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Modified logit life table system: principles, empirical validation, and application

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Despite its widespread use, the Coale–Demeny model life table system does not capture the extensive variation in age-specific mortality patterns observed in contemporary populations, particularly those of the countries of Eastern Europe and populations affected by HIV/AIDS. Although relational mortality models, such as the Brass logit system, can identify these variations, these models show systematic bias in their predictive ability as mortality levels depart from the standard. We propose a modification of the two-parameter Brass relational model. The modified model incorporates two additional age-specific correction factors (γ_x and θ_x) based on mortality levels among children and adults, relative to the standard. Tests of predictive validity show deviations in age-specific mortality rates predicted by the proposed system to be 30–50 per cent lower than those predicted by the Coale–Demeny system and 15–40 per cent lower than those predicted using the original Brass system. The modified logit system is a two-parameter system, parameterized using values of l_5 and l_{60}.

Keywords: adult mortality; age-specific death rates; Brass techniques; indirect estimation techniques; model life table systems; relational logit models

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1. Introduction

Model life table systems (Coale and Demeny 1966; Brass 1971; UN 1981) are used extensively in demographic, epidemiological, and economic analyses. Probably the most common reason for using them is to infer age patterns of adult mortality, about which comparatively little is known in less developed countries, from levels of child mortality, which are much more reliably documented (Ahmad et al. 2000). Yet, substantial evidence has accumulated that these model life table systems do not adequately represent the range of age-specific patterns that are empirically observed. One manifestation of this inadequacy of the original models for current estimation purposes is the routine use of split-level modifications of the Coale–Demeny and the United Nations (UN) model life table systems. Fortunately, over the past 30 years there has been a major increase in the availability of empirically observed data on age-specific mortality in countries with complete or very nearly complete registration systems (Lopez et al. 2002). These data provide an opportunity to improve the widely used model life table systems through a reappraisal of age patterns of mortality that have been observed in these countries.

In this paper, we report the development and testing of a new model life table system based on a modification of the Brass logit life table system (Brass 1971). Section 2 briefly reviews some of the main uses of model life tables and the requirements of a good model life table system. Section 3 reviews the main two-parameter model life table systems, emphasizing the Coale–Demeny, UN, and Brass systems. In Section 4, we present the logic and mathematical foundation for a modification of the Brass logit life table system. This is followed, in Section 5, by a review of the data-set of high-quality life tables which provides the empirical basis for the development of the modified system. Details of the empirical estimation of this system, as well as basic information on the robustness of the model, are given in Section 6. Section 7 presents a direct empirical assessment of the adequacy and predictive power of the Coale–Demeny system, Brass logit

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system, and the modified logit system together with a discussion of the limitations and implications of this work.

2. Uses and required properties of model life table systems

Understanding the strengths and weaknesses of model life table systems and thus the requirements of an improved system should start with a clear articulation of their many uses. Model life tables are used extensively for the following purposes: smoothing data; incorporating age-specific mortality patterns in various indirect estimation techniques such as those based on sibling or parental survival; and for forecasting age-specific mortality rates (Coale and Demeny 1966; UN 1981, 1983). One of the most important applications of model life tables is for routine demographic estimation in settings where complete vital registration is not in operation. Often a complete life table is estimated with information on child mortality only or on child mortality and some measure of adult mortality experience derived from censuses or surveys. Another important application is in the economic appraisal of health interventions when the benefits of an intervention must be modelled in the context of general levels of mortality.

Model life tables are not models in the usual sense of the word. They do not embody causal theories or statistical models. Rather, a model life table can be thought of as a representation theorem. The central thesis is that the complex phenomenon of age-specific mortality rates can be adequately represented by two or three parameters, such as the family to which the model belongs and the mortality level. Being able to represent a full schedule of mortality by age with two or three pieces of information simplifies the understanding of mortality patterns and has proved to have numerous analytical uses in many fields. Thinking of model life tables in these terms can help when formulating appropriate empirical tests of the adequacy of a system.

