Taiwan. March 1953.

In Fig. 4 the two original and the three additional schedules of proportions ever-married are plotted, using a common origin and individual horizontal and vertical scales chosen, as before, to yield the same proportions ultimately ever-married, and the same average pace of increase in proportions. The conformity of the additional schedules is remarkable. Note that the dip originally at age 30 in the United States (surely the effect of less 'age heaping' at 30 among the married) is the most deviant point. It is clear from Fig. 4 that the common age pattern of proportions ever-married that characterized western European populations in the nineteenth and early twentieth centuries is also characteristic of non-European populations with very different marriage customs.

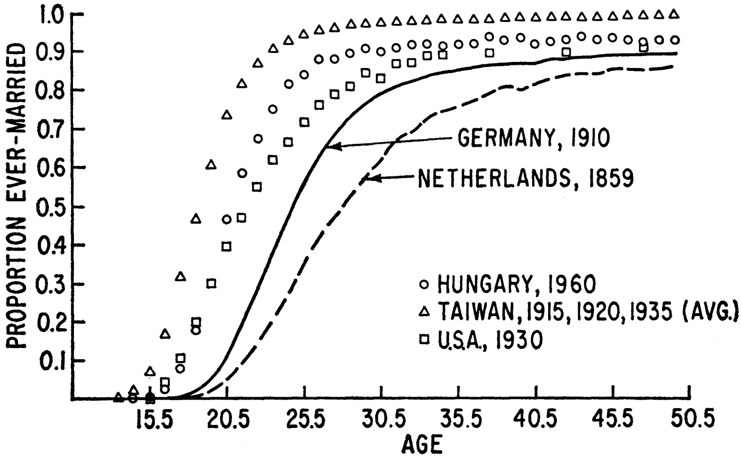

FIGURE 3. *Proportions ever-married, selected populations*

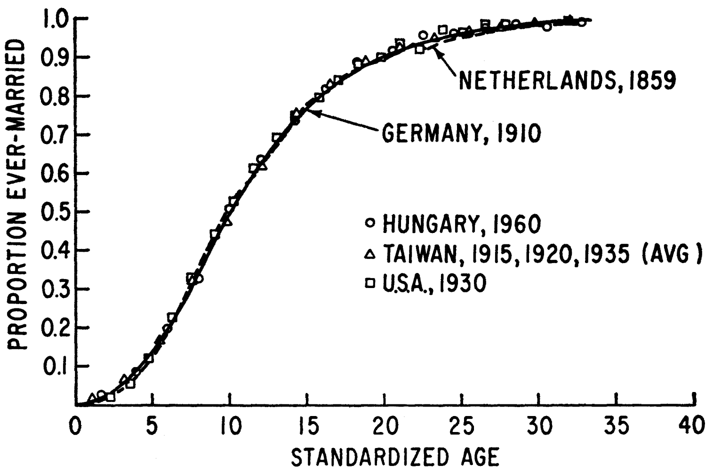

FIGURE 4. *Proportions ever-married, adjusted scale and origin selected populations*

UNIFORMITY OF THE AGE PATTERN OF FIRST-MARRIAGE FREQUENCIES

If frequency of first marriage is defined as the number of first marriages in an age interval divided by the number of women (regardless of marital status) in the interval, it follows that the cumulation of first-marriage frequencies from the earliest age of marriage to a given age is (in a closed cohort) the proportion ever-married at the given age.[2] Therefore, the existence of a common pattern of proportions ever-married by age in different populations implies that there is a common pattern of first-marriage frequencies in these populations. Specifically, first-marriage frequency curves for different populations should differ only in origin, horizontal scale and total area (or vertical scale).

[2] This relation is exact only if the ever-married and the single populations are subject to the same probabilities of survival. Trial calculations show that the effect of differential mortality on the proportion ever-married is small, at least until age 50. See J. Hajnal, 'Age at marriage and proportions marrying', *Population Studies*, 7, 2, November 1953, pp. 111–136.

Cumulation diminishes the influence of small differences and irregularities, so that a comparison of first-marriage frequencies is a more sensitive test of the similarity of structure of nuptiality in different populations than a comparison of proportions ever-married.

The relation between first-marriage frequencies and proportion ever-married logically applies to a cohort, and need not hold at all for a population at a moment of time. In fact, when the incidence of marriage is increasing, it is not uncommon for the proportion ever-married to be substantially greater at a particular age than at a more advanced one, a relation that would be impossible in a cohort. Moreover, calculations for Sweden show, as a matter of empirical fact, that first-marriage frequencies in cohorts conform more closely to a common pattern than first-marriage frequencies for cross-sections.

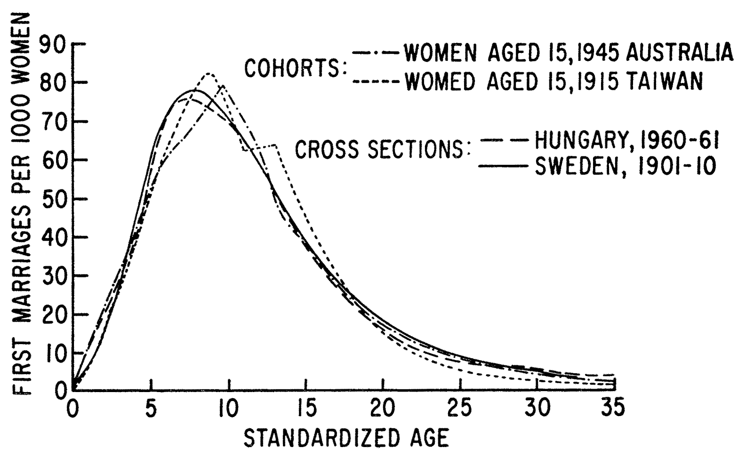

FIGURE 5. *First-marriage frequency (first marriages per thousand women) by single years of age, selected populations*

It would be preferable, then, to examine the age pattern of first-marriage frequencies in cohorts drawn from a number of populations. However, reliable data on first-marriage frequencies by single years of age for cohorts are scarce. Our first test of the age pattern of marriage frequencies is consequently based on a somewhat unsatisfactory collection of examples: the cohort of women who were aged 15 in 1915 in Taiwan, the Australian cohort aged 15 in 1944, and two cross-sections – the Swedish population in 1901–10, and the Hungarian population in 1960–61. The schedules are depicted in Fig. 5. The Swedish schedule was included as typical of first-marriage frequencies in a population with the west European pattern of late marriage, and the Hungarian schedule as an accurate record with relatively early marriage. In Sweden before World War I marriage schedules were quite stable so that a cross-sectional schedule is adequately representative of cohort experience. The Hungarian schedule is less satisfactory from this point of view. The first-marriage frequencies shown from Taiwan are taken from the population register, which has a reputation for remarkable completeness and accuracy. However, the registration of marriages was *not* complete as we know from the recording of married women in the Census of 1935: women whose marriages had not been registered were recorded in a separate column, and nearly 5% of the total had unregistered marriages. The dip in first marriages at age 18 evident in Fig. 5 is possibly the effect of delayed registration. Nevertheless, the Taiwanese data were the best records we found of first-marriage frequencies by single years of age in a population with such early marriage.

The data for the cohort in Australia incorporate not only the post-war marriage boom but are also affected by a very substantial gain in the size of the cohort through in-migration. In all, the schedules shown in Fig. 5 are based on less than ideal statistics, but they are at least quite diverse. It is thus surprising to find that by choice of starting point, vertical and horizontal scale, these curves can be brought as nearly in line as they are in Fig. 6. The approximate conformity to a common pattern occurs in spite of the profound differences among the populations whose first-marriage frequencies are recorded. In the Swedish population, about 25% of women remain single at age 50; in Taiwan the proportion remaining single was less than 1%.

The first-marriage frequencies experienced by a series of Swedish cohorts from early in the century to after World War II provide our next test. In Fig. 7 the recorded first-marriage frequencies are plotted, and in Fig. 8 the same data are shown when each curve is drawn from the same origin, with appropriately chosen horizontal and vertical scales for each curve. Again, the

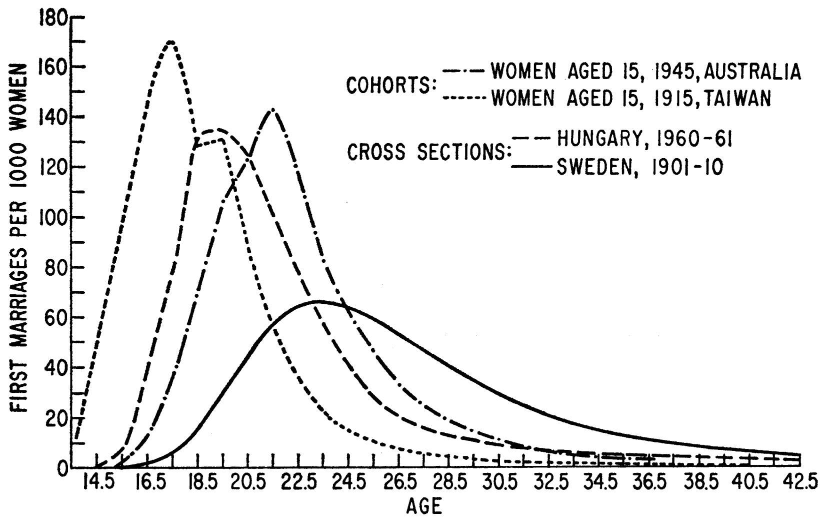

FIGURE 6. *First-marriage frequency, adjusted scale and origin, selected populations*

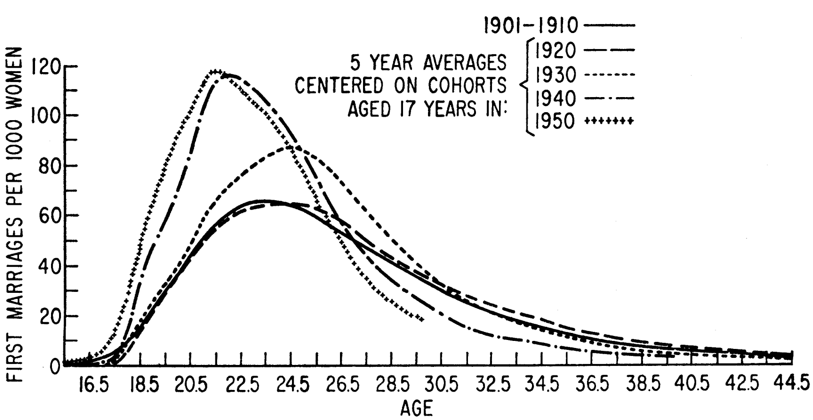

FIGURE 7. *First-marriage frequency, selected cohorts in Sweden*

closeness of fit is surprising. The unadjusted first-marriage frequencies clearly delineate a revolution in nuptiality in Sweden. The later cohorts married at an earlier age and the first-marriage frequencies cumulate to a proportion ever-married of over 90% rather than only some 75% at the beginning of the century. Yet, aside from distortions under extreme circumstances, such as a dip in marriages that occurred in the early 1930's, the first-marriage frequencies of the different cohorts follow a very similar pattern when adjusted to a common scale and starting point.

