The Process of Demographic Translation

N. B. Ryder

Demography, Volume 1, Issue 1 (1964), 74-82.

Stable URL:
http://links.jstor.org/sici?sici=0070-3370%281964%291%3A1%3C74%3ATPODT%3E2.0.CO%3B2-K

THE PROCESS OF DEMOGRAPHIC TRANSLATION

N. B. RYDER
University of Wisconsin

RESUMEN

"Traslación demográfica" es el establecimiento de interrelaciones entre una serie cronológica de observaciones en sección transversal en períodos de tiempo sucesivos y las series cronológicas del mismo índice, referido a cohortes sucesivas. Éste documento presenta dos tipos de soluciones a este problema y dá algunos ejemplos de cada uno.

THE NATURE OF THE PROBLEM

The question with which this paper is concerned begins with a table of measurements which are specific for age and time. To summarize this table with respect to changes in the measurements through time, it is customary to compute some index which combines the measurements over the entire span of ages and study the time series of such indices year by year. But there are two ways of organizing such a table into temporal segments: first, as age-specific measurements for a series of years; second, as age-specific measurements for a series of birth cohorts. The problem to be solved is the relationship between a time series of some index for successive periods, and the time series of the same index for successive cohorts. This paper presents two directions of general solution of the problem, and some elementary examples of each. It then attempts to argue the merits of the translation process, as it is here named, for various problems in demography.

PREVIOUS WRITING ABOUT TRANSLATION

The author first became interested in the problem while doing graduate work at Princeton in the late 1940's. In his doctoral dissertation [1] he demonstrated the tendency for period and cohort total fertility rates and mean ages of fertility to diverge in the short run and in the long run, and located the source of these divergences in the changing distribution through time of the childbearing of successive cohorts. His first use of specific translation formulae [2] concerned the analyses of two centuries of Swedish fertility history and of recent American experience. These studies were both based on age-specific fertility rates. Subsequently he has published two associated papers [3, 4] on contemporary American age-parity-specific fertility, using somewhat more complicated formulae than before, and applying them to truncated cohort age spans. Most recently he has used translation formulae at various junctures in designing a model of demographic transition [5] and extended the application to other demographic variables. The present paper undertakes to direct attention to the process of translation itself, in the belief that others may find it useful. It also presents the mathematical argument as far as the author has been able to develop it.

CHARACTERISTICS OF THE BASIC TABLE

The conventional table of demographic measurements which are age-specific and time-specific has rows for each age of observation and columns for each time of observation. What are here called period indices are summaries of all observations in a particular column (e.g., their sum or mean age). Since a cohort is identified by a fixed difference between time of observation and age at observation, its measurements are located along a diagonal in such a table. If year of observation is t, year of age is a, and year of birth is T, then the identity $t = T + a$ holds throughout the table.[1] The following observations

[1] To represent this identity more faithfully in a geometric sense, the table might better be constructed with columns at 60° and 120°, per-

about the basic table are pertinent to the argument of this paper: (1) Indices for the measurements in each cohort diagonal may be computed and studied as a time series which has as much formal legitimacy as the series for successive periods. (2) Although the period and the cohort indices summarize the same body of basic data, they need not move identically. (3) In the practical situation in which intervals must be used, only two of the three variables (t, T, a) can be defined precisely, and it is most common for the cohort variable (T) to be residually identifiable only as the approximate difference between the other two. (4) The ordinary table will have measurements at each age for each period. Accordingly the older cohorts will be truncated in their younger ages, and the younger cohorts in their older ages.