We propose that a model life table system requires at least three properties. The first of these is that it be simple and easily used. In practice this means that, at most, two parameters should be needed to define a unique life table. More complicated systems may perform better on the second and third criteria described below but such systems have not been widely used in applied work. We include in the category of two-parameter systems: the Coale–Demeny family of life tables, the UN models, the Brass logit system, and the Ledermann system (Ledermann 1969). The Coale–Demeny and UN systems are de facto two-parameter systems, with the choice of family being one parameter and the level being the second. The Brass logit system has two parameters, α and β, when a single global standard is used. When multiple standards are used, it becomes a three-parameter system.

Second, any two-parameter model life table system should also adequately capture the true range of age-specific mortality patterns seen in real populations. In other words, model life table systems should not under-represent the extent to which mortality by age can vary across populations. For example, if one looks at child mortality measured using ${}_5q_0$ plotted against adult mortality measured using ${}_{45}q_{15}$ in populations with good vital registration data, the extent to which the diversity of the mortality pattern is captured in the model life table system is a measure of its adequacy.

Third, when a model life table system is used to select a life table to represent mortality by age for a population, there needs to be a close a fit between the predicted mortality rates and actual mortality rates. The fit between predicted and actual rates can be assessed by many measures, including the root mean square error (RMSE) in the death rates (or log of death rates), the explained variance, and the average relative error in age-specific death rates. Formal assessment of the predictive power of a model life table system should be an absolute requirement in judgements of its adequacy.

There are other uses of model life table systems and therefore other criteria that can be proposed for their evaluation. In this paper, however, we focus on two-parameter systems and formally assess the range of age-specific mortality patterns they capture and their predictive power.

3. Two-parameter model life table systems

The basic objective in the creation of any model life table is to construct a system that gives mortality rates by sex and age, and is defined by a small number of parameters that capture the level as well as the age pattern of mortality. If a particular model adequately represents reality, the characteristics of a given population can be summarized by the parameters of that model, thereby facilitating the study of variation among populations or within a population over time. The principles underlying each of the existing model life tables are discussed below.

3.1. United Nations

The first set of model life tables was published by the UN in 1955 (UN 1955). This was a relatively simple one-parameter system indexed on infant mortality levels. Subsequently, in 1981, the UN published a revised set of model life tables that included an attempt to construct regional models using data from less developed countries judged adequate for inclusion in the empirical data-set. Five families of models were identified, each with a set of tables with a life expectancy ranging from 35 to 75 years for each sex. Although technically the revision remains a one-parameter system, it can be argued that the choice of a family constitutes a separate dimension.

The revised UN model life tables for less developed countries, while clearly an improvement over the one-parameter system, also have important limitations (Menken 1977). The relatively small number of empirical life tables (72) limits the applicability of the models to other populations.

3.2. Coale and Demeny

Perhaps the most widely used model life table system has been the Coale–Demeny regional model life tables (Coale and Demeny 1966). First published in 1966, they were derived from a set of 326 life tables for both sexes from actual populations. Four typical age patterns of mortality were identified, determined largely by the shape of their mortality schedules (corresponding to the geographical location of the population), but also on the basis of their patterns of deviations from previously estimated regression equations. Those patterns were called: North, South, East, and West. As with the revised UN life tables, we consider the Coale–Demeny system to have two parameters, with the second parameter being choice of family. The system was updated in 1989, primarily to include extensions of the model life tables to age 100+ (Coale and Guo 1989).

Strict standards of accuracy imposed in the construction of the Coale–Demeny model life tables limited the number of non-European countries represented. For this reason, the Coale–Demeny tables may not cover patterns of mortality existing in the contemporary developing world. In fact, there are well-documented examples of mortality patterns that lie outside the range of the Coale–Demeny tables (Demeny and Shorter 1968). The fact that one of the parameters of the Coale–Demeny system (the 'family') is discrete restricts the flexibility of the system, certainly in comparison with other systems in which both parameters are continuous.

3.3. Ledermann

The Ledermann system of model life tables was first published in 1959 and was subsequently revised over the course of the following decade (Ledermann 1969). This system is based on a factor analysis of some 157 empirical tables. The method of selection was less rigid than that of the Coale–Demeny tables, and more experiences of less developed countries are represented.