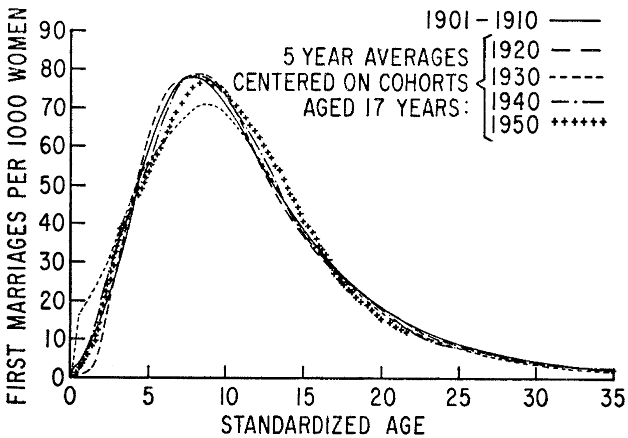

FIGURE 8. *First-marriage frequency, adjusted scale and origin, selected cohorts in Sweden*

AN IDEALIZED CURVE OF FIRST-MARRIAGE FREQUENCIES, OF PROPORTIONS EVER-MARRIED BY AGE, AND OF PERSON–YEARS LIVED IN THE EVER-MARRIED STATE

When the existence of a common pattern of first-marriage frequencies was first noticed, it seemed advantageous for various purposes to construct a curve that best represents the common pattern – a 'standard form' that might serve as a basis for various kinds of estimation and computation. The 'standard' was based on first-marriage frequencies recorded in Sweden in 1965–69. These recorded frequencies were first subjected to minor adjustments to remove irregularities that were evident upon the calculation of first differences. When this slightly modified curve was compared with the first-marriage frequencies in subsequent Swedish cohorts, it was found that first-marriage frequencies above age 35 had been uncharacteristically low in 1865–69 in Sweden. A moderate upward adjustment in the marriage rates was made at these ages. By curvilinear interpolation within single years of age, the resultant standard curve has been calculated at intervals of hundredths of a year, and cumulated (integrated) from the origin to each age to provide a standard curve of proportions ever-married by age, then integrated a second time to provide a standard curve of person–years lived in the ever-married state by a hypothetical cohort not subject to mortality. The last function was calculated to provide a means for determining the proportion ever-married in various age-intervals by subtraction of the person–years lived ever-married at the beginning of the interval from the person–years lived ever-married at the end.

Table 2 presents values of the standard first-marriage frequencies at intervals of one-tenth of the year. Table 3 gives values of the proportions ever-married according to the standard schedule, and Table 4 values of the person–years lived ever-married by a hypothetical cohort not subject to mortality. The curves are adjusted to a vertical scale that yields 100% ever-married by the end of life.

TABLE 2. *Standard schedule of first-marriage frequencies (per 1,000 women) at intervals of 0·1 years*

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0	0.0	0.72	1.44	2.16	2.88	3.60	4.34	5.08	5.82	6.56
1.0	7.30	8.04	8.78	9.52	10.26	10.78	11.72	12.71	13.73	14.80
2.0	15.91	17.05	18.24	19.47	20.73	22.04	23.38	24.77	26.19	27.65
3.0	29.14	30.68	32.26	33.88	35.53	37.23	38.73	40.23	41.73	43.22
4.0	44.71	46.20	47.68	49.16	50.64	52.12	53.54	54.94	56.34	57.72
5.0	59.09	60.44	61.78	63.11	64.43	65.74	66.81	67.82	68.79	69.70
6.0	70.56	71.37	72.12	72.82	73.47	74.06	74.64	75.16	75.64	76.08
7.0	76.46	76.80	77.09	77.34	77.54	77.69	77.87	78.01	78.12	78.20
8.0	78.25	78.26	78.24	78.19	78.10	77.98	77.81	77.59	77.34	77.06
9.0	76.73	76.37	75.97	75.54	75.06	74.55	74.08	73.58	73.06	72.52
10.0	71.96	71.37	70.75	70.12	69.46	68.77	68.10	67.42	66.71	65.99
11.0	65.26	64.50	63.74	62.95	62.15	61.33	60.59	59.85	59.11	58.38
12.0	57.64	56.91	56.17	55.44	54.71	53.98	53.27	52.57	51.87	51.18
13.0	50.49	49.81	49.13	48.45	47.79	47.12	46.46	45.81	45.16	44.52
14.0	43.88	43.24	42.61	41.99	41.37	40.75	40.17	39.59	39.02	38.45
15.0	37.90	37.36	36.82	36.29	35.78	35.27	34.74	34.22	33.70	33.18
16.0	32.67	32.16	31.66	31.16	30.67	30.17	29.70	29.23	28.76	28.30
17.0	27.85	27.40	26.96	26.52	26.09	25.67	25.26	24.87	24.49	24.11
18.0	23.74	23.39	23.04	22.70	22.37	22.04	21.70	21.36	21.03	20.70
19.0	20.38	20.06	19.74	19.42	19.12	18.81	18.51	18.21	17.92	17.63
20.0	17.34	17.06	16.78	16.51	16.23	15.97	15.71	15.45	15.19	14.94
21.0	14.69	14.45	14.21	13.98	13.75	13.52	13.29	13.06	12.84	12.62
22.0	12.40	12.19	11.98	11.77	11.57	11.36	11.16	10.96	10.76	10.56
23.0	10.36	10.16	9.97	9.78	9.59	9.40	9.22	9.05	8.87	8.70
24.0	8.54	8.37	8.21	8.05	7.89	7.74	7.60	7.46	7.34	7.21
25.0	7.09	6.98	6.86	6.76	6.66	6.56	6.46	6.36	6.27	6.18
26.0	6.09	6.00	5.92	5.83	5.76	5.68	5.60	5.53	5.46	5.38
27.0	5.31	5.25	5.18	5.12	5.06	5.00	4.94	4.88	4.83	4.78
28.0	4.73	4.68	4.63	4.59	4.55	4.51	4.46	4.41	4.36	4.31
29.0	4.26	4.21	4.16	4.11	4.07	4.02	3.96	3.91	3.85	3.79
30.0	3.73	3.68	3.62	3.55	3.49	3.43	3.38	3.32	3.27	3.22
31.0	3.17	3.12	3.08	3.03	2.98	2.94	2.89	2.84	2.79	2.74
32.0	2.69	2.65	2.60	2.55	2.50	2.45	2.41	2.36	2.32	2.28
33.0	2.24	2.20	2.16	2.13	2.09	2.06	2.02	1.98	1.94	1.90
34.0	1.86	1.82	1.78	1.74	1.70	1.67	1.63	1.60	1.57	1.54
35.0	1.51	1.48	1.45	1.42	1.40	1.37	1.35	1.32	1.30	1.28
36.0	1.26	1.24	1.22	1.21	1.19	1.18	1.16	1.14	1.12	1.10
37.0	1.08	1.06	1.04	1.02	1.00	0.98	0.96	0.94	0.92	0.90
38.0	0.88	0.86	0.84	0.82	0.80	0.78	0.76	0.74	0.72	0.71
39.0	0.69	0.67	0.65	0.63	0.61	0.59	0.57	0.55	0.53	0.51

TABLE 3. *Standard schedule of proportions ever-married (per 1,000 women) at intervals of 0·1 years*

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0	0.0	0.04	0.14	0.32	0.58	0.90	1.30	1.77	2.31	2.93
1.0	3.62	4.39	5.23	6.15	7.14	8.20	9.32	10.54	11.87	13.29
2.0	14.83	16.48	18.24	20.12	22.13	24.27	26.54	28.95	31.50	34.19
3.0	37.03	40.02	43.17	46.47	49.94	53.58	57.38	61.33	65.42	69.67
4.0	74.07	78.61	83.31	88.15	93.14	98.28	103.56	108.98	114.55	120.25
5.0	126.09	132.07	138.18	144.42	150.80	157.31	163.93	170.67	177.50	184.42
6.0	191.43	198.53	205.71	212.95	220.27	227.64	235.08	242.57	250.11	257.70
7.0	265.32	272.99	280.68	288.40	296.15	303.91	311.69	319.48	327.29	335.10
8.0	342.93	350.75	358.58	366.40	374.21	382.02	389.81	397.58	405.32	413.04
9.0	420.73	428.39	436.01	443.58	451.11	458.59	466.02	473.41	480.74	488.02
10.0	495.24	502.41	509.51	516.56	523.54	530.45	537.29	544.07	550.77	557.41
11.0	563.97	570.46	576.87	583.21	589.46	595.63	601.73	607.75	613.70	619.57
12.0	625.37	631.10	636.75	642.34	647.84	653.28	658.64	663.93	669.14	674.29
13.0	679.37	684.39	689.33	694.20	699.01	703.76	708.43	713.04	717.59	722.07
14.0	726.49	730.84	735.13	739.35	743.51	747.61	751.66	755.64	759.57	763.44
15.0	767.25	771.01	774.72	778.37	781.97	785.52	789.02	792.46	795.86	799.20
16.0	802.49	805.73	808.91	812.05	815.14	818.18	821.17	824.11	827.01	829.86
17.0	832.66	835.42	838.14	840.81	843.43	846.02	848.56	851.07	853.53	855.96
18.0	858.35	860.70	863.02	865.30	867.55	869.77	871.95	874.10	876.22	878.30
19.0	880.35	882.37	884.36	886.31	888.24	890.13	891.99	893.82	895.63	897.40
20.0	899.15	900.86	902.55	904.21	905.85	907.45	909.03	910.59	912.12	913.62
21.0	915.10	916.55	917.98	919.39	920.77	922.13	923.47	924.79	926.08	927.35
22.0	928.60	929.82	931.03	932.22	933.38	934.52	935.65	936.75	937.83	938.89
23.0	939.94	940.96	941.96	942.95	943.91	944.86	945.79	946.70	947.59	948.47
24.0	949.33	950.17	951.00	951.81	952.60	953.38	954.14	954.89	955.63	956.35
25.0	957.07	957.77	958.45	959.13	959.80	960.46	961.11	961.74	962.37	962.99
26.0	963.60	964.20	964.79	965.38	965.96	966.52	967.09	967.64	968.18	968.72
27.0	969.26	969.78	970.30	970.81	971.32	971.82	972.31	972.80	973.28	973.76
28.0	974.23	974.70	975.16	975.62	976.07	976.52	976.97	977.41	977.84	978.27
29.0	978.70	979.12	979.53	979.94	980.35	980.75	981.15	981.54	981.92	982.30
30.0	982.67	983.04	983.40	983.76	984.11	984.45	984.79	985.12	985.45	985.77
31.0	986.08	986.40	986.70	987.00	987.30	987.60	987.88	988.17	988.45	988.72
32.0	988.99	989.25	989.51	989.76	990.01	990.26	990.50	990.73	990.96	991.19
33.0	991.41	991.63	991.85	992.06	992.27	992.47	992.67	992.87	993.06	993.25
34.0	993.44	993.62	993.79	993.97	994.14	994.30	994.46	994.62	994.78	994.93
35.0	995.08	995.22	995.37	995.51	995.65	995.78	995.91	996.04	996.17	996.30
36.0	996.42	996.55	996.66	996.78	996.90	997.02	997.13	997.24	997.35	997.46
37.0	997.56	997.67	997.77	997.87	997.97	998.06	998.15	998.25	998.34	998.42
38.0	998.51	998.60	998.68	998.76	998.84	998.91	998.99	999.06	999.13	999.20
39.0	999.27	999.33	999.39	999.45	999.51	999.57	999.62	999.67	999.72	999.77