ALGEBRAIC PRELIMINARIES

Represent the measurement for cohort T in age x by $b_x(T)$. Assume that the time series of measurements in any particular age may be represented by the nth-order polynomial in T. Thus $b_x(T) = c_0 + c_1T + c_2T^2 + \ldots + c_nT^n$. Each age x would have its own polynomial, i.e., the coefficients $c_0, c_1, \ldots, c_n$ are functions of x. The author has chosen to work with polynomials because he is interested in summarizing the age distribution of measurements with indices of the moment family (like the mean and the variance) and these flow from polynomial fitting. It is useful to note that the following statement, which will be needed in the next paragraph, may be proved by induction:

$$b_x(T-x) = b_x(T) - xb_x'(T) + \frac{x^2}{2!}b_x''(T) - \ldots + (-1)^n\frac{x^n}{n!}b_x^{(n)}(T)$$

where the superscript "(n)" signifies the nth-order derivative with respect to T. If $\beta(r, T)$ is defined as $\Sigma x^r b_x(T)$, using a standard definition of moments, then $\beta^{(k)}(r, T) = \Sigma x^r b_x^{(k)}(T)$, where the parenthetical superscript again indicates a time derivative. $\beta(r, T)$ is the rth (absolute) moment of the age distribution of the measurements for cohort T. The Σ sign, where not otherwise specified, indicates summation over the entire age span. The zero moment is the sum of the measurements for a cohort over the range of ages. Unlike moments as the term is ordinarily used in statistics, these are absolute rather than relative to the zero moment. In the discussion that follows the functional letter β (and later μ) will be used to refer to a cohort calculation, and the second term within parentheses will refer to the date of birth.

When the functional letter B (and later M) are used, they will refer to a period calculation, and the second term within parentheses will refer to the date of observation, i.e., the period. It may also be of interest to note that any set of $(n + 1)$ observations may be represented exactly by an nth-order polynomial. Such a polynomial has no more than n nonvanishing derivatives. If there are x ages, there are at most x moments. If $\beta(r, T)$ is an nth-order polynomial in T, then $\beta^{(k)}(r, T)$ is an $(n\text{-}k)$th-order polynomial in T.

THE FIRST TYPE OF GENERAL FORMULA

In terms of its cohort components, age by age, the rth moment of the period distribution, $B(r, T)$, for the period corresponding to the time when cohort T is age 0, can be expressed as $\Sigma x^r b_x(T - x)$. Using the expansion of $b_x(T - x)$ given in the preceding paragraph, we have:

$$B(r, T) = \Sigma x^r b_x(T) - \Sigma x^{r+1} b_x'(T) + \ldots + (-1)^n \Sigma \frac{x^n}{n!} b_x^{(n)}(T).$$

---

haps employing hexagonal cells. The author sometimes thought that the predisposition to ignore cohort indices may derive in part from the convention of presenting a period-by-age configuration in a rectangle, thus veiling the cohort vector.

Following the expression for the kth derivative of the rth moment of β, above, we have

$$B(r, T) = \beta(r, T) - \beta'(r+1, T) + \dots + (-1)^n \frac{1}{n!} \beta^{(n)}(r+n, T)$$
$$= \sum_{i=0}^{n} \frac{(-1)^i}{i!} \beta^{(i)}(r+i, T).$$

In words, the rth moment of the period function has been expressed as the sum of the rth moment of the cohort function and successively higher derivatives of successively higher moments of the cohort function, with alternating signs. The development proceeds in the same way, when the interest is in expressing the rth moment of the cohort function in terms of moments of the period function. The result is:

$$\beta(r, T) = \sum_{i=0}^{n} \frac{1}{i!} B^{(i)}(r+i, T).$$

DISCUSSION OF THE FORMULAE

These formulae provide one way of expressing the relationship between a time series of cohort indices and a time series of period indices. Intuitive meaning will be provided in the discussion of a simple example, below. The treatment is admittedly ponderous, and becomes more so with the second type of general formula to be discussed later. The mathematical development might be cleaner if infinitesimal rather than finite calculus were used, and if the approach to period and cohort functions were made through solid geometry, relating sections of a surface at one angle to sections of the same surface at another angle. Although the general solution seems satisfactory from a formal standpoint, it rapidly approaches the realm of impracticality in application, because of the well-known instability of higher moments and higher derivatives, especially when the basic data betray such patterned irregularities as those produced by age misstatement and misenumeration.