The primary shortcoming of the Ledermann system is its relative complexity, a feature that essentially precludes its use in most less developed countries. While it does provide some flexibility through a wider variety of entry values, in practice most of these values are not easily estimated for most less developed countries. A second major limitation is that the independent variables used in deriving the model refer, with only one exception, to parameters obtained from data on both sexes combined. The user is therefore forced to accept the relationships between male and female mortality embodied in the model even when there is evidence to the contrary. For instance, it is nearly impossible to estimate a Ledermann model life table in which the male expectation of life exceeds that of females.

3.4. Brass

A different approach to constructing life table systems was first proposed by Brass in 1971 (Brass 1971). The Brass logit life table system belongs to a category of mortality models called relational models. It features a standard life table and two parameters which, through a mathematical transformation, relate any life table to the standard. The general shape of the survivorship functions is captured through the mortality standard while the parameters help to capture deviations from the standard.

The Brass system is based on the assumption that two distinct age patterns of mortality can be related to each other by a linear relationship between the logits of their respective survivorship probabilities. Thus for any two observed survivorship functions, l_x and l_x^s, where the latter is the standard, it is possible to find constants α and β such that

$$\text{Logit}(l_x) = \alpha + \beta\text{Logit}(l_x^s)$$

where

$$\text{Logit}(l_x) = 0.5\ln\left(\frac{1.0 - l_x}{l_x}\right)$$

for all ages x greater than 0. If the above equation holds for every pair of life tables, then any life table can be generated from a single standard life table by changing the pairs of (α, β) values used.

In reality the assumption of linearity is only approximately satisfied by pairs of actual life tables. Deviations from linearity appear to be particularly large when the observed mortality of a population is far from that of the standard. Thus, the complexity of variations in levels and age patterns of mortality is not fully captured by the logit system. This observation led others to modify Brass's original model by including additional parameters that allow for bends in the survivorship function (Zaba 1979; Ewbank et al. 1983). These modifications, however, are of limited practical use, because the additional parameters are difficult to estimate empirically and complicate the application of the models.

It is clear from the foregoing review that there are serious technical difficulties in the use of existing empirical models to describe mortality patterns in contemporary less developed countries. In response to these difficulties, we are proposing a new modified two-parameter system of model life tables anchored in the logit system. The latter was chosen after a careful comparative evaluation of the logit and the Coale–Demeny systems. This evaluation is presented in a subsequent section.

4. Modification of the Brass logit system

We can generalize the principle underlying Brass's approach to postulate that there is some transformation of the survivorship function such that all transformed survivorship functions are linear functions of each other. Formally

$$\Gamma(l_x) = \alpha + \beta\Gamma(l_x^s). \quad (1)$$

If the transformation can be identified, then all survivorship functions can be derived simply from the parameters α and β. Brass's original proposal was that this transformation is a variant of a logit transformation such that:

$$\Gamma(l_x) = 0.5\ln\left(\frac{1 - l_x}{l_x}\right) \text{ for all } x > 0 \text{ (and } l_0 = 1.0). \quad (2)$$

The problem is that the logit transformation does not make completely linear the relationship between many survivorship functions. In developing the modified logit system, we sought to identify a transformation that would improve the linearity of the relationships between most survivorship curves without adding the complexity of additional parameters as in previous extensions of the Brass system (Zaba 1979; Ewbank et al. 1983).

Our modification of Brass's transformation is based upon some simple but powerful empirical observations. The basic observation is that deviations from linearity follow some specific regularities which can be modelled in relation to the amount of mortality change between the standard and the observed life table. These shifts in the structure of mortality can be illustrated by plotting a series of logit life table values against logit values taken from an earlier life table, and examining how the resulting curves depart from linearity. This is shown in Figure 1, which presents data for US males. In this figure, annual logit life table values from 1900 to 1995 are plotted against logit values for 1900, taken as the standard. It is clear that mortality change over time leads to a change in the age pattern of mortality that is not fully captured by the logit relational model. Indeed, if the logit transformation were fully appropriate, the successive plots in the figure would remain linear over time. Our modification of Brass's transformation is based on the observation that differences between observed and predicted logit values follow a pattern that is predictable as the mortality level of the observed life table deviates from that of the standard. That is, deviations from linearity in the original Brass model are linked to the relative difference between the mortality rate of the standard and the mortality of the actual life table being estimated.