TABLE 4. *Standard schedule of person–years ever-married (per 1,000 women) at intervals of 0·1 years*

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0	0.0	0.00	0.01	0.03	0.08	0.15	0.26	0.41	0.62	0.88
1.0	1.20	1.60	2.09	2.65	3.32	4.08	4.96	5.95	7.07	8.33
2.0	9.73	11.30	13.03	14.95	17.06	19.38	21.92	24.69	27.72	31.00
3.0	34.56	38.41	42.57	47.05	51.87	57.04	62.59	68.52	74.86	81.61
4.0	88.80	96.43	104.52	113.09	122.16	131.73	141.82	152.44	163.62	175.36
5.0	187.67	200.58	214.09	228.22	242.98	258.38	274.44	291.17	308.58	326.67
6.0	345.47	364.96	385.17	406.11	427.77	450.16	473.30	497.18	521.81	547.20
7.0	573.35	600.27	627.95	656.40	685.63	715.63	746.40	777.96	810.29	843.41
8.0	877.31	911.99	947.45	983.70	1020.72	1058.53	1097.12	1136.49	1176.63	1217.54
9.0	1259.23	1301.68	1344.90	1388.88	1433.61	1479.09	1525.32	1572.29	1619.99	1668.43
10.0	1717.59	1767.47	1818.06	1869.36	1921.36	1974.06	2027.45	2081.51	2136.25	2191.66
11.0	2247.73	2304.44	2361.81	2419.81	2478.44	2537.69	2597.56	2658.03	2719.10	2780.76
12.0	2843.01	2905.83	2969.22	3033.17	3097.68	3162.73	3228.32	3294.45	3361.10	3428.27
13.0	3495.95	3564.14	3632.82	3702.00	3771.65	3841.79	3912.40	3983.47	4055.00	4126.98
14.0	4199.40	4272.26	4345.56	4419.28	4493.42	4567.98	4642.94	4718.30	4794.05	4870.20
15.0	4946.73	5023.65	5100.93	5178.58	5256.59	5334.97	5413.69	5492.76	5572.17	5651.92
16.0	5732.01	5812.41	5893.14	5974.19	6055.54	6137.21	6219.17	6301.43	6383.99	6466.83
17.0	6549.95	6633.35	6717.03	6800.97	6885.18	6969.65	7054.38	7139.36	7224.58	7310.06
18.0	7395.77	7481.72	7567.90	7654.31	7740.95	7827.82	7914.90	8002.20	8089.71	8177.44
19.0	8265.37	8353.50	8441.83	8530.36	8619.09	8708.00	8797.11	8886.40	8975.87	9065.51
20.0	9155.34	9245.34	9335.50	9425.84	9516.34	9607.00	9697.82	9788.80	9879.94	9971.22
21.0	10062.65	10154.23	10245.96	10337.82	10429.83	10521.95	10614.18	10706.55	10799.04	10891.66
22.0	10984.41	11077.27	11170.26	11263.38	11356.61	11449.96	11543.41	11636.98	11730.65	11824.44
23.0	11918.33	12012.33	12106.42	12200.62	12294.91	12389.31	12483.79	12578.37	12673.03	12767.79
24.0	12862.63	12957.55	13052.55	13147.64	13242.82	13338.06	13433.38	13528.78	13624.26	13719.81
25.0	13815.41	13911.11	14006.88	14102.69	14198.59	14294.56	14390.57	14486.67	14582.83	14679.03
26.0	14775.32	14871.66	14968.06	15064.52	15161.02	15257.62	15354.23	15450.93	15547.67	15644.47
27.0	15741.31	15838.21	15935.17	16032.17	16129.23	16226.33	16323.50	16420.70	16517.95	16615.25
28.0	16712.60	16810.00	16907.43	17004.93	17102.45	17200.05	17297.65	17395.34	17493.04	17590.81
29.0	17688.61	17786.43	17884.33	17982.23	18080.22	18178.22	18276.25	18374.35	18472.45	18570.63
30.0	18668.83	18767.04	18865.34	18963.64	19061.98	19160.38	19258.78	19357.21	19455.71	19554.21
31.0	19652.74	19751.34	19849.94	19948.54	20047.24	20145.94	20244.64	20343.40	20442.20	20541.00
32.0	20639.80	20738.69	20837.59	20936.49	21035.40	21134.40	21233.40	21332.40	21431.40	21530.48
33.0	21629.58	21728.68	21827.78	21926.91	22026.11	22125.31	22224.51	22323.71	22422.94	22522.24
34.0	22621.54	22720.84	22820.14	22919.44	23018.83	23118.23	23217.63	23317.03	23416.43	23515.83
35.0	23615.28	23714.78	23814.28	23913.78	24013.28	24112.78	24212.28	24311.82	24411.42	24511.02
36.0	24610.62	24710.22	24809.82	24909.42	25009.02	25108.63	25208.33	25308.03	25407.73	25507.43
37.0	25607.13	25706.83	25806.53	25906.23	26005.93	26105.70	26205.50	26305.30	26405.10	26504.90
38.0	26604.70	26704.50	26804.30	26904.10	27003.90	27103.70	27203.50	27303.38	27403.28	27503.18
39.0	27603.08	27702.98	27802.88	27902.78	28002.68	28102.58	28202.48	28302.38	28402.28	28502.18

A STANDARD SCHEDULE OF THE RISK OF FIRST MARRIAGE

The most puzzling feature of the common pattern of first-marriage frequencies is its very prevalence. We have seen evidence of the same basic curve of first marriages in cohorts that marry early and cohorts that marry late, in cohorts in which marriage is virtually universal, and in cohorts in which one-quarter remain single. Moreover, the uniform age structure of nuptiality occurs in societies in which most marriages are arranged by families with little regard for the preference of bride and groom, and in societies in which marriages typically result from the self-selection of mutually preferred partners.

A natural step in considering what might account for the puzzling conformity of first-marriage frequencies and proportions ever-married to standard patterns is to examine how the *risk of marriage* evolves with age in the cohorts that exhibit such conformity. The risk of marriage may be defined as the rate of first marriage at each age of persons eligible for first marriage (those still single).

It is obvious that cohorts experiencing a common pattern of first-marriage frequency (relative to all women at each age, not just the single) would also experience a common pattern of risk of marriage (as just defined) only if different cohorts had the same proportion single at higher ages (as first-marriage frequency approaches zero). But such is conspicuously not the case. In Taiwan before World War II first marriage was almost universal (over 99% ever-married by age 50), and in turn-of-the-century Sweden a substantial fraction (about 25%) remained unmarried. Thus the common pattern of first-marriage frequencies combined with different proportions remaining single implies very different patterns of risk of marriage especially at the later ages of marriage. Rates of first marriage are usually calculated and published in the form of *risks* relative to the single population. This practice has, I suspect, prevented general notice of the common pattern of nuptiality.[3]

It is indeed mystifying that the presence of a large reservoir of women remaining single in Sweden did not produce a pattern of first-marriage frequencies essentially different from the pattern in Taiwan. It is as if those destined to remain unmarried had been designated at birth as ineligible, and the experience of the remainder – those fated for marriage – was unaffected by their existence.

If a cohort were divided into the two groups just suggested – into a group all of whom marry (if they survive to old age), and a group none of whom marry – the risk of marriage of the eligible group would follow a standard pattern, defined as the standard first-marriage frequency at each age divided by the standard proportion single or one minus the standard proportion ever-married. (The standard schedules, as tabulated, imply that the proportion ever-married approaches 1·0 as first-marriage frequency approaches zero.) Thus the standard first-marriage frequency at each age in Table 2 divided by one minus the proportion ever-married in Table 3 may be considered as the standard risk of marriage at each age of the single persons destined ever to marry. The values calculated in this way are shown in Fig. 9. Note that the standard risk of marriage rises monotonically for the first 22 years after the earliest age at marriage, appearing from 14 to 22 to approach a horizontal asymptote, but thereafter follows an erratic further upward course. However, the values of the calculated standard first-marriage frequencies beyond 22 years from the origin are of dubious validity because (*a*) the later risks of marriage are sensitive to errors in the cumulated values of first-marriage frequencies; the proportion remaining single (the denominator of the 'risk') 22 years after the origin in the standard schedule in only 0·071 and only 0·026 after 28 years; and (*b*) the first-marriage frequencies at the higher ages in the standard schedules are derived by somewhat arbitrary adjustments of cross-sectional data. The standard risk of marriage may thus be characterized as steadily rising from zero at the earliest age of marriage to an approximately constant

[3] Gösta Carlsson has independently noted that first-marriage frequencies 'beyond the peak value' in Sweden would imply a constant risk of marriage among those ever-marrying. G. Carlsson, 'Marriage rates as social indicators'. In: The Scandinavian Demographic Society, *The Second Scandinavian Demographic Symposium* (Stockholm, 1970), pp. 49–64.

maximum value when most of the cohort who will ever marry have married. The risk of marriage is near a constant maximum from about the 14th year of marriage on the standard scale, until, at higher ages, its value is uncertain.

A little trial-and-error calculation showed that the standard risk of marriage is very closely fitted indeed by a double exponential curve, $r_S(x) = 0{\cdot}174\, e^{-4{\cdot}411\, e^{-0{\cdot}309x}}$. The fit of this function to the standard values derived from adjusted Swedish data is seen in Fig. 9. For a cohort in which marriages begin at age a_0, and for whom the time scale of marriage is compressed by a factor k, the risk of marriage $r(a)$ among those who ever marry is $r(a) = (0{\cdot}174/k)\, e^{-4{\cdot}411\, e^{-(0{\cdot}309/k)(a-a_0)}}$. When nuptiality is compressed within a short span, first-marriage risks rise more quickly to a higher value. Fig. 10 shows first-marriage risks calculated from the double exponential for Taiwan (cohort aged 15 in 1915) and Sweden (1901–10). The values of a_0 and k are 13·2 and 0·48, and 16·5 and 0·89 respectively.

