Models for the Estimation of the Probability of Dying between Birth and Exact Ages of Early Childhood

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INTRODUCTION

In the developing nations of the world, deaths to children under five years of age constitute a large fraction – sometimes, a majority – of all deaths. Childhood mortality statistics are, therefore, of considerable descriptive value. In addition, when known with accuracy, they can provide insight into a population's demographic characteristics by facilitating the analysis and understanding of existing fertility and nuptiality patterns. Moreover, childhood mortality statistics are useful as a sensitive index of a nation's health conditions and as guides for the structuring of public health programmes. Nevertheless, childhood mortality levels are typically known only approximately, if at all, in developing nations.

In general, the absence of accurate mortality statistics for the childhood populations of developing countries is due to a scarcity of reliable data. Vital registration is often incomplete or totally absent so that mortality statistics must be computed or estimated from survey and census data. These sources usually provide data on the age and sex composition of a population and of tabulations of vital events during the past calendar year (reference period data) or during the lifetime of the respondents (retrospective data). Reliable reference period data are, of course, an appropriate source of mortality statistics. However, for well-known reasons, the accuracy of data obtained in a single survey or census in a developing country is often questionable.[1] Age misreporting, underenumeration at certain ages and misconception of the duration of a reference period are the principal causes of inaccurate census data. These errors distort reference period data, particularly the data applicable to the childhood ages, and, accordingly, diminish confidence in age-specific death rates computed directly from them.[2] Nevertheless, problematic reference period data can still be used to estimate mortality conditions for some ages by use of techniques such as model life tables,[3] stable population theory[4] and, if at least two census tabulations are available, cohort survival rates.[5] Unfortunately, the first two techniques provide only very approximate estimates of childhood mortality rates and the third technique is not applicable to the childhood ages.[6]

Notwithstanding the typical errors found in reference period data, censuses often provide relatively accurate tabulations of the female population by five-year age intervals and of retrospective data on their children, namely: (1) the total number of children ever-born and (2) the number of

[1] D. Goldberg, and A. Adlakha, 'Notes on infant mortality based on surveys in the Ankara Area', in F. Shorter and B. Güvenic (eds.), *Turkish Demography: Proceedings of a Conference* (Haceteppe University Press, Ankara, 1966).

[2] United Nations, Department of Social Affairs, Population Studies, No. 13 (United Nations, New York, 1954); Department of Economic and Social Affairs, Population Bulletin of the United Nations, No. 6 (United Nations, New York, 1963).

[3] United Nations, Department of Economic and Social Affairs, *Methods of Estimating Basic Demographic Measures from Incomplete Data*, Population Studies, No. 42 (United Nations, New York, 1967); Department of Economic and Social Affairs, *The Concept of a Stable Population. Application to the Study of Populations of Countries with Incomplete Demographic Statistics*, No. 39 (United Nations, New York, 1968).

[4] Eduardo E. Arriaga, *New Life Tables for Latin American Populations in the Nineteenth and Twentieth Centuries*, Population Monograph Series, No. 3 (University of California, Berkeley, 1968).

[5] Paul Demeny, and F. C. Shorter, *Estimating Turkish Mortality, Fertility and Age Structure: Application of Some New Techniques* (Faculty of Economics Pub. No. 218, University of Istanbul, 1968).

[6] United Nations, *Methods of Estimating Basic Demographic Measures from Incomplete Data*. Cf. also Sullivan's review of Arriaga's monograph in *Social Biology*, June 1971.

children surviving at the time of the inquiry.[7] In particular, women who are young or newly married are able accurately to report such events because the relevant events have only recently occurred. From these data, statistics on the proportion dead of children ever-born, by age interval of female respondents, may be computed. A technique for converting these retrospective mortality statistics to more precise mortality measures was recently developed by William Brass. Essentially, the technique provides estimates of the probability of dying between birth and various exact childhood ages.[8] Since the errors often found in reference period data severely limit their usefulness as a source of childhood mortality statistics, the Brass technique, which relies on retrospective data, is of considerable value. Indeed, it is the only procedure which will provide relatively accurate estimates of childhood mortality conditions in many populations of the world to-day.

This paper deals with the problem of converting retrospective mortality data into precise mortality measures and offers some modifications and adaptations of the Brass solution to this problem. Section I discusses the assumptions underlying the construction of the Brass model. If accurate estimates of current mortality conditions are to be obtained from that model, those assumptions must hold approximately. In this section, particular stress is placed on the model's assumption of static conditions and on its use of mathematical functions to depict the degree of variation found among empirical fertility and mortality schedules. Section II develops two additional models for converting retrospective data into mortality estimates. They are, alternatively, designed for use with retrospective data tabulated by age-intervals of women (the age model) and by marital-duration intervals (the duration model). As with the Brass model, these models assume static conditions. Their development differs from the Brass model in two ways. Regression analysis is used to determine the relationships of interest, and empirically based fertility and mortality schedules provide the data for that analysis. In this section, we also compare the reliability of mortality estimates obtained from the Brass model with those obtained from the age model. This comparison is essentially a test of the appropriateness of the Brass procedure of representing empirical fertility and mortality schedules with analytical expressions. Finally, in Section III, we compare and evaluate the models developed in this paper, the age and duration models. Their performance was found to differ because of the fundamental differences in the structure of age-specific fertility schedules on the one hand, and marital-duration schedules on the other hand.

It is appropriate in this introduction to mention Appendix I, which deals with a problem of special note. It contains a discussion of the underlying determinants of retrospective mortality data, i.e. fertility and mortality conditions. The framework of the discussion is a general dynamic model in which fertility and mortality schedules are permitted to shift in level and configuration over time. The Brass model is then reviewed as a special (static) case of this dynamic model. We do not discuss this material because its treatment is not an immediate concern of this paper. Nevertheless, it is included in an appendix for several reasons. First, the existing literature does not include a dynamic model of the process by which retrospective mortality statistics are generated. Secondly, the structural assumptions of the Brass model and of the age and duration models, as well, may be shown to be special cases of the more general dynamic model. Finally, explicit contrast of the dynamic and static models will clearly reveal the conditions under which use of the latter is inappropriate.

SECTION I

William Brass developed a table of multipliers, appropriate under certain conditions, for converting statistics on the proportion dead of children ever-born[9] reported by women in the age intervals 15 to 19, 20 to 24, etc., into estimates of the probability of dying before attaining certain exact

[7] William Brass, Ansley J. Coale, *et al.*, *The Demography of Tropical Africa* (Princeton, New Jersey, 1968); Ansley J. Coale, and Paul Demeny, *Regional Model Life Tables and Stable Populations* (Princeton, New Jersey, 1966).

[8] *Ibid.*

[9] Hereafter, we will use the term children ever-born, c.e.b., to mean all the births experienced by a woman or group of women. Age distributions of c.e.b. will then mean the distributions which would result in the absence of mortality.

childhood ages. Following the notation of the literature, we will indicate the proportion dead of c.e.b. to women in successive five-year age intervals by D_i, where $i = 1$ signifies the interval 15–19, etc. Thus, in life-table terminology, the Brass multipliers convert D_i values into estimates of $q(a)$. Selection of a multiplier from the Brass table for converting a D_i statistic to its corresponding $q(a)$ estimate depends on factors peculiar to the population for which mortality estimation is desired. Since the rationale underlying the development of the Brass multipliers is presented in detail elsewhere, this discussion will only indicate the factors which determine the precise value of a selected multiplier and the principal assumptions underlying the model employed to develop those multipliers. Those assumptions are the conditions which a population must meet if the Brass model is to provide accurate estimates of current mortality levels.