Empirical investigation of the differences between observed and predicted age-specific mortality rates using the Brass logit transformation with a global standard revealed that this systematic error at each age was related to both the level of child mortality relative to the standard and the level of middle-aged adult mortality relative to the standard. Based on this finding, a variety of alternative transformations were investigated. Ultimately, based on numerous tests, the transformation that we have selected is

$$\Gamma(l_x) = \text{Logit}(l_x) + \gamma_x\left(1 - \left(\frac{\text{Logit}(l_5)}{\text{Logit}(l_5^s)}\right)\right) + \theta_x\left(1 - \left(\frac{\text{Logit}(l_{60})}{\text{Logit}(l_{60}^s)}\right)\right) \quad (3)$$

Thus, the modified transformation includes three

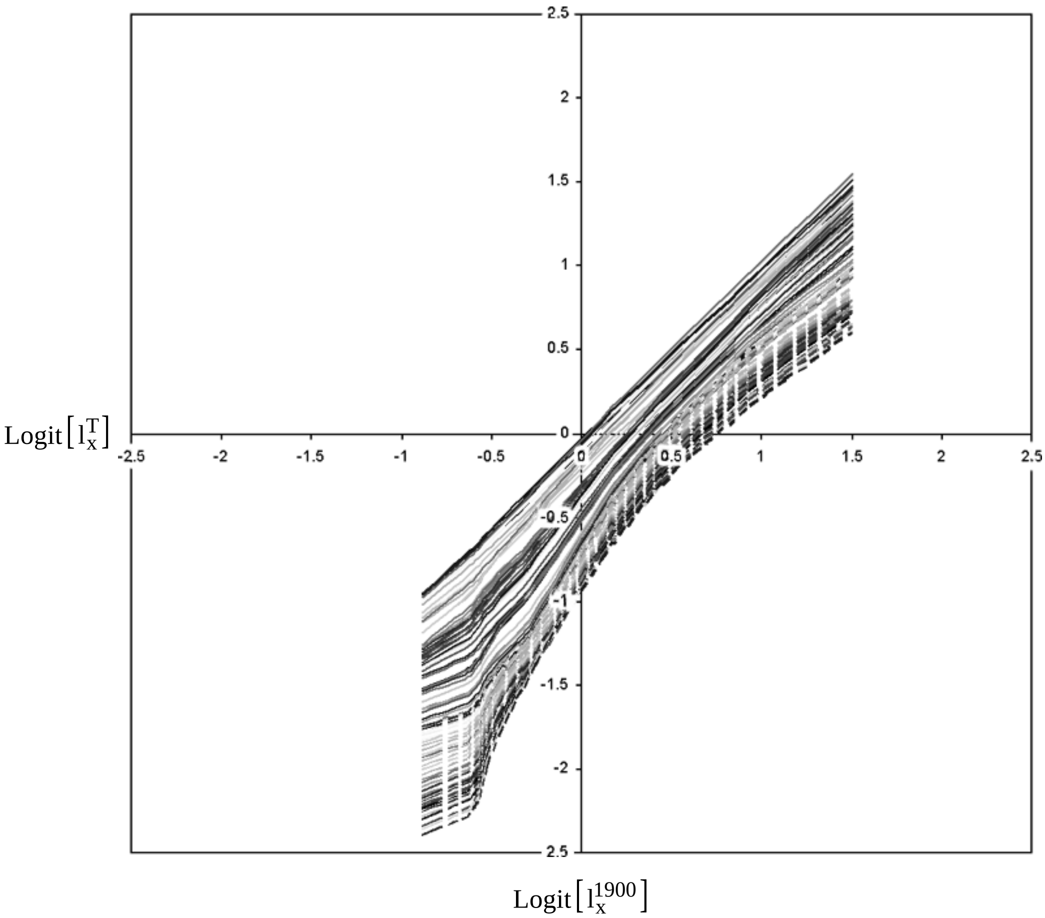

Figure 1 Annual logit l_x values (1900–95) vs. logit l_x values for 1900 taken as standard, US males

standard functions, l_x^s, θ_x, and γ_x, which are age-specific and sex-specific, but invariant across populations. The following sections describe the estimation of these functions.