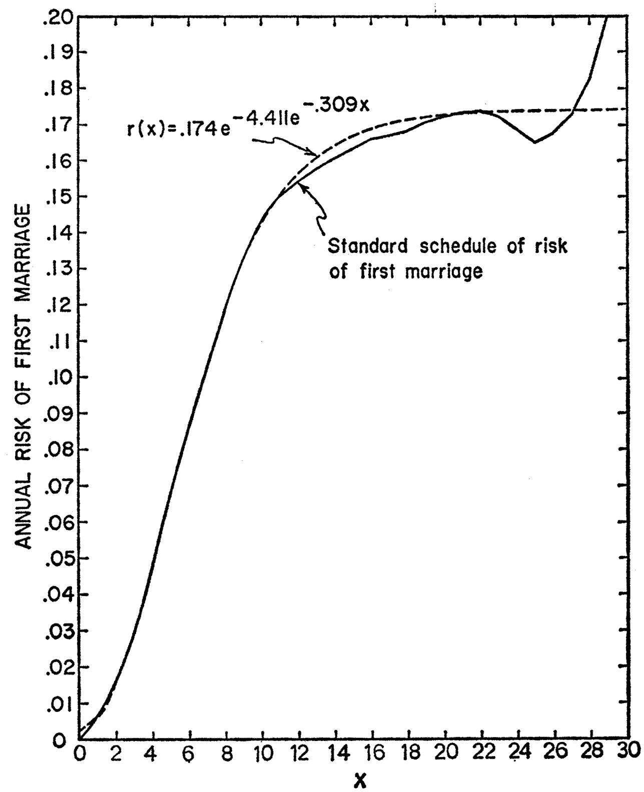

FIGURE 9. *Standard risk of first marriage derived from Swedish data, 1965–69, and fitted by double exponential*

The risk of first marriage in any cohort can be summarized as follows: (1) There is a convention that effectively prevents first marriage in the cohort before a minimum age defined by law, religion, or custom. (2) A fraction of the cohort is not in the marriage pool, because of such forces as the relative numbers of men and women when members of the cohort marry, restrictions on marriage, including the provision of a dowry, and traditions of celibacy. (3) The portion of the cohort in the marriage pool experiences a risk of marriage that rises from zero at the conventionally

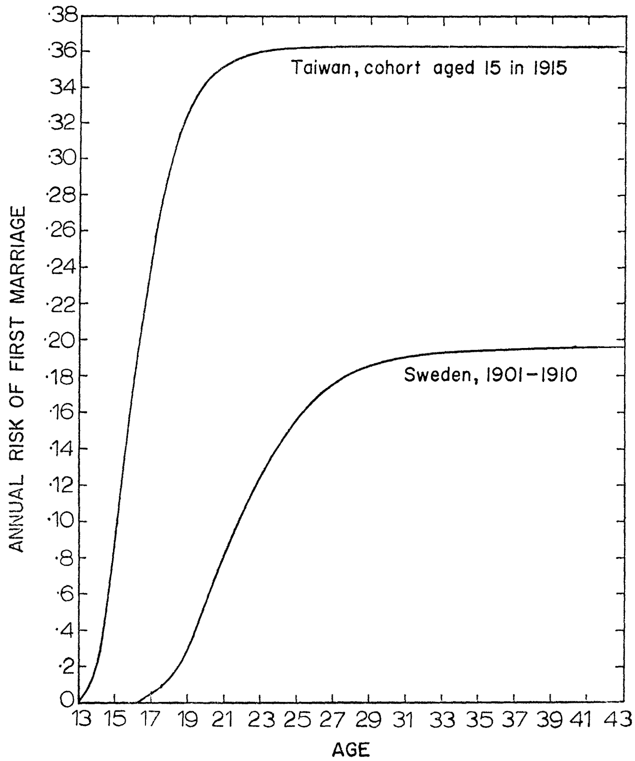

FIGURE 10. *Risk of first marriage for two populations (fitted by double exponentials)*

defined minimum age to a maximum risk that is maintained until the last marriages that occur to the cohort. (4) The more rapid the rise in the risk of marriage the higher the maximum risk. The curve is a double exponential given above.

Nuptiality in Taiwan prior to World War II was characterized by an early start, a rapid rise to a high maximum risk of marriage, and nearly 100% participation in the marriage pool. Nuptiality in Sweden prior to World War I was characterized by a later start, a gradual rise in the risk of marriage to a moderate maximum, and far from complete participation in marriage. The relation between a swift rise in the risk of marriage and a high maximum risk is inherent in conformity to the standard pattern; the tendency for a rapid rise and a high maximum to be associated with an early start and a high proportion participating is common but not universal in the recorded experience we have examined.

SIGNIFICANCE OF THE SIMILARITY OF FIRST-MARRIAGE SCHEDULES

In this article I will attempt only to mention some of the implications and practical uses of the discovery that first-marriage frequencies follow a similar pattern under widely different circumstances. More extensive development of these ideas will appear in subsequent publications by the author and his colleagues at the Office of Population Research.

Reconstitution of female populations by age and marital status

The existence of a standard form of first-marriage frequencies (and hence of proportions ever married) means that the first-marriage experience of a cohort can be expressed by three numbers (parameters): the origin of the curve (a_0—approximately the earliest age of a significant number of first marriages), the proportion ever-married when first marriages have effectively ceased (C), and the time-scale (short or long) according to which nuptiality occurs. The time-scale can be expressed as the time scale of the 'standard' curve times k; k is the requisite third parameter. (If the interval between the origin and the mode of the standard curve is x years, for another curve the interval is kx.) Since only three parameters need to be determined, the estimation of first-marriage frequencies and of proportions ever married by any desired age intervals can sometimes be based on data classified only by broad categories. For example, from data on population classified by age (but not marital status) and by marital status (but not age), together with data on deaths similarly classified, it is possible to estimate the proportion ever married by age. (Such limited data exist for Sweden prior to 1870.)

Etienne van de Walle has made extensive use of standard nuptiality schedules in an ambitious reconstitution of the female population of the 96 *départements* of France from 1831 to 1901 by age and marital status. In making estimates for after 1851 he is able to use marriages by age in estimating the needed parameters; for the earlier years he is restricted to cruder data.[4]

ESTIMATION OF FUTURE MARRIAGES FOR COHORTS WHOSE FIRST MARRIAGES HAVE ALREADY BEGUN

At a time when a cohort has achieved some 60% or more of its ultimate proportion ever-married – has progressed just beyond the modal years of marriage – the first marriages still to be experienced can be estimated by determining the three parameters on the basis of marital experience to the given point, and using a standard schedule characterized by these parameters for the marriages still to come. The evolution of parameters from cohort to cohort provides a rational way of recording (and extrapolating) trends in nuptiality.

THE AGE PATTERN OF NUPTIALITY AND THE AGE PATTERN OF FERTILITY

The age-specific fertility schedule of any population can be factored into two components: the proportion of women at each age living in a sexual union, and the fertility at each age of cohabiting women. Thus $f(a) = C(a) \cdot m(a)$, where $f(a)$ is the proportion of women at age a experiencing a live birth, $C(a)$ is the proportion cohabiting among women at age a, and $m(a)$ is the proportion of cohabiting women experiencing a live birth. In many populations fruitful cohabitation is almost wholly limited to married women. In such populations $C(a)$ is virtually identical with the proportion currently married; and $m(a)$ with marital fertility. $C(a)$ in turn is the proportion ever-married minus the proportion currently widowed or divorced. The proportion widowed and divorced is usually very small until ages beyond 30; the proportion divorced in a cohort often follows a rising straight line from about age 20 to 50; the proportion widowed in a cohort can be roughly approximated to age 50 by the appropriate multiple of a standard curve of widowhood. The most important element causing differences in $C(a)$ below age 50 in cohorts in different times and places is usually the age pattern of proportions ever-married.

The age pattern of marital fertility (above age 20; marital fertility below 20 is often influenced strongly by the frequency of pre-marital conceptions leading to marriage) follows a strongly

[4] The French censuses in fact published tabulations by age and marital status beginning in 1851, but in some years the figures are characterized by erratic and extensive errors, so that reconstitution is still needed to provide usable data.

typical pattern, called 'natural fertility' by Louis Henry, in populations in which deliberate birth control is not practised.[5] I have found that there is a typical age pattern of the departure of marital fertility from the pattern of natural fertility, as follows: Let $n(a)$ be 'natural' fertility (above age 20) and $m(a)$ be marital fertility. There is a function $v(a)$ expressing the typical age pattern of the effect of the voluntary control of fertility, such that $m(a) = M \,.\, n(a)\, e^{m \cdot v(a)}$. M is the ratio of marital fertility at an early age (say 20–24) to fertility at that age in the schedule representing natural fertility, and m is a factor expressing the degree to which $m(a)$ departs from the *pattern* of natural fertility. Values of $v(a)$ are as follows, when $n(a)$ is taken as the marital fertility of the Hutterites, 1921–30, one of the highest marital fertility schedules on reliable record:

Age	20–24	25–29	30–34	35–39	40–45	45–49
$v(a)$	0·000	−0·252	−0·623	−1·094	−1·519	−2·437

The calculation of $v(a)$ required a preliminary set of calculations in which M was set equal to the ratio of marital fertility at ages 20–24 in a given population to that of the Hutterites, and m was set at 1·0. Then an individual set of values of $v(a)$ – say $v_i(a)$ – was calculated for each of 43 schedules of marital fertility selected from those listed in the UN *Demographic Yearbook* for 1965. Schedules were omitted when the quality of the data was suspect. The value of $v(a)$ at each age tabulated above is the arithmetical mean of the 43 values of $v_i(a)$. For most populations a constant value of m in $M \,.\, n(a)\, e^{m \cdot v(a)}$ gives a satisfactory approximation of $m(a)$.

The approximation $m(a) = M \,.\, n(a)\, e^{m \cdot v(a)}$ expresses marital fertility in terms of two parameters: M, the ratio of fertility at ages 20–24 to that of the Hutterites, and m, the extent to which control of fertility causes a systematic deviation from the age pattern of natural fertility. If $m = 0$ the resultant schedule is simply a constant multiple at every age of 'natural' fertility (represented by the Hutterite schedule); if $m = 1$ the schedule deviates from natural fertility to an extent that is the average degree of deviation of 43 schedules in the early 1960's; if m is very large the schedule has very rapidly diminishing ratios of fertility relative to the Hutterite schedule as age increases. Only the second of the parameters (m) affects the age structure of fertility; the other (M) only helps determine the level of fertility.

When fertility among the non-married is low enough to be statistically negligible, the age structure of fertility depends essentially on how proportions married vary with age, and on a parameter expressing the degree to which marital fertility departs from the age pattern of natural fertility. The dominant influence of nuptiality is on the rising portion of the fertility schedule – the crucial element being the age at which marriage begins (a_0), and the degree to which marriages are concentrated (k). The descending portion of the schedule is dominated by m, or the degree to which marital fertility is altered from the age structure of 'natural' fertility. These dominant influences are modified by variations in widowhood and divorce, and by failure of $M \,.\, n(a)\, e^{m \cdot v(a)}$ to be an accurate representation of marital fertility. But the modifications are not usually large.

The age-specific fertility schedules of two populations are shown in Fig. 11 together with schedules fitted by assuming (1) that first-marriages follow the standard pattern, (2) that proportions widowed and divorced to age 50 can be represented as the simple multiple of a single curve, (3) that marital fertility can be represented as $M \,.\, n(a)\, e^{m \cdot v(a)}$, and (4) that illegitimate fertility is negligible. One schedule (Hungary, 1965) represents the combined effect of early marriage and a very high degree of control; the other (Ireland, 1961) a combination of late marriage and fertility differing only moderately from natural fertility. The synthetic estimates reproduce the actual differences in the schedules more than adequately.

[5] Louis Henry, 'Some data on natural fertility', *Eugenics Quarterly*, **8**, 1961, pp. 81–91.