AN EXAMPLE OF THE FIRST (MORE SIMPLE) TYPE OF FORMULA

Suppose the polynomial fitted to the measurements in each age is a straight line. Then all derivatives beyond the first vanish, and the formula for the zero moment (for the sum of the observations in all ages) becomes $B(0, T) = \beta(0, T) - \beta'(1, T)$. The period to which this formula applies is that corresponding to the time cohort T is born, i.e., year T. Suppose we now consider the value of the zero moment for the period when cohort T reaches its mean age, say μ_1, of some type of activity.

$$B(0, T+\mu_1) = \beta(0, T+\mu_1) - \beta'(1, T+\mu_1)$$
$$= \beta(0, T) + \beta'(0, T) \times \frac{\beta(1, T)}{\beta(0, T)} - \beta'(1, T)$$
$$= [\beta(0, T)] \cdot [1 - \mu_1'(T)]$$

since

$$\mu_1(T) = \frac{\beta(1, T)}{\beta(0, T)}.$$

Under the assumed conditions we have the result that the period sum, for the period representing the time the cohort reaches its mean age of some type of activity, exceeds the cohort sum by a factor which is the complement of the annual change in the cohort's mean age. Let us term this factor "distributional distortion." When the measurements concerned are age-specific birth rates, this formula is an expression of the tendency for the period total fertility rate to exceed the cohort total fertility rate whenever and to the extent that the mean age of childbearing is declining from cohort to cohort. This is one component of explanation of the baby boom.

DATING AND SIMPLIFICATION

Formulae like the one in the preceding paragraph are advantageous not only for reducing computations in application but also for explication. The essence of the relationship is expressed in familiar terms, at the cost of some inaccuracy. It would appear that any curve which would represent empirical reality adequately would have to be a rather high-order polynomial, implying high-order moments and derivatives. But such calculations are notoriously susceptible to errors in the original measurements by age. These problems have been avoided in my empirical investigations by using linear or sometimes quadratic fits, but bringing them into close correspondence with reality by fitting them to a moving series, using each new equation to provide a value only for the central cohort or period of the series. Perhaps the best argument for this practice is that it works [2, 4].

Another procedure which achieves both simplification and approximation to reality is dating. As in the preceding paragraph, the formulae adopt a more familiar look when the period toward which translation is occurring is located at the mean of the cohort distribution which provides the basis for translation. Such a choice maximizes the tendency for period and cohort parameters to be alike. Indeed in the example of the preceding paragraph, period and cohort totals are always equal when this dating practice is followed, provided there is no distributional variation through time (provided $\mu_1' = 0$).

THE SECOND TYPE OF GENERAL FORMULA

The first type of general formula proved clumsy for other than linear fitting and for attempts to translate the mean. Because the basic statistical measures are relative moments, the formulae were developed again, this time using a separate polynomial fit for the total, and proportion of that total in each separate age. As before

$$B(0,T) = \Sigma b_x(T-x) = \Sigma b_x(T) - \Sigma x b_x'(T) + \ldots + (-1)^n \Sigma \frac{x^n}{n!} \times b_x^{(n)}(T).$$

If $b_x(T) = \beta(0, T)\cdot p_x(T)$, where $p_x(T)$ is the proportion of the total in age x, then

$$b_x^{(n)}(T) = \sum_{i=0}^{n} \binom{n}{i} \beta^{(i)}(T)\cdot p_x^{(n-i)}(T).$$

$$B(0,T) = [\beta]\cdot\left[\Sigma p - \Sigma x p' + \Sigma \frac{x^2 p''}{2!} - \ldots + (-1)^n \Sigma \frac{x^n p^{(n)}}{n!}\right] - [\beta']\cdot\left[\Sigma x p - \Sigma x^2 p' + \Sigma \frac{x^3 b''}{2!} - \ldots + (-1)^{n-1} \Sigma \frac{x^n p^{(n-1)}}{(n-1)!}\right] + \left[\frac{\beta''}{2!}\right]\cdot\left[\Sigma x^2 p - \Sigma x^3 p' + \Sigma \frac{x^4 p''}{2!} - \ldots + (-1)^{n-2} \Sigma \frac{x^n p^{(n-2)}}{(n-2)!}\right]$$

and so forth, where all functions are for cohort T, and β is the zero absolute moment. Symbolize $\Sigma x^r p_x(T)$ by $\mu_r(T)$, the conventional rth moment about the origin. Then