Brass found that the relation between corresponding pairs of D_i and $q(a)$ is primarily influenced by fertility conditions and, in particular, by the age at onset of childbearing. In general, the earlier that age in a given population, the older are the children of the women in each age interval. The older the children, the longer their exposure to the force of mortality and the smaller the multiplier required to convert the proportion dead to a point on the mortality schedule ($q(a)$ estimate) which the children have experienced. Therefore, the appropriate value of a conversion multiplier depends on fertility conditions. Accordingly, the selection of a Brass multiplier is keyed to a fertility parameter, the ratio of the average parity of women aged 15 to 19 to women aged 20 to 24, P_1/P_2. This statistic has the virtue of being correlated with the age at onset of childbearing and of being easily obtained from the data necessary for calculation of statistics on the proportion dead of c.e.b. to women of five-year age intervals.

Two sets of assumptions underlying the Brass model can be distinguished. The first set pertains to the structure of the model. The second set pertains to the functions which were used to determine the relationship between corresponding pairs of D_i and $q(a)$, on the one hand, and the selection factor, P_1/P_2, on the other. The structural assumptions are:

(1) static conditions; age-specific fertility and infant and childhood mortality are assumed to have been constant in recent years, and

(2) a uniform age distribution of women within each five-year age interval considered.

The assumptions implicit in employing mathematical functions are:

(1) empirical mortality schedules can be closely approximated by an analytic function of age and a scale factor, and

(2) empirical fertility schedules can be approximated by an analytic function of age with a single parameter, the age at onset of childbearing, and a scale factor. The role of the parameter, age at onset of childbearing, is to slide the fertility function along the age axis (i.e. a linear translation) without altering the relative magnitude of the age-specific rates which are a fixed number of years removed from the age at onset of childbearing.

For reasons integral to the Brass technique, the required degree of similarity between Brass functions and empirical schedules is limited to their rate of change with respect to age and not their level. One of the major concerns of this paper is the investigation of how accurately the Brass functions approximate empirical schedules and, in particular, to investigate the accuracy of the procedure of sliding the fertility function along the age axis in representing the variability among empirical schedules characterized by different ages at onset of childbearing. There is evidence, based on a limited number of observations, that the Brass functions do represent empirical schedules closely and that the technique estimates mortality accurately.[10] However, a more general test of the performance of the Brass technique is needed. At the end of the next section of this paper such a general test is undertaken by employing the Brass model over a wide, but not exhaustive, range of fertility and mortality conditions.

[10] United Nations, *Methods of Estimating Basic Demographic Measures.*

SECTION II

In this Section I present two models which are designed to estimate the level of childhood mortality. Each model is developed with the use of regression analysis. Unfortunately, few empirical data are available on the variables included in the models. However, quasi-empirical data, data generated from empirically observed fertility and mortality schedules, can be used for the analysis. The data of the first model were generated from age-specific fertility and mortality schedules. The data of the second model were generated from marital-duration fertility schedules and age-specific mortality schedules. All the schedules were specific for single years of age or marital duration. To the extent that the generated data preclude respondent error, they are actually preferable to fully empirical data.

The Age Model

Our objective is to develop a set of equations for estimating multipliers ($q(a)/D_i$ ratios) to convert observed D_i values into estimates of $q(a)$. The age model is a simple linear regression model relating the ratio of selected pairs of $q(a)$ and $D_i(q(a)/D_i$ ratios for fixed values of a and i) to a fertility schedule parameter. The variables of the model are similar to the Brass variables and are readily calculated from retrospective data obtainable in surveys and censuses. The statistical characteristics of the regression equations of the model will help to determine their accuracy when used for estimation purposes.

Three types of data are necessary for the regression analysis: (1) fertility schedule parameters, (2) the probability of dying between birth and various ages ($q(a)$ values), and (3) the proportion dead of c.e.b. to women in five-year age intervals (D_i values). A single observation on these three sets of data was obtained from each possible combination of the fertility and mortality schedules listed in Appendix II. In all, 65 fertility and 40 mortality schedules were used to generate the data. The mortality schedules were taken from the Coale and Demeny *Regional Model Life Tables and Stable Populations*. The 40 schedules represent four distinct mortality patterns (West, North, South and East) at levels of life expectancy at birth from 30 to 52·5 years. Table 1 depicts the data generated for various age intervals of women.

TABLE 1. *Data generated by each pair of age-specific fertility and mortality schedules*

Mortality data:	$q(a)$ for $a = 1, 5$			
	Age intervals of women			
	15–20	20–24	25–29	30–34
Average parity data, P_i:	P_1	P_2	P_3	
Proportion dead of c.e.b., D_i:	D_1	D_2	D_3	D_4

Hereafter we will report our results for only those $q(a)/D_i$ ratios for which the regression analysis was most successful: $q(2)/D_2$, $q(3)/D_3$ and $q(5)/D_4$.[11] The fertility parameter which best explains variation in the value of these $q(a)/D_i$ ratios, and which should be employed in the regression model, cannot be determined *a priori*. Certainly, the desired parameter must reflect differences in the relationship among age-specific fertility rates of young women. Table 2 presents the results of correlation analysis between $q(a)/D_i$ values and two such parameters: the ratio of the average parity of women aged 15–19 to women aged 20–24 (P_1/P_2, the Brass parameter) and the ratio of the

[11] Other ratios are, of course, equally valid and were investigated. Nevertheless, these particular results are displayed because they yield smaller percentage errors in estimating $q(a)$ values than do other relations, say, for example, $q(1)/D_1$ or $q(1)/D_2$. We should note, in addition, that alternative forms of the regression model, logarithmic and second-degree polynomial, produced no better results than the simple linear model for regression with $q(2)/D_2$, $q(3)/D_3$ and $q(5)/D_4$ and improved the regressions with other $q(a)/D_i$ ratios very little.

TABLE 2. *Correlation coefficients between the* $q(a)/D_i$ *values and fertility parameters*

$q(a)/D_i$	Mortality pattern	Fertility parameter P_1/P_2	P_2/P_3
$q(2)/D_2$	West	−0·970	−0·970
	North	−0·973	−0·967
	East	−0·970	−0·971
	South	−0·966	−0·968
$q(3)/D_3$	West	−0·868	−0·988
	North	−0·869	−0·990
	East	−0·859	−0·988
	South	−0·852	−0·985
$q(5)/D_4$	West	−0·747	−0·920
	North	−0·749	−0·922
	East	−0·659	−0·897
	South	−0·737	−0·919

average parity of women aged 20–24 to women aged 25–29 (P_2/P_3). Since P_2/P_3 correlates more highly with $q(a)/D_i$ in eleven of the twelve data sets of Table 2, it was used as the explanatory variable of the age model which was specified as:

$$\frac{q(a)}{D_i} = A + B(P_2/P_3). \tag{1}$$

Using this model, a regression was calculated separately with the data generated from the mortality schedules of each of the four mortality patterns. The resulting regression equations incorporate all the variations in the shape of the fertility schedules but only the variation in the shape of the mortality schedules that is found within a single mortality pattern over a range of life expectancy at birth from 30 to 52·5 years. Reconciliation of the regression equations associated with the four different mortality patterns, for a particular $q(a)/D_i$ ratio, is accomplished at a later point. The regression coefficients and the statistical characteristics of the regressions of $q(2)/D_2$, $q(3)/D_3$ and $q(5)/D_4$ on P_2/P_3 for all four mortality patterns are displayed in Table 3. Values of R^2