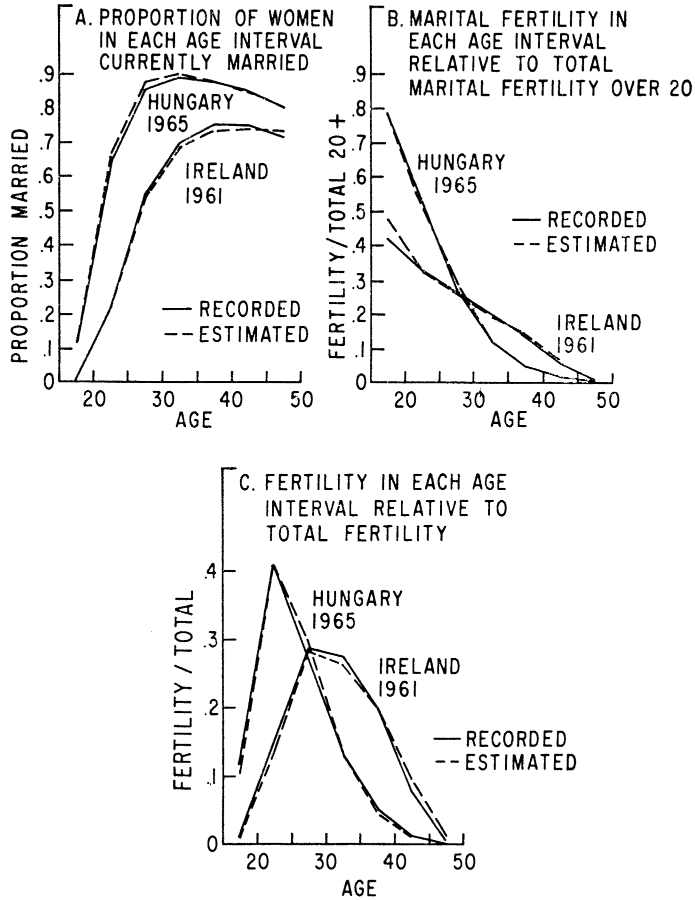

FIGURE 11. *Estimation of age-specific fertility and its components (proportion married and marital fertility), Hungary, 1965, and Ireland, 1961*

NOTES: The estimation of proportions married in 11A is based on determining k, a_0 and C as outlined in the Appendix, and estimating proportions widowed and divorced as a constant times the schedule of proportions widowed and divorced in France in 1896. The constant is the proportion widowed and divorced in the given population at ages 40–44 divided by the corresponding proportion for France.

Hungary, 1965: $k=0{\cdot}469$, $a_0=15{\cdot}75$, $C=0{\cdot}935$, proportion widowed and divorced at $40\text{–}44=0{\cdot}091$.

Ireland, 1961: $k=0{\cdot}754$, $a_0=17{\cdot}16$, $C=0{\cdot}782$, proportion widowed and divorced at $40\text{–}44=0{\cdot}034$.

The estimation of marital fertility in 11B is based on $m(a)=M\,.\,n(a)\,e^{m.v(a)}$, where $v(a)$ has the values given in the text, $m(a)$ is the marital fertility of the Hutterites (0·550, 0·502, 0·447, 0·406, 0·222, 0·061, at ages 20–24 to 45–49), M is the ratio of fertility at 20–24 to that of the Hutterites, and m is a factor (the 'degree of control') determining the steepness of the decline of marital fertility, or more exactly, the extent to which a modification of pattern of the form $v(a)$ occurs. The value of m is the average of the values that would yield the observed $m(a)$ at 30–34 and 35–39. Marital fertility at ages 15–19 is taken as 1·49 times marital fertility at ages 20–24 – the mean value for European countries listed in Table 25 of the UN *Demographic Yearbook* for 1965. For Hungary, 1965, $m=1{\cdot}97$; for Ireland, 1961, $m=0{\cdot}465$. The effect of illegitimate births on the age pattern of fertility is ignored.

SOURCES: For Ireland, UN *Demographic Yearbook*, 1963 and 1965.
For Hungary, Központi Statisztikai Hivatal, *Demografiai Évkonyo*, 1965.

APPENDIX

Fitting the standard schedule of first-marriage frequencies to empirical data; determining the origin, the vertical scale, and the horizontal scale relative to the standard

The standard schedule of first-marriage frequencies is presented in Tables 2, 3 and 4 (as marriage frequencies, proportions ever-married, and person–years lived ever-married). The ages serving as the basis of the tabulations run from zero to 39·9 years. The origin is not at birth, and hence 'age' is not ordinary chronological age. The origin is the earliest age of marriage, and 'age' is chronological age less this value.

Let (x_S) be age on the standard scale, $g_S(x_S)$ be standard first-marriage frequency at age x_S, $G_S(x_S)$ the standard proportion ever-married at age x_S, and $Z_S(x_S)$ the average number of person–years lived ever-married at age x_S in a cohort not subject to mortality.

$$G_S(x_S') = \int_0^{x_S'} g_S(x_S)dx_S \quad \text{and} \quad Z_S(x_S') = \int_0^{x_S'} G_S(x_S)dx_S.$$

Now suppose there is a cohort subject to a schedule of first-marriage frequencies of the standard form, with an origin a_0, a proportion ultimately ever-married C (rather than 1·0), and a horizontal scale k times the standard. Thus the distance from the origin to age a is k times x_S, or $a - a_0 = kx_S$; hence

$$x_S = \frac{a - a_0}{k}.$$

The proportion ever-married at age a is

$$G(a) = C \,.\, g_S \frac{(a - a_0)}{k}.$$

The 'annual' rate of marriages at age a (annual on the time scale of x_S) is

$$C \,.\, g_S \frac{(a - a_0)}{k}.$$

But one year on the x_S scale is k years for the given cohort, hence

$$g(a) = \frac{C}{k} G_S \frac{(a - a_0)}{k}.$$

Similarly

$$Z(a) = C \,.\, kZ_S \frac{(a - a_0)}{k}.$$

Estimating nuptiality from the standard tables when a_0, k and C are known

It follows from the relations given in the preceding paragraph that first-marriage frequencies and proportions ever-married can readily be estimated from Tables 2, 3 and 4 for any desired age intervals, once a_0, k and C are known. For example, if first-marriage frequencies are desired for a cohort in which a_0 is 15·8, k=0·62 and C=0·91, the following table is calculated:

a	(1) $\frac{a-a_0}{k} = x_S$	(2)* $G_S(x_S)$	(3) $CG_S(x_S)$	(4) (Differences in Column (3)) divided by 5·0
20	6·77	0·2478	0·2255	0·0451
25	14·84	0·7611	0·6926	0·0934
30	22·90	0·9389	0·8544	0·0324
35	30·97	0·9860	0·8973	0·0086
40	39·03	0·9993	0·9094	0·0024
45	47·10	1·0000	0·9100	0·0001
50	55·16	1·0000	0·9100	0·0000

* Interpolated from Table 3.

TABLE 1A. *Ratios of proportion married in Nth to (N+1)th age interval*

A0 =	0.0			0.5			1.0			1.5			2.0			2.5		
K	R1	R2	R3	R1	R2	R3	R1	R2	R3	R1	R2	R3	R1	R2	R3	R1	R2	R3
0.30	0.400	0.943	0.999	0.322	0.924	0.997	0.249	0.900	0.996	0.183	0.869	0.994	0.125	0.830	0.991	0.078	0.779	0.987
0.32	0.376	0.929	0.997	0.301	0.908	0.995	0.231	0.881	0.993	0.169	0.847	0.990	0.115	0.804	0.986	0.071	0.751	0.982
0.34	0.353	0.914	0.995	0.281	0.891	0.992	0.215	0.861	0.989	0.156	0.824	0.986	0.106	0.779	0.981	0.065	0.723	0.975
0.36	0.333	0.899	0.992	0.264	0.873	0.989	0.201	0.841	0.985	0.145	0.802	0.980	0.098	0.753	0.975	0.060	0.696	0.968
0.38	0.314	0.883	0.988	0.248	0.855	0.984	0.188	0.821	0.980	0.135	0.779	0.975	0.091	0.729	0.968	0.055	0.670	0.960
0.40	0.297	0.867	0.984	0.234	0.837	0.979	0.176	0.800	0.974	0.127	0.756	0.968	0.085	0.705	0.961	0.052	0.645	0.952
0.42	0.281	0.850	0.979	0.221	0.818	0.974	0.166	0.779	0.968	0.119	0.734	0.961	0.080	0.681	0.953	0.048	0.621	0.943
0.44	0.267	0.833	0.974	0.209	0.799	0.969	0.157	0.759	0.962	0.112	0.712	0.954	0.075	0.658	0.945	0.046	0.599	0.934
0.46	0.254	0.816	0.969	0.198	0.780	0.963	0.149	0.739	0.956	0.106	0.691	0.947	0.071	0.637	0.937	0.043	0.577	0.925
0.48	0.242	0.799	0.963	0.188	0.762	0.957	0.141	0.719	0.949	0.101	0.670	0.940	0.067	0.616	0.928	0.041	0.556	0.915
0.50	0.231	0.782	0.958	0.180	0.743	0.950	0.135	0.700	0.942	0.096	0.650	0.932	0.064	0.596	0.919	0.039	0.537	0.905
0.52	0.221	0.765	0.952	0.172	0.725	0.944	0.129	0.681	0.935	0.092	0.631	0.923	0.061	0.577	0.910	0.038	0.518	0.894
0.54	0.212	0.748	0.946	0.165	0.708	0.937	0.123	0.663	0.927	0.088	0.613	0.915	0.059	0.558	0.900	0.036	0.501	0.884
0.56	0.203	0.732	0.940	0.158	0.691	0.930	0.118	0.645	0.919	0.084	0.595	0.906	0.057	0.541	0.890	0.035	0.484	0.872
0.58	0.196	0.716	0.933	0.152	0.674	0.923	0.114	0.628	0.910	0.081	0.578	0.896	0.055	0.525	0.880	0.034	0.468	0.861
0.60	0.189	0.700	0.926	0.147	0.658	0.915	0.110	0.612	0.902	0.079	0.562	0.887	0.053	0.509	0.870	0.033	0.454	0.850
0.62	0.182	0.685	0.919	0.142	0.642	0.907	0.106	0.596	0.893	0.076	0.547	0.877	0.052	0.494	0.859	0.032	0.440	0.838
0.64	0.176	0.670	0.911	0.137	0.627	0.898	0.103	0.581	0.884	0.074	0.532	0.867	0.050	0.480	0.848	0.032	0.427	0.827
0.66	0.171	0.655	0.903	0.133	0.613	0.890	0.100	0.567	0.875	0.072	0.518	0.857	0.049	0.467	0.837	0.031	0.414	0.815
0.68	0.166	0.641	0.896	0.129	0.599	0.881	0.097	0.553	0.865	0.070	0.504	0.847	0.048	0.454	0.827	0.031	0.402	0.804
0.70	0.161	0.628	0.888	0.126	0.585	0.873	0.095	0.540	0.856	0.068	0.492	0.837	0.047	0.442	0.816	0.030	0.391	0.792
0.72	0.157	0.615	0.879	0.122	0.572	0.864	0.092	0.527	0.847	0.067	0.479	0.827	0.046	0.430	0.806	0.030	0.381	0.781
0.74	0.153	0.602	0.871	0.119	0.560	0.855	0.090	0.515	0.837	0.066	0.468	0.817	0.045	0.420	0.795	0.029	0.371	0.770
0.76	0.149	0.590	0.863	0.117	0.548	0.846	0.088	0.503	0.828	0.064	0.457	0.807	0.045	0.409	0.785	0.029	0.362	0.759
0.78	0.146	0.578	0.855	0.114	0.536	0.838	0.087	0.492	0.819	0.063	0.446	0.798	0.044	0.400	0.774	0.029	0.353	0.749
0.80	0.143	0.567	0.846	0.112	0.525	0.829	0.085	0.481	0.810	0.062	0.436	0.788	0.044	0.390	0.764	0.029	0.345	0.738
0.82	0.140	0.556	0.838	0.110	0.514	0.820	0.084	0.471	0.800	0.062	0.427	0.779	0.043	0.382	0.754	0.028	0.337	0.728
0.84	0.137	0.545	0.830	0.108	0.504	0.812	0.083	0.461	0.791	0.061	0.418	0.769	0.043	0.373	0.744	0.028	0.330	0.717
0.86	0.135	0.535	0.822	0.106	0.494	0.803	0.081	0.452	0.783	0.060	0.409	0.760	0.042	0.366	0.735	0.028	0.323	0.707
0.88	0.133	0.525	0.814	0.105	0.485	0.795	0.080	0.443	0.774	0.059	0.401	0.751	0.042	0.358	0.725	0.028	0.316	0.698
0.90	0.131	0.516	0.806	0.103	0.476	0.786	0.079	0.435	0.765	0.059	0.393	0.742	0.042	0.351	0.716	0.028	0.310	0.688
0.92	0.129	0.507	0.798	0.102	0.467	0.778	0.079	0.427	0.756	0.058	0.386	0.733	0.042	0.345	0.707	0.028	0.304	0.679
0.94	0.127	0.498	0.790	0.101	0.459	0.770	0.078	0.419	0.748	0.058	0.379	0.724	0.041	0.339	0.698	0.028	0.299	0.669
0.96	0.126	0.489	0.783	0.100	0.451	0.762	0.077	0.412	0.740	0.058	0.372	0.715	0.041	0.333	0.689	0.028	0.294	0.660
0.98	0.124	0.481	0.775	0.099	0.443	0.754	0.076	0.405	0.731	0.057	0.366	0.707	0.041	0.327	0.680	0.028	0.289	0.651
1.00	0.123	0.474	0.767	0.098	0.436	0.746	0.076	0.398	0.723	0.057	0.360	0.698	0.041	0.322	0.671	0.028	0.285	0.643
1.02	0.121	0.466	0.760	0.097	0.429	0.738	0.075	0.392	0.715	0.057	0.354	0.690	0.041	0.317	0.663	0.028	0.280	0.634
1.04	0.120	0.459	0.752	0.096	0.423	0.731	0.075	0.386	0.707	0.057	0.349	0.682	0.041	0.312	0.655	0.028	0.276	0.626
1.06	0.119	0.453	0.745	0.095	0.416	0.723	0.075	0.380	0.699	0.056	0.344	0.674	0.041	0.308	0.647	0.028	0.272	0.618
1.08	0.118	0.446	0.738	0.095	0.411	0.716	0.074	0.375	0.692	0.056	0.339	0.666	0.041	0.303	0.639	0.028	0.269	0.610
1.10	0.117	0.440	0.731	0.094	0.405	0.708	0.074	0.369	0.684	0.056	0.334	0.659	0.041	0.299	0.632	0.028	0.265	0.603
1.12	0.116	0.434	0.724	0.094	0.399	0.701	0.074	0.365	0.677	0.056	0.330	0.651	0.041	0.295	0.624	0.028	0.262	0.596
1.14	0.116	0.428	0.717	0.093	0.394	0.694	0.073	0.360	0.670	0.056	0.326	0.644	0.041	0.292	0.617	0.028	0.259	0.588
1.16	0.115	0.423	0.710	0.093	0.389	0.687	0.073	0.355	0.663	0.056	0.322	0.637	0.041	0.288	0.610	0.028	0.256	0.582
1.18	0.114	0.418	0.703	0.092	0.384	0.680	0.073	0.351	0.656	0.056	0.318	0.630	0.041	0.285	0.603	0.029	0.253	0.575
1.20	0.114	0.413	0.696	0.092	0.380	0.674	0.073	0.347	0.649	0.056	0.314	0.624	0.041	0.282	0.597	0.029	0.251	0.568