$$B(0,T) = [\beta]\cdot\left[1 - \mu_1' + \frac{\mu_2''}{2!} - \ldots + (-1)^n \frac{\mu_n^{(n)}}{n!}\right] - [\beta']\cdot\left[\mu_1 - \mu_2' + \frac{\mu_3''}{2!} - \ldots + (-1)^{n-1} \frac{\mu_n^{(n-1)}}{(n-1)!}\right] + \left[\frac{\beta''}{2!}\right]\cdot\left[\mu_2 - \mu_3' + \frac{\mu_4''}{2!} - \ldots + (-1)^{n-2} \frac{\mu_n(n-2)}{(n-2)!}\right]$$

and so forth. Accordingly,

$$B(0,T) = \sum_{i=0}^{n} \left\{ \frac{\beta^{(i)}(0,T)}{i!} \times \left[\sum_{j=0}^{n-i} \frac{(-2)^{i+j}}{i!} \mu_{i+j}{}^{(j)}(T) \right] \right\}$$

and $B(r, T)$ is the same formula with the order of the moment increased to $(i + j + r)$.

EXEMPLIFICATION OF THE SECOND TYPE OF GENERAL FORMULA

Consider first the period total as a function of the cohort total, under the assumptions that $\beta(0, T)$ is linear, and all proportions are fixed. For the year when cohort T is age 0, we have $B(0, T) = \beta(0, T) - \mu(T)\cdot\beta'(0, T)$. Using the same dating trick as before, the total for the year when cohort T is at its mean age μ is $B(0, T+\mu) = \beta(0, T+\mu) - \mu(T+\mu)\cdot\beta'(0, T+\mu) = \beta(0, T) + \mu(T)\cdot\beta'(0, T) = \beta(0, T)$. This should not be a surprise. It is the same result as obtained in paragraph 7, because the assumptions here are a special case of the assumption of linearity there. If, however, we assume that the proportions $p_x(T)$ as well as $\beta(0, T)$ are linear, this becomes a special case of the quadratic in $b_x(T)$. The assumption that all $p_x(T)$ are linear means that $\mu_1(T)$ and $\mu_2(T)$ are also linear, and that their second and higher order derivatives vanish. The formula becomes

$$B(0, T) = [\beta(0, T)]\cdot[1 - \mu_1(T)] + [\beta'(0, T)]\cdot[\mu_2'(T) - \mu_1(T)].$$

Substituting $(T+\mu_1)$ for T in the above equation, and simplifying, we obtain $B(0, T+\mu_1) = \beta(1 - \mu_1' + \delta\cdot\gamma)$ where all functions are for cohort T, γ stands for cohort variance, and δ is $[\beta'(0, T)]/[\beta(0, T)]$, i.e., the proportional change in the cohort total. The difference between this and the result for the linear assumption with the general formula of the first type is $\delta\cdot\gamma'$. This term, which is of the second order of smalls, is helpful for indicating the consequence of departure from linearity.

DERIVATION OF TRANSLATION FORMULAE FOR MEAN AND VARIANCE

With the formula obtained in paragraph 9, we can translate values for the first and second moments of the period distribution, and thus obtain for the period distribution the mean (M_1) and the variance $(V = M_2 - M_1^2)$. Intuition and experimentation suggest that the principal source of discrepancies between period and cohort mean, and between period and cohort variance, would be temporal variations in the cohort total. Therefore the author considered the case of linear change in the zero moment, and fixed proportions, again dating at $(T+\mu_1)$. $B(1, T) = \beta(0, T)\cdot\mu_1(T) - \beta'(0, T)\cdot\mu_2(T)$. Substituting $(T+\mu_1)$ for T, and simplifying, we obtain $B(1, T+\mu_1) = (\beta + \mu_1\beta')\cdot(\mu_1) - (\beta')\cdot(\mu_2) = \beta\mu_1 - (\beta + \mu_1(\beta')\cdot(\mu_1) - (\beta')\cdot(\mu_2) = \beta\mu_1 - \beta'(\mu_2 - \mu_1^2) = \beta\mu_1 - \beta'\gamma$. Using the same assumptions in the preceding paragraph we obtained $B(0, T+\mu_1) = \beta(0, T)$. Accordingly,