TABLE 3. *Results of regression: age model*

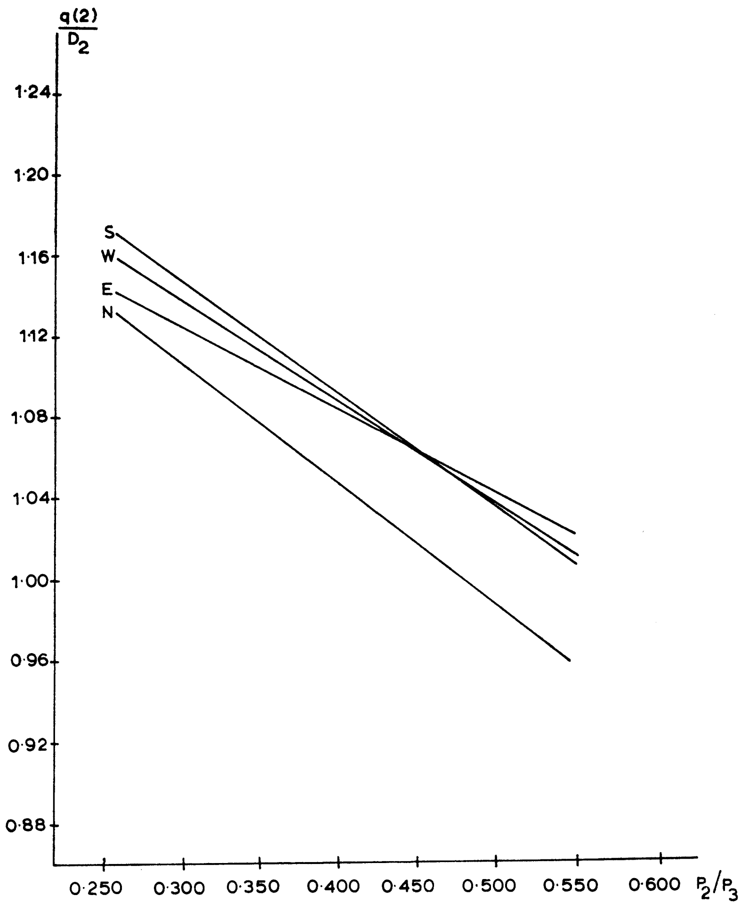
Regression equation*	Mortality pattern	Regression coefficients A	B	Standard error regression	R^2
$q(2)/D_2 = A + B(P_2/P_3)$	West	1·30	−0·54	0·008	0·942
	North	1·30	−0·63	0·010	0·936
	East	1·26	−0·44	0·007	0·943
	South	1·33	−0·61	0·009	0·938
$q(3)/D_3 = A + B(P_2/P_3)$	West	1·17	−0·40	0·004	0·977
	North	1·17	−0·50	0·004	0·980
	East	1·14	−0·33	0·003	0·977
	South	1·20	−0·44	0·005	0·970
$q(5)/D_4 = A + B(P_2/P_3)$	West	1·13	−0·33	0·009	0·846
	North	1·15	−0·42	0·011	0·851
	East	1·11	−0·26	0·007	0·839
	South	1·14	−0·32	0·009	0·845

* Each regression is based on 650 observations on the data.

are substantial (universally > 0·80). The standard error of each regression is less than 1 part in 100. Since, over the range of P_2/P_3 investigated the regression equation centers approximately on unity, this implies that estimates of $q(a)$ are characterized by a standard error at approximately 1%.

A comment on the reason for the deviation of data points from the regression line is appropriate here. From the resemblance among the mortality schedules of a given family, it can be inferred that the deviations of $q(a)/D_i$ values about a fitted regression line are almost wholly caused by differences in fertility schedules. This was verified by a study of the residuals.

The regression equations associated with $q(2)/D_2$ are shown diagrammatically in Fig. 1. In magnitude, the West regression equation lies between those of the other mortality patterns. This suggests that, if the prevailing mortality pattern is unknown, the use of the West equation for estimation is the safest course. A statistical analysis pertinent to the problem is contained in Table 4. In that table we present the standard errors of estimate of the multipliers estimated by the four



Panel A. $q(2)/D_2 = A + B(P_2/P_3)$

FIGURE 1. *Regression equations, age model.*

regression equations for each $q(a)/D_i$ ratio with the data from the four mortality patterns.[12] Reading across each line of Table 4 reveals how well a given estimation equation fits the data of the four mortality patterns. Of course, the West regression equation best estimates multipliers for the West mortality pattern, etc. The largest standard error of estimate of each regression equation indicates the maximum expected error caused by employing that regression equation when the true mortality pattern is unknown. Minimization of these maximum errors is the safest approach to estimation in those circumstances. The min–max solution for each $q(a)/D_i$ relation is indicated in Table 4. The West equation is the best overall performer and is thus recommended for use in estimating multipliers when the mortality pattern is unknown.[13]

TABLE 4. *Standard error of estimate, age model*

(Standard error of estimating actual $q(a)/D_i$ values generated by the West, North, East and South mortality schedules by employing the regression equations of the West, North, East and South mortality patterns. 16 combinations for each $q(a)/D_i$ relationship.)

Relation estimated	Regression model	Mortality pattern of data: West	North	East	South	Min–max solution
$q(2)/D_2$	West	0·008	0·043	0·009	0·016	
	North	0·043	0·010	0·041	0·046	
	East	0·010	0·042	0·007	0·016	0·042
	South	0·010	0·046	0·014	0·009	
$q(3)/D_3$	West	0·004	0·045	0·007	0·018	0·045
	North	0·045	0·004	0·051	0·063	
	East	0·006	0·049	0·003	0·016	
	South	0·018	0·062	0·014	0·005	
$q(5)/D_4$	West	0·008	0·025	0·008	0·015	0·025
	North	0·025	0·011	0·026	0·037	
	East	0·009	0·027	0·007	0·015	
	South	0·010	0·039	0·019	0·008	

Evaluation of the Brass Model and the West Regression Model

Both the Brass model and the West regression model are evaluated here in terms of the accuracy with which they estimate $q(2)$, $q(3)$ and $q(5)$. The data generated for the preceding regression analysis (P_1/P_2 and P_2/P_3 ratios and D_i and $q(a)$ values) are used in this evaluation. Brass multipliers were determined from P_1/P_2. These multipliers were then used in conjunction with data on children dying, D_i values, to estimate $q(a)$ probabilities.[14] In the same way, the parity ratio P_2/P_3 was used to make estimates with the West regression model. Deviations between estimated and actual $q(a)$

[12] The standard error of estimate pertains to the difference between actual $q(a)/D_i$ values (data generated for the regression analysis) and estimates of $q(a)/D_i$ obtained from the regression equations in conjunction with P_2/P_3 values from the data of the regression analysis.

[13] It is interesting to note that the data used by Coale and Demeny to produce the West Model Life Tables were a residual collection of 130 life tables remaining after the life tables used as data for the North, East and South Model Life Tables were removed from the original collection of reliable life tables. The West mortality pattern is an average of non-distinctive life tables which comprise the great bulk of the data collected by Coale and Demeny.