TABLE 1A—*continued*

A0 =	2.5			3.0			3.5			4.0			4.5			5.0		
K	R1	R2	R3	R1	R2	R3	R1	R2	R3	R1	R2	R3	R1	R2	R3	R1	R2	R3
0.30	0.078	0.779	0.987	0.042	0.717	0.982	0.018	0.645	0.976	0.005	0.565	0.968	0.001	0.482	0.957	0.0	0.400	0.943
0.32	0.071	0.751	0.982	0.038	0.687	0.975	0.016	0.615	0.968	0.005	0.536	0.958	0.001	0.455	0.945	0.0	0.376	0.929
0.34	0.065	0.723	0.975	0.035	0.658	0.968	0.015	0.586	0.959	0.005	0.508	0.947	0.001	0.430	0.932	0.0	0.353	0.914
0.36	0.060	0.696	0.968	0.032	0.631	0.959	0.014	0.559	0.949	0.004	0.483	0.936	0.001	0.406	0.919	0.0	0.333	0.899
0.38	0.055	0.670	0.960	0.030	0.605	0.950	0.013	0.533	0.938	0.004	0.459	0.924	0.001	0.385	0.906	0.0	0.314	0.883
0.40	0.052	0.645	0.952	0.028	0.580	0.941	0.012	0.509	0.928	0.004	0.437	0.911	0.001	0.365	0.891	0.0	0.297	0.867
0.42	0.048	0.621	0.943	0.026	0.556	0.931	0.012	0.487	0.917	0.004	0.416	0.899	0.001	0.347	0.877	0.0	0.281	0.850
0.44	0.046	0.599	0.934	0.025	0.534	0.921	0.011	0.466	0.905	0.004	0.397	0.885	0.001	0.330	0.862	0.0	0.267	0.833
0.46	0.043	0.577	0.925	0.023	0.513	0.910	0.011	0.446	0.893	0.004	0.380	0.872	0.001	0.315	0.846	0.0	0.254	0.816
0.48	0.041	0.556	0.915	0.022	0.493	0.899	0.010	0.428	0.880	0.003	0.363	0.857	0.001	0.301	0.831	0.0	0.242	0.799
0.50	0.039	0.537	0.905	0.022	0.474	0.888	0.010	0.411	0.867	0.003	0.348	0.843	0.001	0.287	0.815	0.0	0.231	0.782
0.52	0.038	0.518	0.894	0.021	0.457	0.876	0.010	0.395	0.854	0.003	0.334	0.829	0.001	0.275	0.799	0.0	0.221	0.765
0.54	0.036	0.501	0.884	0.020	0.441	0.864	0.010	0.380	0.841	0.003	0.321	0.814	0.001	0.264	0.783	0.0	0.212	0.748
0.56	0.035	0.484	0.872	0.020	0.425	0.852	0.009	0.366	0.828	0.003	0.309	0.800	0.001	0.254	0.768	0.0	0.203	0.732
0.58	0.034	0.468	0.861	0.019	0.411	0.839	0.009	0.353	0.814	0.003	0.297	0.785	0.001	0.245	0.753	0.0	0.196	0.716
0.60	0.033	0.454	0.850	0.019	0.397	0.827	0.009	0.341	0.801	0.003	0.287	0.771	0.001	0.236	0.738	0.0	0.189	0.700
0.62	0.032	0.440	0.838	0.018	0.385	0.814	0.009	0.330	0.788	0.003	0.277	0.757	0.001	0.228	0.723	0.0	0.182	0.685
0.64	0.032	0.427	0.827	0.018	0.373	0.802	0.009	0.319	0.774	0.003	0.268	0.743	0.001	0.220	0.708	0.0	0.176	0.670
0.66	0.031	0.414	0.815	0.018	0.361	0.790	0.009	0.310	0.762	0.003	0.260	0.730	0.001	0.213	0.694	0.0	0.171	0.655
0.68	0.031	0.402	0.804	0.018	0.351	0.778	0.009	0.300	0.749	0.003	0.252	0.716	0.001	0.207	0.681	0.0	0.166	0.641
0.70	0.030	0.391	0.792	0.018	0.341	0.766	0.009	0.292	0.736	0.003	0.245	0.703	0.001	0.201	0.667	0.0	0.161	0.628
0.72	0.030	0.381	0.781	0.017	0.332	0.754	0.009	0.284	0.724	0.003	0.238	0.691	0.001	0.196	0.654	0.0	0.157	0.615
0.74	0.029	0.371	0.770	0.017	0.323	0.743	0.009	0.276	0.712	0.003	0.232	0.678	0.001	0.191	0.642	0.0	0.153	0.602
0.76	0.029	0.362	0.759	0.017	0.315	0.731	0.009	0.269	0.700	0.003	0.226	0.666	0.001	0.186	0.630	0.0	0.149	0.590
0.78	0.029	0.353	0.749	0.017	0.307	0.720	0.009	0.263	0.689	0.003	0.221	0.655	0.001	0.182	0.618	0.0	0.146	0.578
0.80	0.029	0.345	0.738	0.017	0.300	0.709	0.009	0.257	0.678	0.003	0.216	0.643	0.001	0.178	0.606	0.0	0.143	0.567
0.82	0.028	0.337	0.728	0.017	0.293	0.698	0.009	0.251	0.667	0.003	0.211	0.632	0.001	0.174	0.595	0.0	0.140	0.556
0.84	0.028	0.330	0.717	0.017	0.287	0.688	0.009	0.246	0.656	0.003	0.207	0.621	0.001	0.171	0.584	0.0	0.137	0.545
0.86	0.028	0.323	0.707	0.017	0.281	0.678	0.009	0.241	0.645	0.003	0.203	0.611	0.001	0.167	0.574	0.0	0.135	0.535
0.88	0.028	0.316	0.698	0.017	0.275	0.668	0.009	0.236	0.635	0.003	0.199	0.601	0.001	0.164	0.564	0.0	0.133	0.525
0.90	0.028	0.310	0.688	0.017	0.270	0.658	0.009	0.232	0.625	0.003	0.195	0.591	0.001	0.162	0.554	0.0	0.131	0.516
0.92	0.028	0.304	0.679	0.017	0.265	0.648	0.009	0.228	0.616	0.003	0.192	0.581	0.001	0.159	0.545	0.0	0.129	0.507
0.94	0.028	0.299	0.669	0.017	0.261	0.639	0.009	0.224	0.606	0.003	0.189	0.572	0.001	0.157	0.536	0.0	0.127	0.498
0.96	0.028	0.294	0.660	0.017	0.256	0.630	0.009	0.220	0.597	0.003	0.186	0.563	0.001	0.154	0.527	0.0	0.126	0.489
0.98	0.028	0.289	0.651	0.017	0.252	0.621	0.009	0.217	0.588	0.003	0.183	0.554	0.001	0.152	0.519	0.0	0.124	0.481
1.00	0.028	0.285	0.643	0.017	0.248	0.612	0.009	0.214	0.580	0.003	0.181	0.546	0.001	0.151	0.510	0.0	0.123	0.474
1.02	0.028	0.280	0.634	0.017	0.245	0.604	0.009	0.211	0.572	0.004	0.179	0.538	0.001	0.149	0.503	0.0	0.121	0.466
1.04	0.028	0.276	0.626	0.017	0.241	0.596	0.009	0.208	0.564	0.004	0.176	0.530	0.001	0.147	0.495	0.0	0.120	0.459
1.06	0.028	0.272	0.618	0.017	0.238	0.588	0.009	0.205	0.556	0.004	0.174	0.523	0.001	0.146	0.488	0.0	0.119	0.453
1.08	0.028	0.269	0.610	0.018	0.235	0.580	0.009	0.203	0.548	0.004	0.173	0.515	0.001	0.144	0.481	0.0	0.118	0.446
1.10	0.028	0.265	0.603	0.018	0.232	0.573	0.009	0.201	0.541	0.004	0.171	0.508	0.001	0.143	0.475	0.0	0.117	0.440
1.12	0.028	0.262	0.596	0.018	0.229	0.566	0.009	0.198	0.534	0.004	0.169	0.502	0.001	0.142	0.468	0.0	0.116	0.434
1.14	0.028	0.259	0.588	0.018	0.227	0.559	0.010	0.196	0.527	0.004	0.168	0.495	0.001	0.141	0.462	0.0	0.116	0.428
1.16	0.028	0.256	0.582	0.018	0.225	0.552	0.010	0.195	0.521	0.004	0.166	0.489	0.001	0.140	0.456	0.0	0.115	0.423
1.18	0.029	0.253	0.575	0.018	0.222	0.545	0.010	0.193	0.515	0.004	0.165	0.483	0.001	0.139	0.451	0.0	0.114	0.418
1.20	0.029	0.251	0.568	0.018	0.220	0.539	0.010	0.191	0.508	0.004	0.164	0.477	0.001	0.138	0.445	0.0	0.114	0.413