$$M_1(T+\mu_1) = \mu_1\left(1 - \delta\,\frac{\gamma}{\mu_1}\right).$$

Thus the distortion of the period mean from the cohort mean depends on the realtive annual change in the sum, weighted by the coefficient of variation. By similar procedures we can determine that $M_2(T+\mu_1) = \mu_2 - \delta(\mu_3 - \mu_1\mu_2)$. Then the variance of the period dated at $T+\mu_1$ is $V(T+\mu_1) = M_2(T+\mu_1) - M_1^2(T+\mu_1) = (\gamma)\cdot(1 - \delta a_3\sigma - \delta^2\gamma)$ where σ is the standard deviation (the square root of the variance) and a_3 is a common measure of skewness (the third moment about the mean divided by the cube of the standard deviation). The principal reason for providing this last formula is to correct a published error (1960, p. 121, n. 6). In general, these formulae document the assertion the writer has made at various times that the period mean is a distorted verison of the cohort mean, because of temporal variations in the cohort sum, just as the period sum is a distorted version of the cohort sum, because of temporal variations in the cohort mean. Empirically these formulae are not as satisfactory as the ones provided for translation of the zero moment.

USES OF THE FORMULAE IN ANALYSIS

An obvious and important question at this point is why one should go to the trouble of determining the relationship between time series of period and cohort parameters when the only situation in which the parameters can be computed on a cohort basis is one in which the period parameters are themselves available. In attempting to answer this question, the writer's first assertion is that translation formulae can be used to help determine the level of current fertility. It is only true in a special sense that data are equally available from both cohort and period viewpoints. No cohort parameter can be computed accurately until that cohort has completed the activity being studied, and that might be long past the time of maximum activity. Since a complete new period function by age is provided each year as the records are processed, there is an understandable tendency to use period parameters for analysis, even if it is well understood (and it doesn't seem to be yet) that they are distorted reflections of cohort behavior. If cohort parameters were just as convenient, they would probably be preferred. Now the translation formulae provide an indication of when it is plausible to use a period time series as if it referred to cohorts, when such an inference is unwarranted, and what the magnitude of distortion is.

It is also possible to use the translation formulae to make estimates of cohort parameters from period parameters, and thus provide a basis for completing the truncated experience of younger cohorts. In two papers [3, 4] the writer showed how to use this procedure to escape from the analytic dilemma of recent but distorted period indices, on the one hand, and old or incomplete cohort indices, on the other. The procedure does not give something for nothing. The problem is that it is difficult to complete a cohort function in the higher ages, while taking into account possible distributional changes which are themselves difficult to see when one is looking at differentially incomplete experience. Application of the translation formulae to period parameters is a way of using all the information available in a manner which does not waste data. Furthermore, the indices used in the formulae are less erratic from one period to the next than are the distributional components from one cohort to the next.

USES OF THE FORMULAE IN PROJECTION

The customary mode of projection uses cohort or age components. As noted, assumptions about temporal changes in cohort distributions are difficult to formulate on a piece-by-piece basis. Projections of age components tend to be blind to the implied distributional modifications for constituent cohorts. There is little logic to the relationship between the performance of one cohort in one period and the performance of the next cohort in the next period. On the contrary, translation projections can use assumptions about the time path of the cohort mean and variance which are designed to reflect what we may know about changing patterns of childbearing. The proposal is that we determine, by methods suggested in the preceding paragraph, the present trends in cohort distributions, project these into the future on the basis of assumptions about movements of the cohort total, mean and variance, and then employ the translation formulae to derive directly from these the movements of the period parameters, as required for policy purposes. This will not necessarily produce more accurate projections, but the projections will be more self-conscious. That is to say, the assumptions, and the reasons for them, will be stated in terms of cohort behavior, rather than in terms of arbitrary operational procedures.