[14] Estimates of $q(5)$ from the Brass model based on the fertility index P_1/P_2 vice $\overline{m}$ or $\overline{m}'$ (the mean and the median age respectively of the fertility schedule) are reported here because the P_1/P_2 index actually yielded much better estimates (lower average percentage error) than $\overline{m}$ or $\overline{m}'$.

values were used to compute two statistics: the standard error and the average percentage error of estimate. These are shown in Table 5.

A comparison of the performance of the two models with the West mortality data is biased in favour of the regression model – the regression model being the best linear estimator of multiplying factors for the data of the West mortality pattern. Of course, no advantage is enjoyed by either model when assessing their accuracy in estimating $q(a)$ values with the data of the North, East or South mortality patterns.

TABLE 5. *Evaluation of mortality estimating techniques*

Brass Model: $\hat{q}(a) = (\text{multiplier})\ D_i$
West Regression Model: $\hat{q}(a) = (A + B\ P_2/P_3)\ D_i$

Mortality pattern	Standard error of estimate		Average % error of estimate	
	Regression model	Brass model	Regression model	Brass model
	Estimate of $q(2)$			
West	0·0018	0·0021	0·6	0·7
North	0·0086	0·0094	4·0	4·4
East	0·0021	0·0018	0·7	0·6
South	0·0025	0·0032	0·8	1·0
	Estimate of $q(3)$			
West	0·0009	0·0078	0·3	2·9
North	0·0109	0·0184	4·6	7·6
East	0·0020	0·0066	0·5	2·3
South	0·0045	0·0055	1·6	1·6
	Estimate of $q(5)$			
West	0·0024	0·0091	0·6	3·1
North	0·0066	0·0157	2·3	5·5
East	0·0024	0·0088	0·6	3·0
South	0·0046	0·0064	1·3	2·0

In estimating $q(2)$, the two models perform equally well. With the exception of the data associated with the North mortality pattern, both provide estimates which are accurate, on average, within 1%. However, the regression model is more accurate in estimating $q(3)$ and $q(5)$. For three mortality patterns, it provides estimates which are accurate, on average, within 1·5%. The exception is the North mortality pattern, where the average percentage error is several times greater. In most instances the average percentage errors of estimate of $q(3)$ and $q(5)$ of the Brass model are several times greater than those of the regression model.

Table 6 helps to elucidate the cause of the differential performance of the models in estimating $q(3)$ and $q(5)$. The table displays the mean value and the range of multipliers estimated by the regression equations of the four mortality patterns and by the Brass models for various $q(a)/D_i$ ratios. Each regression equation fits the data of its own mortality pattern exceedingly well. Thus, the accuracy of the Brass technique depends on whether the estimated multipliers have a mean value and a range similar to that estimated by the equations of regression model. The mean values of the multipliers estimated by the Brass model and by the West, East and South equations are close together. However, mean values associated with the North equations lie somewhat apart.

In contrast to the general agreement of mean values, there are substantial differences in the range of multipliers estimated by the regression model and the Brass model. The multipliers of the

Brass model – particularly those applicable to values of D_3 and D_4 – span a considerably smaller range. This difference stems from two possible sources. First, P_2/P_3 of the age model reflects the fertility of women aged 30 and below, while P_1/P_2 reflects the fertility experience of women aged less than 25. Thus, P_2/P_3 is more sensitive to fertility variations of older women and, since data from older women are used to estimate $q(3)$ and $q(5)$, P_2/P_3 should be a superior index for selecting a multiplier for estimating those values of $q(a)$. Secondly, the Brass fertility function, particularly the procedure of sliding that function along the age axis, may not simulate the variability of empirical fertility schedules as completely as do the schedules of the age model.

TABLE 6. *Mean and range of multipliers estimated from the regression equations and from the Brass model for the parity ratios of the fertility schedules employed in the study*

(Range for P_1/P_2=0·045 to 0·175 and P_2/P_3=0·264 to 0·552)

	Mean	Range
Multipliers ($q(2)/D_2$)		
Regression models		
West	1·07	0·15
North	1·03	0·18
East	1·07	0·13
South	1·07	0·18
Brass model	1·08	0·10
Multipliers ($q(3)/D_3$)		
Regression models		
West	1·00	0·12
North	0·95	0·15
East	1·00	0·10
South	1·01	0·13
Brass model	1·03	0·05
Multipliers ($q(5)/D_4$)		
Regression models		
West	1·00	0·10
North	0·98	0·12
East	0·99	0·08
South	1·00	0·10
Brass model	1·03	0·04

In conclusion, although the West regression model has some advantages, both models perform extremely well. Those errors on the order of a few percentage points due to structural deficiencies in the models are tolerable. Since even retrospective data collected in censuses and surveys in the developing nations are somewhat in error, more elaborate structural relations do not appear justified. Nevertheless, the simplicity of the West regression model and its advantages in the estimation of $q(3)$ and $q(5)$ would seem to recommend it over the Brass model for the estimation of childhood mortality conditions when the underlying mortality pattern is unknown.

The Duration Model

The model of this section differs from the age model in a single characteristic. It is structured in terms of marital-duration intervals rather than age intervals. The important differences in the structure of age-specific and marital-duration-specific fertility schedules need elaboration. First, they differ in degree of variation displayed by the parameters, age at onset of childbearing and duration at onset of childbearing. Among age-specific fertility schedules, the age at onset of childbearing has a range of several years. Among marital fertility schedules, the duration at onset of childbearing varies over a range of less than one year. Secondly, age-specific fertility schedules peak and begin declining several years after the age at onset of childbearing, while marital fertility schedules usually peak within a year of the duration at onset of childbearing. Since the age distributions of the c.e.b. to women of a particular age or marital duration are equivalent to their respective fertility schedules written backwards, these age distributions reflect the characteristic differences between these two types of fertility schedules. When aggregates of c.e.b. to all the women of an age or duration interval are composed, once again the distributions differ markedly. Since the proportion dead of c.e.b. is partially determined by the age distributions of those children, one would expect that estimation models based on age-specific and duration-specific fertility schedules exhibit different properties. This conjecture, soon to be examined, is the motivation for constructing the duration model.

Since the age and duration models are conceptually similar, a lengthy description of the latter is unnecessary. The twelve fertility and the 40 mortality schedules which were employed to generate the data for the model are listed in Appendix II. Table 7 summarizes the data obtained from a single pairing of a fertility and mortality schedule.

TABLE 7. *Data generated by each pair of duration-specific fertility and age-specific mortality schedules*

Mortality data:	$q(a)$ for $a=1,\ \ 5$		
	Duration intervals of women		
	0–4	5–9	10–14
Average parity data, P_i:	P_1	P_2	P_3
Proportion dead of c.e.b., D_i:	D_1	D_2	D_3

The appropriateness of a linear regression model was substantiated by correlation analysis between $q(a)/D_i$ values and fertility parameters (P_1/P_2, P_2/P_3 and P_1/P_3).[15] The correlations between $q(a)/D_i$ values and each of these fertility parameters were all in excess of 0·87. Moreover, the correlation between a particular $q(a)/D_i$ ratio and each of the three parity ratios did not differ enough to establish one particular parity ratio as a superior explanatory variable. Development of a model designed to use statistics based on respondents' reports about events experienced as recently as possible will minimize recall error. Hence, P_1/P_2 was chosen to serve as the explanatory variable of the duration model.[16] This choice resulted in the following regression equation:

$$\frac{q(a)}{D_i} = A + B(P_1/P_2). \tag{2}$$

Regressions were calculated separately for data generated from each of the four mortality patterns. Table 8 displays the most successful results obtained with the model, that is, those

[15] The use of the symbols P_i and D_i to represent average parity and proportion dead of c.e.b. in this model should not lead to confusion with their use in the age model. For the remainder of this paper, it will be clear from the context of their use whether these symbols represent the experience of women of an age or duration interval.