5. Life table data-set

Since the 1960s, the World Health Organization (WHO) has systematically collected vital registration data on causes of death in countries, making every effort to complete the series back to 1950. For most countries, the most recent data refer to the period 1998–2000 (Lopez et al. 2002). The data for most countries contain the number of deaths by age, sex, and cause, classified according to the Revision of the International Classification of Diseases in use. Data are collected by the conventional 5-year age groups (0, 1–4, 5–9, . . ., 85+), although in recent years the terminal age group has been extended to 100+. For each year, mid-year population estimates by age and sex are also provided by reporting countries. These data have been screened for completeness using standard demographic tests, and only those country-years for which mortality was considered complete have been retained for this analysis.

This data-set was supplemented by life tables from two other sources. The historical life tables compiled by Preston, Keyfitz, and Schoen (Preston et al. 1972) were added for years not covered by the WHO mortality data-set. The mortality data underlying these life tables had been adjusted, where necessary, for under-reporting.

To improve the coverage of less developed countries in the data-set, the adjusted national life tables used by the UN (UN 1981) to produce their model life tables were also added. As more high-quality data for less developed countries become available, the model life table parameters can be re-estimated. One potentially important source of age-specific mortality rates for defined populations, primarily in sub-Saharan Africa, is the INDEPTH network (International Network for the Continuous Demographic Evaluation of Populations and their Health in less developed countries). The first results of this collaboration have recently been published (IDRC 2002). While the levels of adult mortality in many sites are undoubtedly under-reported, the Network offers

Table 1 Life tables used to test and develop the modified logit life table system

Country	Year(s)	Total number
Argentina	1966–70, 1977–79, 1982–97	48
Australia	1911, 1921, 1950–97	100
Austria	1955–99	90
Belarus	1981–98	36
Belgium	1954–98	90
Bangladesh (Matlab Region)	1975	2
Bulgaria	1964–98	70
Canada	1921, 1950–97	98
Chile	1909, 1920, 1930, 1940, 1950, 1955–82, 1984–98	96
Colombia	1960, 1964	4
Costa Rica	1956–83, 1985–98	84
Croatia	1982–98	34
Cuba	1970–98	58
Czech Republic	1934, 1982–99	38
Denmark	1921, 1930, 1952–98	98
El Salvador	1950, 1971	4
Estonia	1981–98	36
Finland	1952–98	94
France	1900–13, 1920–39, 1946–97	172
Georgia	1981–96	30
Germany	1969–98	58
Greece	1928, 1956–98	88
Guatemala	1961, 1964	4
Honduras	1961, 1974	4
Hungary	1955–99	90
India	1971	2
Iran	1974	2
Ireland	1950–98	98
Israel	1975–98	48
Italy	1901, 1910, 1921, 1931, 1951–97	102
Japan	1950–98	98
Korea, Rep. of	1973	2
Latvia	1980–98	38
Lithuania	1981–98	36
Macedonia	1982–97	32
Mauritius	1990–98	18
Mexico	1958–59, 1969–73, 1981–83, 1985–98	48
Netherlands	1950–98	98
New Zealand	1901, 1911, 1950–98	102
Norway	1910, 1920, 1951–98	100
Panama	1960	2
Peru	1970	2
Philippines	1964, 1970	4
Poland	1959–98	80
Portugal	1920, 1930, 1940, 1955–98	94
Republic of Moldova	1981–98	36
Romania	1963, 1969–98	60
Russian Federation	1980–98	38
Singapore	1955–98	88
Slovakia	1982–98	34
Slovenia	1982–98	34
South Africa	1941, 1951, 1960	6
Spain	1930, 1940, 1951–69, 1971–98	98
Sri Lanka	1946, 1953	4
Sweden	1900–17, 1920–98	194
Switzerland	1951–98	96
Thailand	1970	2
Trinidad and Tobago	1990–97	14
Tunisia	1968	2
Ukraine	1981–98	36
United Kingdom	1901, 1911, 1921, 1931, 1950–98	106
United States of America	1900–16, 1920–41, 1945–98	186
Yugoslavia	1982–97	32

considerable promise for rapidly improving knowledge about adult mortality in Africa.