THE FORMULAE AS RELATIONSHIPS

Any decent formula is not only an assistance in statistical analysis or practical projection, but also an expression of relationships. It is a way of distinguishing between the more important and the less important facets of complex reality, in the same way as the mean and the variance are abstractions of essential features from diverse frequency distributions. Regardless of the analytic priority of cohorts or periods in time series analysis, the translation formulae give both more meaning by indicating the implications for each of certain kinds of change in the other. The main task of the formal demographer is essentially the transformation of measurements from one shape into another to accommodate diverse analytic or policy purposes. In the process, there often occurs the important by-product of the revelation of new and interesting topics for substantive inquiry. In the present case, the translation formulae demonstrate clearly the relevance of distributional change as a focus for investigation. The age patterns of cohort fertility, *inter alia,* are now recognized as critical features of short run and long run demographic transition, and this recognition is at least partly attributable to the links between period totals and the changing time distribution from cohort to cohort.

THE TRANSLATION MODEL (*i*)

Perhaps the most important employment of the formulae will prove to be in models of demographic change [5]. Until now there have been few models designed to encompass more than the overt manifestations of population variations through time. One reason for this is that fertility and mortality functions by age are resistant to simple mathematical formulation, and arithmetical assumptions are both unwieldy and particularistic. The translation model of demographic change relies on the distinction between the behavior of a series of cohorts through time, and the manifestations of that behavior in successive periods, as two modes of representation of the same experience. These modes are distinguished by the circumstance that the former considers data in the shape most suited to the analysis of determinants, while the latter considers data in the shape most suited to the analysis of consequences. Model-building is the reverse of the typical demographic posture. Instead of taking a set of data and milking them for information about the determinant vital processes, which then become the topic of "population studies," the model-builder starts with those processes, and shows what consequences flow from them.

In this work, the translation model seems to offer greater economy of statement than is possible with schemes which use age components of fertility, mortality, and population, for each time period. It is simpler, it obviates the implicit detail of assumption of the component technique, and its form seems more elegant. It is in the spirit of Lotka: showing the ultimate dependency of births, deaths and population by age, on the underlying processes of fertility and mortality (which, in his models, were cohort processes).

THE TRANSLATION MODEL (*ii*)

The model begins with a radix of births to establish the initial size of the inital cohort. The relative sizes at age zero of successive cohorts are a net reproductivity relationship, involving three components: (1) the proportion of the cohort surviving to the mean age of childbearing, P; (2) the cohort gross reproduction rate, R; (3) a translation of this net reproduction rate, $R_0 = RP$, which represents births occurring over a span of years, into the appropriate number of births in a particular year. Given assumptions about the time series of P, R, and A (the mean age of net reproductivity), the time series of births year by year may be determined.[2]

Once the initial size of each successive cohort is determined, it is simple to calculate what may be called the population size for each cohort—the sum of its person-years of life—by assuming a time

[2] It is clear from the mathematical argument presented above that the cohort net reproduction rate is not appropriate. It may not be so clear that the conventional period net reproduction rate is likewise inappropriate. The proper survival proportions to use age by age are those for the respective cohorts, and not those for the synthetic cross-section derived from the mortality rates of a period. In short, the cohort survival proportions are translated, and these are only remotely related to the period survival proportions.

path for the cohort expectation of life at birth, E. The cohort size is the product of its initial size, B, and E. Now the age structure in a period is a translation of the cohort age structure. Viewed as an age structure, the population at any point of time is a cross-section of cohorts viewed as age distributions of their person-years of exposure. Thus the number of persons (or, more properly, person-years) in a period is a translated number of cohort person-years. This translation depends on the changing age-distribution of cohort person-years, which is in turn a simple function of the changing age-distribution of cohort mortality.