TABLE 2A. *Ratios of proportion married at end of Nth to end of $(N+1)$th age interval*

AO =	0.0			0.5			1.0			1.5			2.0			2.5	
K	RA1	RA2	RA3	RA1	RA2	RA3	RA1	RA2	RA3	RA1	RA2	RA3	RA1	RA2	RA3	RA1	RA2	RA3
0.30	0.830	0.992	1.000	0.777	0.988	1.000	0.708	0.982	1.000	0.621	0.975	1.000	0.512	0.967	1.000	0.386	0.957	1.000
0.32	0.800	0.987	1.000	0.743	0.981	1.000	0.670	0.975	1.000	0.579	0.967	1.000	0.469	0.957	1.000	0.348	0.945	0.999
0.34	0.771	0.980	1.000	0.710	0.974	1.000	0.633	0.966	1.000	0.539	0.957	0.999	0.431	0.946	0.998	0.315	0.932	0.997
0.36	0.741	0.973	1.000	0.676	0.966	1.000	0.597	0.958	0.999	0.502	0.948	0.998	0.396	0.935	0.996	0.286	0.918	0.994
0.38	0.712	0.966	0.999	0.644	0.958	0.998	0.562	0.949	0.997	0.468	0.937	0.996	0.365	0.922	0.994	0.261	0.902	0.991
0.40	0.683	0.959	0.998	0.612	0.950	0.996	0.530	0.939	0.995	0.437	0.925	0.993	0.338	0.908	0.990	0.239	0.887	0.987
0.42	0.654	0.951	0.996	0.582	0.941	0.994	0.500	0.928	0.992	0.409	0.913	0.989	0.313	0.894	0.986	0.220	0.871	0.981
0.44	0.627	0.943	0.993	0.554	0.931	0.991	0.472	0.917	0.988	0.383	0.900	0.985	0.292	0.879	0.981	0.203	0.854	0.976
0.46	0.600	0.934	0.990	0.527	0.921	0.987	0.446	0.906	0.984	0.359	0.887	0.980	0.272	0.864	0.976	0.188	0.837	0.972
0.48	0.575	0.925	0.987	0.502	0.911	0.983	0.422	0.894	0.980	0.338	0.873	0.976	0.254	0.849	0.972	0.176	0.820	0.967
0.50	0.551	0.915	0.983	0.478	0.900	0.979	0.400	0.881	0.976	0.319	0.859	0.972	0.239	0.833	0.967	0.164	0.802	0.962
0.52	0.528	0.905	0.979	0.456	0.888	0.976	0.379	0.868	0.972	0.301	0.844	0.967	0.224	0.816	0.962	0.154	0.784	0.957
0.54	0.506	0.894	0.976	0.435	0.876	0.972	0.360	0.855	0.968	0.285	0.829	0.963	0.212	0.800	0.957	0.146	0.766	0.951
0.56	0.486	0.883	0.972	0.416	0.864	0.968	0.343	0.841	0.964	0.270	0.814	0.958	0.200	0.784	0.952	0.138	0.749	0.945
0.58	0.466	0.872	0.968	0.398	0.851	0.964	0.327	0.827	0.959	0.257	0.800	0.953	0.190	0.768	0.947	0.131	0.732	0.938
0.60	0.448	0.860	0.965	0.381	0.838	0.960	0.313	0.813	0.955	0.245	0.785	0.948	0.181	0.753	0.941	0.125	0.716	0.932
0.62	0.431	0.848	0.961	0.366	0.826	0.956	0.299	0.800	0.950	0.234	0.771	0.943	0.173	0.737	0.934	0.120	0.700	0.925
0.64	0.416	0.837	0.957	0.352	0.813	0.951	0.287	0.787	0.945	0.224	0.756	0.937	0.166	0.722	0.928	0.115	0.684	0.917
0.66	0.401	0.825	0.953	0.338	0.801	0.946	0.276	0.773	0.939	0.215	0.742	0.931	0.159	0.707	0.921	0.111	0.668	0.909
0.68	0.387	0.813	0.948	0.326	0.788	0.941	0.265	0.760	0.933	0.207	0.729	0.924	0.153	0.693	0.913	0.107	0.653	0.901
0.70	0.374	0.802	0.943	0.314	0.776	0.936	0.255	0.747	0.927	0.199	0.715	0.917	0.148	0.679	0.906	0.103	0.638	0.893
0.72	0.362	0.791	0.938	0.304	0.764	0.930	0.246	0.735	0.920	0.192	0.702	0.910	0.143	0.664	0.898	0.100	0.623	0.885
0.74	0.350	0.779	0.932	0.294	0.752	0.924	0.238	0.722	0.914	0.186	0.688	0.903	0.139	0.651	0.890	0.097	0.609	0.877
0.76	0.340	0.768	0.926	0.285	0.741	0.917	0.231	0.710	0.907	0.180	0.675	0.895	0.135	0.637	0.883	0.095	0.595	0.869
0.78	0.330	0.757	0.921	0.276	0.729	0.911	0.224	0.698	0.900	0.175	0.663	0.888	0.131	0.624	0.875	0.093	0.582	0.860
0.80	0.320	0.746	0.914	0.268	0.718	0.904	0.217	0.686	0.893	0.170	0.650	0.881	0.128	0.611	0.867	0.091	0.569	0.852
0.82	0.312	0.736	0.908	0.260	0.706	0.898	0.212	0.674	0.886	0.166	0.638	0.874	0.125	0.599	0.860	0.089	0.556	0.844
0.84	0.303	0.725	0.902	0.254	0.695	0.891	0.206	0.662	0.880	0.162	0.626	0.866	0.122	0.587	0.852	0.088	0.544	0.836
0.86	0.296	0.715	0.896	0.247	0.684	0.885	0.201	0.651	0.873	0.159	0.614	0.859	0.120	0.575	0.844	0.086	0.533	0.828
0.88	0.289	0.704	0.890	0.241	0.673	0.879	0.197	0.640	0.866	0.155	0.603	0.852	0.118	0.564	0.837	0.085	0.522	0.820
0.90	0.282	0.694	0.884	0.236	0.663	0.872	0.193	0.629	0.859	0.152	0.592	0.845	0.116	0.553	0.829	0.084	0.511	0.812
0.92	0.276	0.684	0.878	0.231	0.652	0.866	0.189	0.618	0.852	0.149	0.582	0.838	0.114	0.543	0.821	0.083	0.501	0.804
0.94	0.270	0.674	0.872	0.226	0.642	0.859	0.185	0.608	0.845	0.147	0.571	0.830	0.112	0.533	0.814	0.082	0.492	0.796
0.96	0.264	0.664	0.866	0.222	0.632	0.853	0.182	0.598	0.839	0.144	0.562	0.823	0.111	0.523	0.806	0.081	0.482	0.788
0.98	0.259	0.655	0.859	0.218	0.623	0.846	0.179	0.589	0.832	0.142	0.552	0.816	0.109	0.514	0.799	0.080	0.474	0.780
1.00	0.255	0.645	0.853	0.214	0.613	0.840	0.176	0.579	0.825	0.140	0.543	0.809	0.108	0.505	0.791	0.080	0.465	0.772
1.02	0.250	0.636	0.847	0.211	0.604	0.834	0.173	0.570	0.819	0.138	0.534	0.802	0.107	0.496	0.784	0.079	0.457	0.764
1.04	0.246	0.628	0.841	0.208	0.595	0.827	0.171	0.561	0.812	0.137	0.525	0.795	0.106	0.488	0.777	0.079	0.449	0.757
1.06	0.242	0.619	0.835	0.205	0.587	0.821	0.169	0.553	0.805	0.135	0.517	0.788	0.105	0.480	0.770	0.079	0.442	0.749
1.08	0.239	0.610	0.829	0.202	0.578	0.815	0.167	0.545	0.799	0.134	0.509	0.781	0.104	0.473	0.762	0.078	0.435	0.742
1.10	0.235	0.602	0.823	0.199	0.570	0.808	0.165	0.537	0.792	0.132	0.502	0.774	0.104	0.466	0.755	0.078	0.428	0.734
1.12	0.232	0.594	0.817	0.197	0.563	0.802	0.163	0.529	0.786	0.131	0.495	0.768	0.103	0.459	0.748	0.078	0.422	0.727
1.14	0.229	0.587	0.811	0.194	0.555	0.796	0.161	0.522	0.779	0.130	0.488	0.761	0.102	0.452	0.741	0.078	0.416	0.720
1.16	0.227	0.579	0.805	0.192	0.548	0.790	0.159	0.515	0.773	0.129	0.481	0.754	0.102	0.446	0.734	0.077	0.410	0.713
1.18	0.224	0.572	0.799	0.190	0.541	0.784	0.158	0.508	0.766	0.128	0.475	0.748	0.101	0.440	0.728	0.077	0.404	0.706
1.20	0.221	0.565	0.794	0.188	0.534	0.777	0.157	0.502	0.760	0.128	0.468	0.741	0.101	0.434	0.721	0.077	0.399	0.699