As well as applying criteria of completeness and adequacy of age-specific and sex-specific detail, our screening criteria also excluded life tables of populations for periods of war and for those affected by the Spanish influenza pandemic of 1918–19. Data for years before 1900 were excluded since the age patterns of mortality tended to be atypical. Small populations with a total size of less than 1 million people (both sexes combined) were also excluded to minimize the effects of random fluctuations in death rates.

The resulting set of 1,802 life tables used to develop and test the model are shown in Table 1. There is, of course, a preponderance of countries from Europe, North America, and Australasia, but among the 63 countries represented, about one-third belong to developing regions. For several developed countries, historical data-sets back to the beginning of the twentieth century have been included. Unfortunately, there are very few empirical data from Africa and most of Asia included in the final life table set used to develop the model. The application of the model to these populations will therefore be more uncertain than it will be elsewhere.

Table 2 summarizes the characteristics of the life tables included in the data-set. The mean life expectancies are relatively high (67.5 years for males, 73.4 for females), reflecting the developed-country bias, although the range of life expectancies (27–77 years for males, 29–84 years for females) encompasses the experience of all countries (Lopez et al. 2002). Average levels of child and adult mortality are not too dissimilar to what are observed in many less developed countries today, and again the range of values more than encompasses estimated levels across all less developed countries, with the exception of a few countries in Africa (Namibia, Botswana, Zambia) where female mortality from HIV is extreme.

6. Empirical estimation of global standard, θ_x and γ_x

6.1. Estimating θ_x and γ_x

By rewriting equations (3) and (1), we can express the age-specific parameters θ_x and γ_x and the country-year-specific parameters α_{ij} and β_{ij} (where i represents country and j year) in a way that allows estimation of the parameter values using ordinary least squares (OLS) regression:

$$\text{Logit}(l_x^{ij}) = \alpha_{ij} + \beta_{ij}\cdot\text{Logit}(l_x^s) + \gamma_x\left(1-\left(\frac{\text{Logit}(l_5^{ij})}{\text{Logit}(l_5^s)}\right)\right) + \theta_x\left(1-\left(\frac{\text{Logit}(l_{60}^{ij})}{\text{Logit}(l_{60}^s)}\right)\right) \quad (4)$$

The last two terms of equation (4) are designed to control for the mortality differential between the standard life table and an observed life table. The first of these captures the effect of differences in child mortality (relative to the standard) while the second captures differences in adult mortality up to age 60. The standard life table used is a sex-specific global standard calculated by taking the average of all sex-specific life tables included in the data-set. As the typical deviation from the standard is neither in the same direction nor of the same magnitude across age groups, θ and γ vary by age but are constant across countries and years.

We have estimated the model parameters by repeated sampling of a randomly selected subset of approximately 70 per cent of the country-years in the full life table data-set (1,261 life tables). The remaining 30 per cent of the empirical observations were reserved for validation purposes, as described below. We ran separate regressions by sex in order to estimate *simultaneously* the α_{ij} and β_{ij} for each

Table 2 Characteristics of life tables listed in Table 1

Sex	Parameter	Mean	Standard deviation	Minimum	Maximum
Males:	e_0	67.46	6.16	26.64	77.29
	${}_5q_0$	0.039	0.047	0.005	0.439
	${}_{45}q_{15}$	0.208	0.076	0.087	0.762
	${}_{20}q_{60}$	0.636	0.078	0.422	0.906
Females:	e_0	73.39	6.81	29.20	84.00
	${}_5q_0$	0.033	0.043	0.003	0.427
	${}_{45}q_{15}$	0.121	0.066	0.049	0.656
	${}_{20}q_{60}$	0.478	0.099	0.222	0.833

country-year life table and the set of θ_x and γ_x, for all ages except 5 and 60, using OLS regression. After comparing the results of several alternatives, we found it marginally advantageous to set θ_5, γ_5, θ_{60}, and γ_{60} to zero in the estimation for identification purposes. The resulting θ_x and γ_x, are shown in Table 3, along with the global standard l_x values. As Figure 2 shows, the values by age for both parameters in males and females follow a consistent pattern.