[16] In the age model P_2/P_3, as opposed to P_1/P_2, was selected as the explanatory variable because of its greater explanatory power. In the duration model P_2/P_3 did not enjoy a similar advantage over P_1/P_2. This difference between the models is most probably due to the consistency with which the women of young ages, on the one hand, and of early marriage durations, on the other, characterize age and duration specific fertility schedules. For the age model, P_1 pertains to women under 20 and sometimes contains small numbers of c.e.b.; fertility in the first five years of marriage, however, is always substantial. Hence, P_1 in the duration model is always a predominant fertility schedule characteristic while it may not be so in the age model.

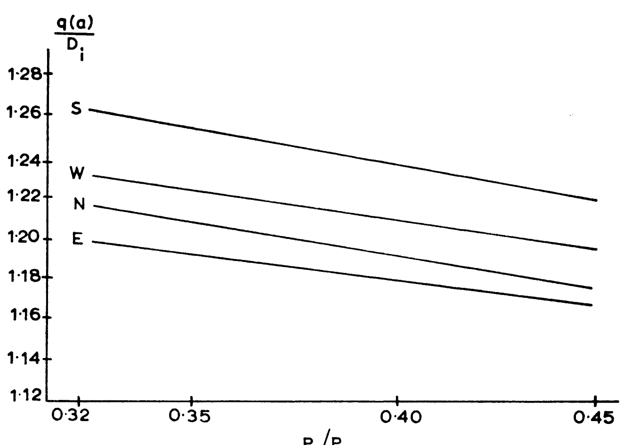
TABLE 8. *Results of regression: duration model*

Regression equation*	Mortality pattern	Regression coefficients A	B	Standard error regression	R^2
$q(2)/D_1=A+B(P_1P_2)$	West	1·34	−0·35	0·011	0·891
	North	1·34	−0·38	0·019	0·774
	East	1·29	−0·28	0·009	0·893
	South	1·40	−0·42	0·021	0·764
$q(3)/D_2=A+B(P_1P_2)$	West	1·18	−0·44	0·009	0·951
	North	1·18	−0·53	0·008	0·974
	East	1·16	−0·37	0·006	0·968
	South	1·22	−0·50	0·008	0·968
$q(5)/D_3=A+B(P_1P_2)$	West	1·17	−0·44	0·019	0·810
	North	1·20	−0·57	0·024	0·824
	East	1·14	−0·36	0·016	0·815
	South	1·19	−0·46	0·019	0·834

* Each regression is based on 120 observations on the data.

$q(a)/D_i$ ratios which, when regressed on P_1/P_2, displayed the smallest standard errors of estimate; namely $q(2)/D_2$, $q(3)/D_2$ and $q(5)/D_3$. Once again the values of R^2 are high and standard errors of regression are small. Fig. 2 graphically represents the regression equations associated with $q(2)/D_1$.

We will now examine the accuracy of the regression equations in converting D_i statistics to $q(a)$ values. Table 9 shows the expected error of the conversion multipliers estimated by each equation. Standard errors are presented both from the application of each equation to data from its own mortality pattern and from the application of each equation to data from three 'alien' mortality patterns. Statistics of the former kind are found along the diagonal of each panel in Table 9 and statistics of the latter variety are the off-diagonal terms. Assuming that nothing is known about the



$q(2)/D_1=A+B(P_1/P_2)$.

FIGURE 2. *Regression equations, duration model*

TABLE 9. *Standard error of estimate, marital duration estimation model*

(Standard error of estimating actual $q(a)/D_i$ values generated by the West, North, East and South mortality schedules by employing the regression equations of the West, North, East and South mortality patterns. 16 combinations for each $q(a)/D_i$ relationship.)

Relation estimated	Regression model	Mortality pattern of data: West	North	East	South	Min–max solution
$q(2)/D_1$	West	0·004	0·017	0·028	0·026	0·028
	North	0·018	0·007	0·013	0·043	
	East	0·028	0·011	0·004	0·053	
	South	0·027	0·043	0·054	0·008	
$q(3)/D_2$	West	0·004	0·039	0·006	0·016	0·039
	North	0·039	0·003	0·044	0·054	
	East	0·007	0·045	0·002	0·011	
	South	0·016	0·054	0·012	0·003	
$q(5)/D_3$	West	0·008	0·027	0·007	0·013	0·027
	North	0·027	0·009	0·030	0·037	
	East	0·008	0·031	0·006	0·010	
	South	0·013	0·038	0·011	0·007	

prevailing mortality pattern, min–max criteria may be employed to minimize the risk of a substantial error. Table 9 indicates that according to min–max criteria, the West equation provides the best solution. Indeed, the maximum standard error of estimate of the West equation with the data from any mortality pattern is less than 0·05, which is equivalent to errors of less than 5% in estimating $q(a)$ values.

SECTION III

Comparison of the Age and Duration Models

Both the models which we have presented are designed to estimate $q(2)$, $q(3)$ and $q(5)$. In addition, the West regressions provide a min–max solution to both models. These similarities allow a convenient pairing of estimation equations between the two models (Table 10). The independent variable of both models, in each case a parity ratio, reflects the same demographic characteristic – the shape of

TABLE 10. *Preferred estimation equations from the age and duration models*

(Based on West Regional mortality pattern)

Mortality value to be estimated	Estimation model	Equation estimating the multiplier applicable to statistics on the proportion dead of c.e.b.
$q(2)$	Age	$1·30–0·54\ (P_2/P_3)$
	Duration	$1·34–0·35\ (P_1/P_2)$
$q(3)$	Age	$1·17–0·40\ (P_2/P_3)$
	Duration	$1·18–0·44\ (P_1/P_2)$
$q(5)$	Age	$1·13–0·33\ (P_2/P_3)$
	Duration	$1·17–0·44\ (P_1/P_2)$

* P_2/P_3 is the parity ratio of women age 20–24 to women age 25–29. P_1/P_2 is the parity ratio of women married 0–4 years to women married 5–9 years.

childhood age distributions. The slope of each equation indicates the relative weight assigned to the parity ratio in developing the estimates of $q(a)$. The slopes of the two models differ most in the estimation of $q(2)$. If, because of respondent error, the parity ratios P_2/P_3 (age model) and P_1/P_2 (duration model) are somewhat in error – say in error by the same absolute amount – it will be advantageous to use the duration model. Indeed, for $q(2)$, the duration model is approximately half as sensitive to erroneous parity ratio values as the age model.