TABLE 2A.—*continued*

3.0			3.5			4.0			4.5			5.0		
RA1	RA2	RA3	RA1	RA2	RA3	RA1	RA2	RA3	RA1	RA2	RA3	RA1	RA2	RA3
0.255	0.943	1.000	0.136	0.925	0.998	0.053	0.902	0.997	0.012	0.870	0.995	0.0	0.830	0.992
0.226	0.929	0.998	0.120	0.908	0.996	0.047	0.880	0.994	0.011	0.845	0.991	0.0	0.800	0.987
0.202	0.913	0.995	0.106	0.889	0.993	0.042	0.858	0.990	0.010	0.820	0.985	0.0	0.771	0.980
0.181	0.896	0.992	0.095	0.869	0.989	0.038	0.836	0.984	0.009	0.794	0.979	0.0	0.741	0.973
0.164	0.879	0.988	0.086	0.849	0.983	0.034	0.813	0.978	0.008	0.768	0.972	0.0	0.712	0.966
0.149	0.861	0.982	0.078	0.829	0.977	0.032	0.790	0.972	0.008	0.741	0.966	0.0	0.683	0.959
0.137	0.843	0.977	0.072	0.808	0.972	0.029	0.766	0.966	0.008	0.715	0.960	0.0	0.654	0.951
0.126	0.824	0.972	0.066	0.786	0.966	0.028	0.742	0.960	0.007	0.689	0.953	0.0	0.627	0.943
0.117	0.804	0.967	0.062	0.765	0.961	0.026	0.719	0.954	0.007	0.664	0.945	0.0	0.600	0.934
0.109	0.785	0.961	0.058	0.744	0.955	0.025	0.696	0.947	0.007	0.640	0.937	0.0	0.575	0.925
0.102	0.765	0.956	0.055	0.723	0.948	0.024	0.673	0.939	0.006	0.616	0.928	0.0	0.551	0.915
0.096	0.746	0.950	0.052	0.702	0.941	0.023	0.652	0.931	0.006	0.594	0.919	0.0	0.528	0.905
0.091	0.727	0.943	0.049	0.683	0.934	0.022	0.631	0.923	0.006	0.572	0.910	0.0	0.506	0.894
0.086	0.709	0.936	0.047	0.663	0.926	0.021	0.610	0.914	0.006	0.551	0.900	0.0	0.486	0.883
0.082	0.691	0.929	0.045	0.644	0.918	0.021	0.591	0.905	0.006	0.531	0.889	0.0	0.466	0.872
0.079	0.674	0.921	0.044	0.626	0.909	0.020	0.572	0.895	0.006	0.512	0.879	0.0	0.448	0.860
0.076	0.657	0.913	0.043	0.608	0.900	0.020	0.554	0.885	0.006	0.494	0.868	0.0	0.431	0.848
0.073	0.640	0.905	0.041	0.590	0.891	0.020	0.536	0.875	0.006	0.478	0.857	0.0	0.416	0.837
0.071	0.623	0.897	0.040	0.574	0.882	0.020	0.520	0.865	0.006	0.462	0.846	0.0	0.401	0.825
0.068	0.607	0.888	0.040	0.558	0.872	0.019	0.504	0.855	0.006	0.447	0.835	0.0	0.387	0.813
0.067	0.592	0.879	0.039	0.542	0.863	0.019	0.489	0.845	0.006	0.432	0.825	0.0	0.374	0.802
0.065	0.577	0.870	0.038	0.528	0.854	0.019	0.475	0.835	0.006	0.419	0.814	0.0	0.362	0.791
0.064	0.563	0.861	0.038	0.513	0.844	0.019	0.461	0.825	0.006	0.407	0.804	0.0	0.350	0.779
0.062	0.549	0.853	0.037	0.500	0.835	0.019	0.448	0.815	0.006	0.395	0.793	0.0	0.340	0.768
0.061	0.536	0.844	0.037	0.487	0.826	0.019	0.436	0.805	0.006	0.384	0.783	0.0	0.330	0.757
0.060	0.523	0.835	0.037	0.475	0.816	0.019	0.425	0.796	0.006	0.373	0.772	0.0	0.320	0.746
0.060	0.511	0.827	0.036	0.463	0.807	0.019	0.414	0.786	0.006	0.363	0.762	0.0	0.312	0.736
0.059	0.499	0.818	0.036	0.452	0.798	0.019	0.404	0.776	0.006	0.354	0.752	0.0	0.303	0.725
0.058	0.488	0.809	0.036	0.442	0.789	0.019	0.394	0.767	0.006	0.345	0.742	0.0	0.296	0.715
0.058	0.478	0.801	0.036	0.432	0.780	0.019	0.385	0.757	0.006	0.337	0.732	0.0	0.289	0.704
0.057	0.468	0.792	0.036	0.423	0.771	0.019	0.376	0.748	0.006	0.329	0.722	0.0	0.282	0.694
0.057	0.458	0.784	0.036	0.414	0.762	0.019	0.368	0.739	0.006	0.322	0.713	0.0	0.276	0.684
0.057	0.449	0.776	0.036	0.405	0.754	0.019	0.360	0.730	0.006	0.315	0.703	0.0	0.270	0.674
0.056	0.440	0.767	0.036	0.397	0.745	0.019	0.353	0.721	0.006	0.308	0.694	0.0	0.264	0.664
0.056	0.432	0.759	0.036	0.389	0.736	0.019	0.346	0.712	0.006	0.302	0.684	0.0	0.259	0.655
0.056	0.424	0.751	0.036	0.382	0.728	0.019	0.339	0.703	0.006	0.297	0.675	0.0	0.255	0.645
0.056	0.417	0.743	0.036	0.375	0.720	0.019	0.333	0.694	0.006	0.291	0.666	0.0	0.250	0.636
0.056	0.410	0.735	0.036	0.369	0.711	0.019	0.327	0.686	0.006	0.286	0.658	0.0	0.246	0.628
0.056	0.403	0.727	0.036	0.362	0.703	0.019	0.322	0.677	0.006	0.281	0.649	0.0	0.242	0.619
0.056	0.396	0.719	0.036	0.357	0.695	0.019	0.317	0.669	0.006	0.277	0.641	0.0	0.239	0.610
0.056	0.390	0.712	0.037	0.351	0.687	0.019	0.312	0.661	0.006	0.273	0.632	0.0	0.235	0.602
0.056	0.384	0.704	0.037	0.346	0.679	0.019	0.307	0.653	0.006	0.269	0.624	0.0	0.232	0.594
0.056	0.378	0.697	0.037	0.341	0.672	0.020	0.303	0.645	0.006	0.265	0.617	0.0	0.229	0.587
0.056	0.373	0.689	0.037	0.336	0.664	0.020	0.298	0.638	0.006	0.262	0.609	0.0	0.227	0.579
0.056	0.368	0.682	0.037	0.331	0.657	0.020	0.295	0.630	0.006	0.259	0.602	0.0	0.224	0.572
0.056	0.363	0.675	0.037	0.327	0.650	0.020	0.291	0.623	0.006	0.256	0.595	0.0	0.221	0.565

Column (3) yields estimated proportions ever-married at the ages in the stub of the table, and Column (4) estimated average first-marriage frequencies for the age intervals 15–20, 20–25, etc. To estimate proportions ever-married by five-year intervals for the same cohort, one would enter $Z_S(x_S)$ in Column (2), $C \cdot k \cdot Z_S(x_S)$ in Column (3), and Column (4) would then yield the desired estimates.

Determining the values of a_0, k and C

The fit of the standard schedules to the nuptiality of an actual cohort is, of course, only approximate. It is desirable to have a procedure that determines a set of parameters insuring a close fit to the whole experience of the cohort from data in conventionally tabulated form. The method of determining parameters proposed here is based on two sets of ratios, one or the other of which can usually be calculated. One set of ratios (designated R_1, R_2 and R_3 in Table 1A) is the proportion ever-married in the first five-year age interval in which marriages occur to the proportion ever-married in the second five-year interval, etc. To define the ratios more succinctly: if a_0 lies between 15 and 20, R_1 is (proportion ever-married (15–20))/(proportion ever-married (20–25)), R_2 is (proportion ever-married (20–25))/(proportion ever-married 25–30)), etc. If a_0 falls between 10 and 15, R_1 is (proportion ever-married (10–15))/(proportion ever-married (15–20)). A given value of any of these ratios (R_1, R_2 and R_3) can occur with different combinations of k and a_0, but if two ratios (e.g. R_1 *and* R_2) are specified, only one combination of k and a_0 is possible. Hence k and a_0 can be estimated by locating (through interpolation in Table 1A) the values that would yield the observed R_1 and R_2, or the observed R_2 and R_3. If the experience of the cohort were perfectly consistent with a transformed standard curve, the values of k and a_0, indicated by R_1 and R_2, and by R_2 and R_3, would be the same. A perfect fit is rare; the recommended procedure is to combine R_2 with R_1 if $R_1 > (1 - R_3)$, and with R_3 if $(1 - R_3) > R_1$.

For example, suppose values of R_1, R_2 and R_3 of 0·108, 0·530 and 0·828, when R_1 is (ever-married (15–19))/(ever-married (20–24)). By the recommended procedure, k and a_0 will be estimated from R_2 and R_3. One method of interpolation is as follows: for one value of a_0 find a value of k (by linear interpolation) that gives the correct R_3 and an R_2 that is too small; for another value of a_0 find a k that gives the correct R_3 and an R_2 that is too large; by linear interpolation of both k and a_0 between these two points estimate values of the parameters yielding correct R_3 *and* R_2. In this example, the combination of $k=0{\cdot}802$ and $a_0=0{\cdot}50$ would give $R_2=0{\cdot}524$, $R_3=0{\cdot}828$, and $k=0{\cdot}845$, $a_0=0{\cdot}0$ would give $R_2=0{\cdot}542$, $R_3=0{\cdot}828$. The final estimate would be one-third of the way between these points, or $k=0{\cdot}186$, $a_0=0{\cdot}33$. The first age group is 15–19, so that the final estimate of a_0 is 15·33 years.

The value of C (proportion ultimately ever-married) is estimated by determining the person–years lived ever-married at the beginning and end of the third five-year age interval (25 and 30 if marriages begin between 15 and 20) in the transformed standard schedule with the estimated values of k and a_0. Specifically, one determines

$$kZ_S\frac{(25-a_0)}{k} \qquad \text{and} \qquad kZ_S\frac{(30-a_0)}{k}.$$

The difference between these two, divided by 5·0, is the proportion ever-married in a cohort subject to a curve characterized by the calculated values of a_0 and k, and with an ultimate proportion ever-married of 1·0. The required estimate of C is, then,

$$\text{(proportion ever-married 25–30)}\Big/\frac{k}{5}\left[Z_S\left(\frac{30-a_0}{k}\right)-Z_S\left(\frac{25-a_0}{k}\right)\right].$$

Table 2A provides another set of three ratios for a range of values of k and a_0. RA_1 is the ratio of proportion ever-married at the end of the first five-year age interval to proportion ever-married at the end of the second, RA_2 the ratio of proportion ever-married at the end of the second interval to the proportion at the end of the third, etc. It is readily seen that RA_1 is $g_1/(g_1+g_2)$, RA_2 is $(g_1+g_2)/(g_1+g_2+g_3)$, etc., where g_1 is the average first marriage frequency in the i^{th} interval. The use of this table to ascertain k and a_0 is the same as the use of Table 1A, described above. To determine C when the data are first-marriage frequencies, calculate the proportion ever-married at the end of the third five-year interval, namely $5(g_1+g_2+g_3)$. Suppose this is $G(25)$ (i.e. suppose that a_0 falls in the interval 10 to 15). Then

$$C=\frac{G(25)}{G_S\left(\dfrac{25-a_0}{k}\right)}.$$