THE TRANSLATION MODEL (*iii*)

Finally, the number of deaths to a cohort is equal to its initial size, B. The number of deaths in a period is a translation of this, with the key role in the translation formula being played by the change in the mean age at death from cohort to cohort. Thus, in summary, the model employs translation at three points: (1) to develop the births from year to year; (2) to develop the person-years (population) series from year to year; (3) to develop the deaths from year to year; in each case relying on time derivatives of the mean of the appropriate cohort age distribution. The three series are formally interdependent, as would be expected from the link between population change and numbers of births and deaths. The decision to base the model on cohort processes is justified by argument for the analytic priority of cohort measures [6]. It is also justified by the simplicity of relationships between cohort numbers and cohort processes. The ability to achieve the chain of consequences in the form of period births, period deaths, and period population sizes, is achieved, in the absence of age distributions of fertility and mortality, by the translation formulae. Translation is a necessary accompaniment of the employment of summary measures of fertility distributions (like R and A) and mortality distributions (like P and E). These in turn permit flexibility of model development and an increased generality of assumptions, because they are simple.

It should also be noted that parameters of the period age distribution, like its mean and variance, can be translated from movements of the like cohort parameters, and that the proportions in particular age spans, from period to period, can be obtained by translating subpopulations bounded by particular cohort (age) limits. In conclusion of this account, the writer proposes that the translation procedure provides a feasible substitute for component models, which can give important results within a flexible framework for experimentation.

TRANSLATION FORMULAE FOR MORTALITY MEASURES

Development of translation procedures has proven more difficult for mortality functions than for fertility functions, because indices of the former are multiplicative whereas indices of the latter are additive. Cohort and period mortality indices are constructed from a common surface which may be variously represented in terms of life table functions like ${}_nm_x$ or ${}_np_x$. The most used index of mortality is the expectation of life at birth. This is a sum, but of ${}_nL_x$, or, in continuous form, the integral of l_x over all ages, where the function l_x is developed by successive multiplication by p_x. Thus the translation formulae which have been developed above are inapplicable. It is possible to make some headway by converting the successive multiplication process into a sum of logarithms, thus deriving an expression which yields $\log l_x$. Following this line, the writer determined that (period l_x) = (cohort $l_x)^{1-a}$, under the assumption that change in colog ${}_1m_x$ for all ages was linear, and the period was dated at the mean age of the function colog ${}_1m_x$. (To do this the catalytic approximation for l_x in terms of m_x was used.) The writer intends to do some experimenting with formulae like this, because it seems to him that the mortality process is a more typically demographic function than the fertility process. The same type of multiplicative procedure is encountered in nuptiality, when it is formulated as attrition of, say, the single population, as first marriages occur, and in parity-specific fertility analysis [4].

THE SCOPE OF APPLICABILITY OF THE TRANSLATION FORMULAE

It may appear to the reader that the above paper has discussed the translation formulae as if they were intended only for fertility measures. Clearly this is not the case. The attempts to make cohort-type inferences from cross-sectional data which are specific for some time interval extend through a wide variety of studies of human and other behavior. In a paper at the Population Association meetings last year the writer attempted to indicate the breadth of the demographer's ken. [7] It appears to him that problems of translation are pertinent in any context in which the concept of a population, in its most general sense, can be applied. One of the principal consolations for a person working in the imaginary realm of formal methodology is the circumstance that any small headway that can be made has the potential for application far outside the substantive area which may have suggested the inquiry. This appears to be the case with the formulae for demographic translation.

REFERENCES

1. Ryder, N. B. "The Cohort Approach." Ph.D. dissertation, Princeton University, 1951.
2. ———. "Problems of Trend Determination during a Transition in Fertility," *Milbank Memorial Fund Quarterly*, XXXIV (1) (January, 1956), 5–21.
3. ———. "An Appraisal of Fertility Trends in the United States," *Thirty Years of Research in Human Fertility: Retrospect and Prospect*, Milbank Memorial Fund, 1959.
4. ———. "The Structure and Tempo of Current Fertility," in *Demographic and Economic Change in Developed Countries*, pp. 117–136. Report of the National Bureau of Economic Research.—(Princeton, N.J.: Princeton University Press, 1960.)
5. ———. "The Translation Model of Demographic Change," *Emerging Techniques in Population Research*, Milbank Memorial Fund, 1963.
6. ———. "Cohort Analysis," *International Encyclopaedia of the Social Sciences*. Forthcoming.
7. ———. "Notes on the Concept of a Population," *American Journal of Sociology*, LXIX (5) (March, 1964), 447–463.