Inspection of the fertility schedules of the models will help to explain the difference in slope between the two equations for the estimation of $q(2)$. Table 11 contains data on the range of values of parity ratios from the fertility schedules used to develop each model. By a factor of two, the parity ratios of the age-specific fertility schedules exhibit the greater range. Fig. 3, which depicts the extent of variation in the childhood age distributions associated with the fertility schedules of the two models, is also pertinent to the explanation of the slope differential. The two curves of Panel A are associated with the marital fertility schedules with the maximum and minimum parity ratios. The curves are quite similar. Accordingly, the function of the parity ratio, to ensure that estimated multipliers are consistent with the distribution of c.e.b., is not crucial, hence the small slope of the regression equation in spite of the narrow range of marital parity ratios. The curves of Panel B are associated with age-specific fertility schedules with the maximum and minimum parity ratios. These age distributions are much more disparate than those of Panel A. Therefore, the burden placed on the parity ratio P_2/P_3 to assure accurate multipliers is relatively heavy. Thus, in spite of the greater range of P_2/P_3, the regression coefficient for $q(2)$ of the age model is greater than that of the duration model.

TABLE 11. *Range of parity ratios from the fertility schedules of the age and duration models*

Model	Parity ratio	Values of parity ratio		
		Min.	Max.	Range
Age	P_2/P_3	0·25	0·55	0·30
Duration	P_1/P_2	0·32	0·45	0·13

These conclusions apply only to the sets of fertility schedules in each model. The universality of the results can be assessed only by considering the representativeness of those schedules. Since that task is undertaken in detail elsewhere[17] only a summary is presented here. Consider first the age model. The listing of fertility schedules in Appendix II reveals that low-fertility populations of the developed world are well represented in the age model. Only one schedule of that collection, the schedule from the Cocos-Keeling Islands, represents a high-fertility population. However, that schedule represents high-fertility populations rather well in the sense of displaying a P_2/P_3 ratio which is quite high, both because of an early age at marriage and because most girls are pregnant at the time of marriage.[18] Consider now the fertility schedules of the duration model. They span a gamut of high and low-fertility populations. The Hutterite schedule represents the high-fertility end of the range, while white-collar workers in England and Wales (1929) represent the low-fertility end. Since the Hutterites exhibit a pattern of marital fertility which is sustained at a level as high as that reliably recorded for any population in the world, it represents parity ratios as low as those of

[17] Jeremiah M. Sullivan, *Estimation of Childhood Mortality Conditions from Childhood Survival Statistics*, Doctoral Thesis, University Microfilms Order No. 71–1636 (Ann Arbor, Michigan, 1970); 'Book Review: Eduardo E. Arriaga, New Life Tables for Latin American Populations in the Nineteenth and Twentieth Centuries' (*Social Biology*, June 1971).

[18] T. E. Smith, 'The Cocos-Keeling Islands: A demographic laboratory', *Population Studies*, **14**, 2 (November 1970).

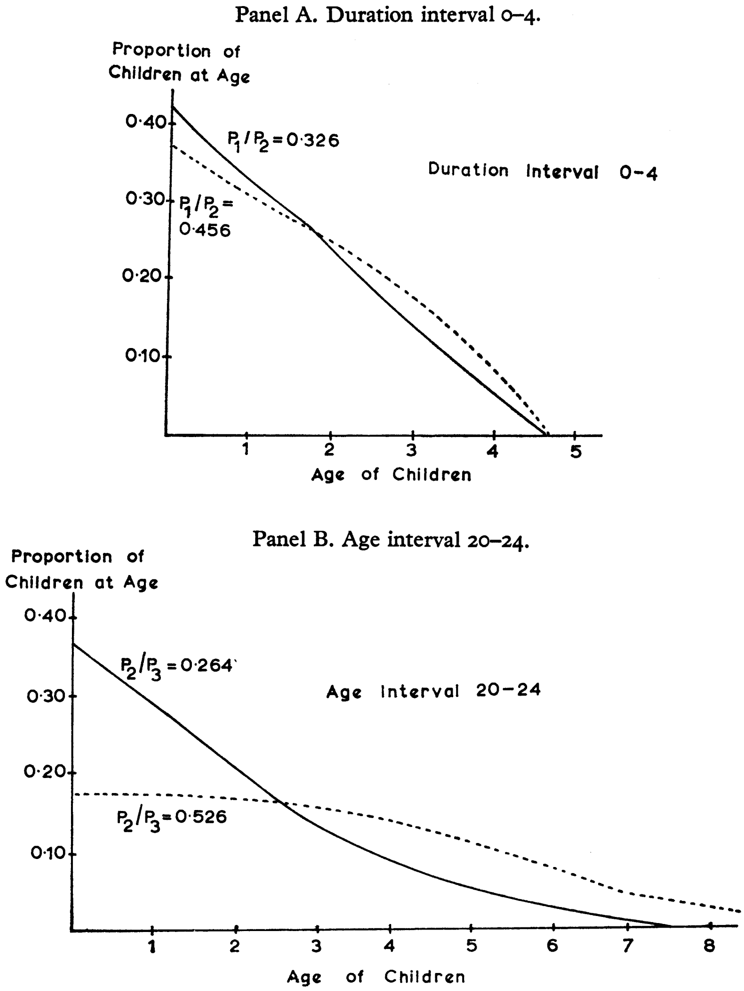


FIGURE 3. *Age distributions of c.e.b. in absence of mortality*

any sizeable current population.[19] It might be argued that if the Hutterites were to marry early rather than late, their associated P_2 values would be somewhat higher. The cumulative fertility of Hutterite women implies an average of 2·86 c.e.b. by the end of the fifth year of marriage and 5·50 by the end of the tenth year.[20] Thus, the childbearing performance of these women did not flag in the second five years of marriage. Of course, the English populations also marry late. But that characteristic extends the P_1/P_2 values at the high end of the range.

[19] In counter-distinction to P_2/P_3 of the age model, the value of P_1/P_2 in the duration model is inversely related to the fertility level. Thus, in the duration model the Hutterite and German populations display the smallest parity ratios while the low-fertility English populations display the highest. Among Hutterite females, high fertility continues well after the first five years of marriage, so that P_2 is high relative to P_1. Among the English population of the early part of this century, contraception tended to impinge more heavily at longer marriage durations leading to higher parity ratios.

[20] Mindel C. Sheps, 'An analysis of reproductive patterns in an American isolate', *Population Studies*, **19**, 1 (July 1965).

Inspection of the fertility schedules implicit in the models reveals that a wide range of fertility conditions is represented. This in turn implies that it is appropriate to employ the models over a wide range of fertility levels and that our conclusion regarding the sensitivity of the models to respondent error is not the result of a selective aggregation of schedules. However, some reservations about the representativeness of the schedules with regard to future fertility conditions should be stated. It has often been observed that the introduction of family planning programmes among the high-fertility population of the developing world results in the adoption of contraceptive practices on an age and marital duration selective basis. Greater reduction of fertility at older ages and longer durations of marriage could result in fertility schedules substantially different from those we have employed. Nevertheless, since family planning programmes take considerable time to initiate, in the near future, few parity ratios pertaining to sizeable national populations will be encountered which are beyond the range of those associated with our models.

SUMMARY

This paper develops two models for the estimation of childhood mortality conditions from statistics on the proportion dead of c.e.b. to women of five-year-age and marriage-duration intervals. The models were developed by employing regression analysis with data generated from empirically observed fertility and mortality schedules. The data were generated on the assumption of unchanging fertility and mortality, and the models will yield estimates of current mortality conditions only if a population's recent demographic history coincides with those assumptions. Consideration was given to the problem of estimation when the configuration of the underlying mortality pattern is unknown. Preferred sets of estimation equations were presented (Table 10) for use in those circumstances. Based on the variation in the shape of the mortality schedules of the mortality patterns isolated by Coale and Demeny, we concluded that in absence of knowledge about the prevailing mortality pattern, the West equations of both models will provide estimates of $q(2)$, $q(3)$ and $q(5)$ with a standard error of estimate of less than 5%. The advantage of having both models available for use is that, on some occasions, the preconditions for the use of only one will be met. In the happy circumstance that the use of either is appropriate, one can serve as a check for the other.

Additionally, in this paper, the reliability of the Brass model in estimating childhood mortality was tested over a wide range of fertility and mortality conditions. Its performance was consistently high – the age model enjoying, within the context of the test we administered, only a slight edge

ACKNOWLEDGMENTS

Financial support for this research was provided by the Milbank Memorial Fund and computer facilities during the terminal phase of this project were provided by the Population Studies Center of the University of Michigan. I am particularly indebted to Ansley J. Coale, Albert I. Hermalin, Ronald Freedman and Robert Johnson who provided support and valuable assistance in the course of this research.

APPENDIX I

GENERAL DYNAMIC MODEL FOR THE DETERMINATION OF RETROSPECTIVE MORTALITY STATISTICS

Retrospective mortality data pertaining to the women of each five-year age interval are determined in an identical manner. Therefore, this discussion will be restricted to women aged 20 to 24 without loss of generality. Notation, consistent with the literature, will be used to indicate statistics relevant to the women of a given age. Thus P_1 and P_2 will indicate the average parity of women 15–19 and 20–24, respectively. $C_2(a)$ represents the age distribution of c.e.b. to women 20–24, while D_2 denotes the proportion dead of c.e.b. to those women. Additional notation applicable to women of an exact age, as opposed to an age interval, will be introduced as we proceed.

Determinants of the Statistic D_2

The proportion dead of the c.e.b. which would be reported by error-free female respondents aged 20–24 is a function of (1) the proportionate age distribution of their children in the absence of mortality, and (2) the mortality conditions experienced by those children.

The roles of fertility and mortality in determining age distributions and the proportion dead of c.e.b. are displayed in Fig. 4. The first two quadrants present probability curves depicting the fertility of individual women of exact ages. In the third quadrant the aggregate experience of all women in the age interval is presented. In particular, the age distribution of c.e.b. to women aged 20–24 and the mortality experience of the children at each exact age is shown. The statistic D_2 is derived from the curves of the third quadrant.

It is customary to represent the fertility schedule of a female cohort as a function of age, x, i.e. $f(x)$. In Fig. 4 that representation is modified. That is, we substitute for x in $f(x)$ the expression $x-t$, where x is a parameter (current exact age of a woman) and t is a variable (time, measured retrospectively from the present). To permit the fertility schedules of women at different exact ages to differ, we employ a subscript, $f_x(x-t)$. Then $f_{20}(20-t)$ represents the probability that a woman currently aged 20 had a child t years ago. Thus, in the first quadrant, each curve displays the fertility of women, who are currently at different exact ages within the interval 20 to 24, as a time series of events during the recent past. In the second quadrant, that fertility history is presented in cross-section as the probability that a woman has a child of a given age. Now, the abscissa represents age, a, and the fertility schedules of the first quadrant, $f_x(x-t)$, appear in minor reflection as age distributions of c.e.b., $c_x(a)$, where $a=t$.

In the third quadrant, the procedure for determining aggregate statistics on the age distribution of c.e.b. ($C_2(a)$) and the proportion dead (D_2) for all women in the age interval 20–24 is displayed. The abscissa again represents age but the ordinate is calibrated in two different scales, the proportionate age distribution of c.e.b. and the proportion dead of children born a years ago. The aggregation process must consider the number of women at each exact age in the interval 20–24, which we represent by N_x where x indicates exact ages of women. Then, the proportion of children, in the absence of mortality, at a particular age, say a', is the sum, over all ages in the interval 20–24, of the product N_x and $c_x(a')$ divided by the total number of children.

$$C_2(a')=\frac{\int_{20}^{25} N_x c_x(a')\,\mathrm{d}x}{\int_{20}^{25}\int_0^{\omega} N_x c_x(a)\,\mathrm{d}a\,\mathrm{d}x} \tag{1}$$

where $C_x(a')$ is the proportion of c.e.b. to women aged 20–24 who are exactly age a';
$c_x(a)$ is the age distribution of c.e.b. to women aged x exactly;
ω is the oldest age of children of the women aged x exactly;
N_x is the number of women at age x; and
a' is an exact childhood age.

In the third quadrant the complete age distribution of c.e.b. is, of course, labelled $C_2(a)$.

To determine D_2, the proportion dead of c.e.b. to women aged 20–24, the mortality experience of the children is introduced. In this dynamic model, mortality must be represented in a general way which allows the level of mortality to shift over time. Thus the mortality schedule of the third quadrant, $q(a)$, is defined somewhat differently than usual; $q(a)$ is the probability that the member of a birth cohort born exactly a years ago will have died before attaining age a. This schedule is a composite one, each point of which represents the mortality experience of different birth cohorts.[21] The product of $C_2(a)$ and $q(a)$ is D_2.

$$D_2=\int_0^{\omega} C_2(a)\,q(a)\,\mathrm{d}a, \tag{2}$$

where $C_2(a)$, a and ω are as earlier defined; and $q(a)$ is the probability of dying before attaining age a of the members of the birth cohort born a years age.

D_2 may be interpreted as a weighted average of $q(a)$, the weights being determined by the proportionate age distribution, $C_2(a)$. Thus, D_2 depends on both fertility and mortality conditions which reveals its weakness as a mortality index.

[21] This concept must be distinguished from the usual period and cohort mortality schedules. According to this definition of $q(a)$, $q(2)$ is the proportion dead among the children born exactly two years ago. It need not be the proportion dying between birth and age two of any other cohort.

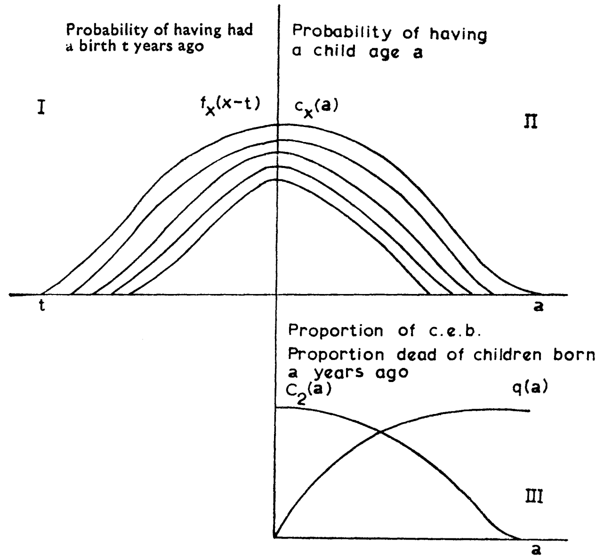


FIGURE 4. *Determinants of D_2, the proportion dead of c.e.b. to women 20–24.*

The Brass Model

The Brass model may be viewed as a simulation process in which fertility and mortality schedules are employed to generate hypothetical values of D_i and $q(a)$ – allowing the relationship between those variables to be determined. The fertility function of the Brass model may be written as

$$f(x) = K' f_s(x, \alpha) \tag{3}$$

where $f(x)$ is the probability of a woman age x bearing a child;

$f_s(x, \alpha)$ is a standard fertility schedule containing one parameter, α, the age at onset of childbearing, which serves to slide the entire schedule along the age axis; and

K' is a scale factor.

The mortality schedule may be written as

$$q(a) = K'' q_s(a), \tag{4}$$

where $q(a)$ is the probability of dying before reaching age a;

$q_s(a)$ is a standard mortality schedule; and

K'' is a scale factor.

The characteristic of the Brass model which differentiates it from the general model is its assumption of static conditions. In the simulation process, by which a pair of hypothetical values of D_2 and $q(2)$ are produced, values are assigned to the parameters (K', K'' and α) of the fertility and mortality functions. Those values, and hence the functions, are fixed throughout the simulation process for a single pair of values. The model is static then, in the sense that all women within the age interval experience, for a given exact age, the same fertility conditions, and all their children are subject to the same mortality conditions.[22]

[22] Note that the fertility function of Equation 3 is sub-scripted only to indicate that it is a standard schedule as opposed to indicating that its form can change, which is the role of the subscript x of $f_x(x-t)$ in the fertility function of the general model. Similarly, the mortality function of the Brass model (Equation (4)) is commonly shared by all children. So $q(2)$ is not only the proportion who have died of the children born two years ago but also the proportion who died before attaining age two of the children born, say, three years ago. These differences between the Brass model and the discussion of the determinants of D_2 emphasize the restrictive nature of that model relative to the dynamic aspects of empirical data.

This is the major limitation of the Brass model and of the age and duration models as well. Because these models are not designed to cope with fluctuations in fertility and mortality conditions, results obtained under those circumstances cannot be considered estimates of current mortality levels but must be interpreted with extreme caution.

Within this framework, the Brass model develops a rationale for estimating $q(a)$ parameters from D_i values. The age distribution of c.e.b. to women aged 20–24 of the Brass model is determined under the assumption of a uniform age distribution of those women and is similar to Equation 1. Thus:

$$C_2(a) = \frac{NK' \int_{20}^{25} f_s(x-a)\,dx}{NK' \int_{0}^{25-\alpha} \int_{20}^{25} f_s(x-a)\,dx\,da} \tag{5}$$

where $C_2(a)$ is the proportionate age distribution of c.e.b. to women aged 20–24.
$f_s(x-a)$ and K' are as defined in (3) above, the parameter α assuming a particular value;
$25-\alpha$ is the age of the oldest children born to women of the interval;
a and x respectively represent the ages of children and the women of the interval 20 to 24; and
N is the number of women at each age x in the uniform age distribution of women.

Since K' and N cancel, $C_2(2)$ depends only on the value of α. The proportion dead of c.e.b. becomes

$$D_2 = K'' \int_{0}^{25-\alpha} C_2(a)\, q_s(a)\, da \tag{6}$$

where $C_2(a)$, $q_s(a)$, K'', a and α are as previously defined; and $25-\alpha$ is the age of the oldest children of women aged 20–24.
D_2 depends on both α and K''.

Now, the relationship between D_2 and $q(2)$ can be expressed as a ratio.

$$M_2 = \frac{q(2)}{D_2} = \frac{K'' q_s(2)}{K'' \int_{0}^{25-\alpha} C_2(a)\, q_s(a)\, da} \tag{7}$$

where M_2 is the multiplier for converting D_2 to $q(2)$; and $q(2)$ and D_2 are as previously defined.
Since K'' is a scale factor in both $q(2)$ and D_2, it cancels, making these multipliers depend only on α. Brass offers a set of multipliers (actually the ratios $q(2)/D_2$) for different values of α to convert D_2 to $q(2)$.[23] Of course, the use of these multipliers requires that they be tabulated in terms of a statistic which is a function of α and which is readily obtained from survey data. The ratio of the average parity of women aged 15–19 to women aged 20–24, P_1/P_2, provides such an index. This statistic is a parameter of a fertility schedule and in the Brass scheme equals

$$P_1/P_2 = \frac{NK' \int_{0}^{20-\alpha} \int_{15}^{20} f_s(x-a)\,dx\,da}{NK' \int_{0}^{25-\alpha} \int_{20}^{25} f_s(x-a)\,dx\,da} \tag{8}$$

where P_1/P_2 is the ratio of the average parity of women 15–19 to women 20–24; and N, K', a, α and $f_s(x-a)$ are as earlier defined.

[23] Note that multipliers could be computed to represent the relationship between D_2 and other values of $q(a)$, say $q(3)$. The association of D_2 and $q(2)$ is not meant to imply that the argument of $q(a)$ and the subscript of D_i must be equal.

APPENDIX II

AGE MODEL

Listing of Age-Specific Fertility Schedules

(Specific for single years of age)

Country	*Year*
Belgium	1935, 1939, 1940, 1945, 1960, 1965
Canada	1931, 1940–41, 1951, 1961
Czechoslovakia	1930
Denmark	1950
Finland	1950, 1960
France	1925–27, 1930–32, 1935–37, 1952, 1960
Germany	1950, 1960
Hungary	1957, 1960
Italy	1951
Latvia	1937, 1938
Norway	1930
Poland	1950, 1955, 1959
Portugal	1942–45, 1945, 1950, 1960
Sweden	1891–00, 1901–10, 1911–20, 1921–30, 1931–40, 1941–50, 1961
Ukraine	1926–27
United States	1960
Yugoslavia	1953, 1960
Cocos-Keeling Islands	Early 1900

SOURCES: The Office of Population Research, Princeton University, has gathered a collection of age-specific fertility schedules by single years of age and 46 of these were used in this study. The P_1/P_2 parameter associated with the fertility schedule from the Cocos-Keeling Islands was 0·175 while the remaining schedules displayed values of P_1/P_2 which were evenly distributed over the range 0·045 to 0·134. To provide continuity of the value of P_1/P_2, an additional 19 fertility schedules were obtained by shifting the original 46 schedules downward one year (i.e. earlier childbearing) and selecting those resultant schedules with a P_1/P_2 value between 0·134 and 0·175.

DURATION MODEL

Listing of Duration-Specific Fertility Schedules

(Specific for single years of marital duration)

Fertility schedule	*Population*
1 schedule	American Hutterite Population.
2 schedules	Bavarian Villages: Tottleben and Anhausen, 19th century.
3 schedules	England and Wales, 1900–09: Blue-collar Workers, All Social Groups and White-Collar Workers.
3 schedules	England and Wales, 1920: Blue-Collar Workers, All Social Groups and White-Collar Workers.
3 schedules	England and Wales, 1929: Blue-Collar Workers, All Social Groups and White-Collar Workers.

SOURCES: Hutterite Population: Mindel C. Sheps, 'An analysis of reproductive patterns in an American isolate', *Population Studies*, July 1965.
Tottleben and Anhausen: Lorenz Scheuenpflug, *Ortssippenbuch Anhausen*, Frankfurt/Main, 1961.
England and Wales: D. V. Glass and E. Grebenik, *The Trend and Pattern of Fertility in Great Britain*, Part II, Tables.