The effect of this transformation on a set of survivorship functions is shown in Figure 3. In the top panel, the deviations (residuals) by age between the logits of the observed l_x and those predicted from the original Brass system using the global standard are plotted for three populations covering a range of mortality experiences. Substantial deviations are evident in the three populations, particularly at ages 0–4 and among older adults. In the bottom panel, the deviations based on this new transformation are shown for the same three populations. Clearly the fit is much better. Because this transformation makes the relationship between survivorship functions more

Table 3 Values of model parameters θ_x, γ_x, and of global standard l_x, by sex

	Males			Females		
Age	γ_x	θ_x	l_x Standard	γ_x	θ_x	l_x Standard
0	0.0000	0.0000	100,000	0.0000	0.0000	100,000
1	0.1607	–0.0097	96,870	0.0855	0.0734	97,455
5	0.0000	0.0000	96,010	0.0000	0.0000	96,651
10	–0.0325	0.0025	95,666	–0.0026	–0.0229	96,370
15	–0.0297	0.0047	95,385	0.0291	–0.0485	96,153
20	0.0427	0.0018	94,782	0.1199	–0.1090	95,795
25	0.1262	–0.0210	93,915	0.1931	–0.1702	95,340
30	0.1877	–0.0518	93,007	0.2352	–0.2117	94,824
35	0.2430	–0.0883	91,949	0.2686	–0.2408	94,197
40	0.2899	–0.1248	90,575	0.3003	–0.2601	93,370
45	0.3148	–0.1482	88,645	0.3203	–0.2594	92,220
50	0.2888	–0.1402	85,834	0.2935	–0.2183	90,569
55	0.1915	–0.0910	81,713	0.1967	–0.1338	88,159
60	0.0000	0.0000	75,792	0.0000	0.0000	84,679
65	–0.2304	0.1170	67,493	–0.2794	0.1859	79,481
70	–0.5523	0.2579	56,546	–0.7066	0.4377	71,763
75	–0.9669	0.4150	42,989	–1.2835	0.7534	60,358
80	–1.5013	0.5936	28,117	–2.0296	1.1360	44,958
85	–2.2126	0.8051	14,364	–2.9576	1.5774	27,123

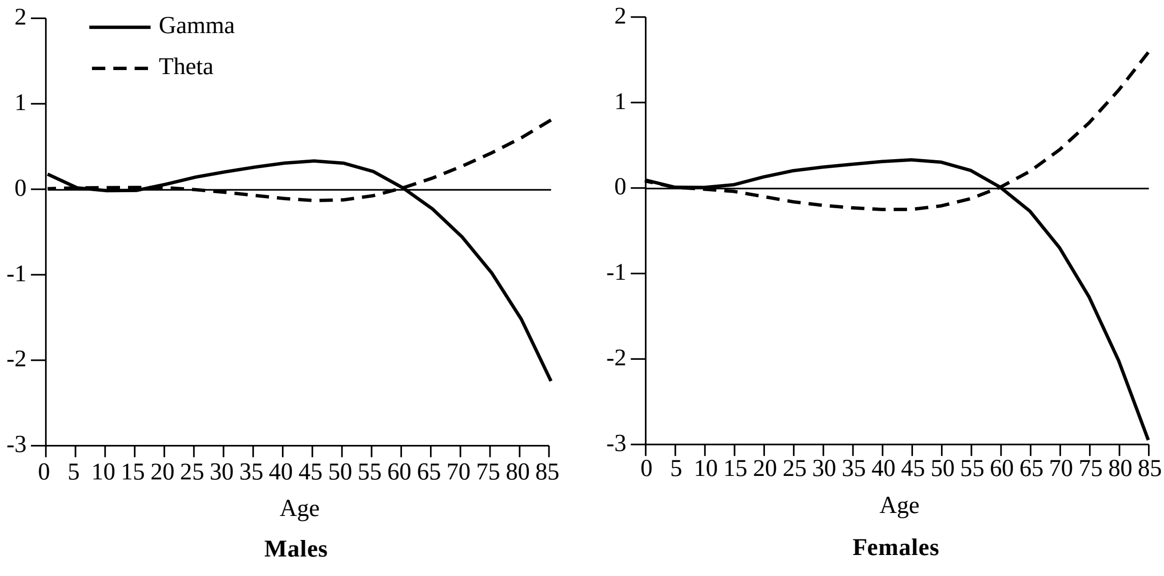

Figure 2 Values of model parameters θ_x, γ_x, by age and sex

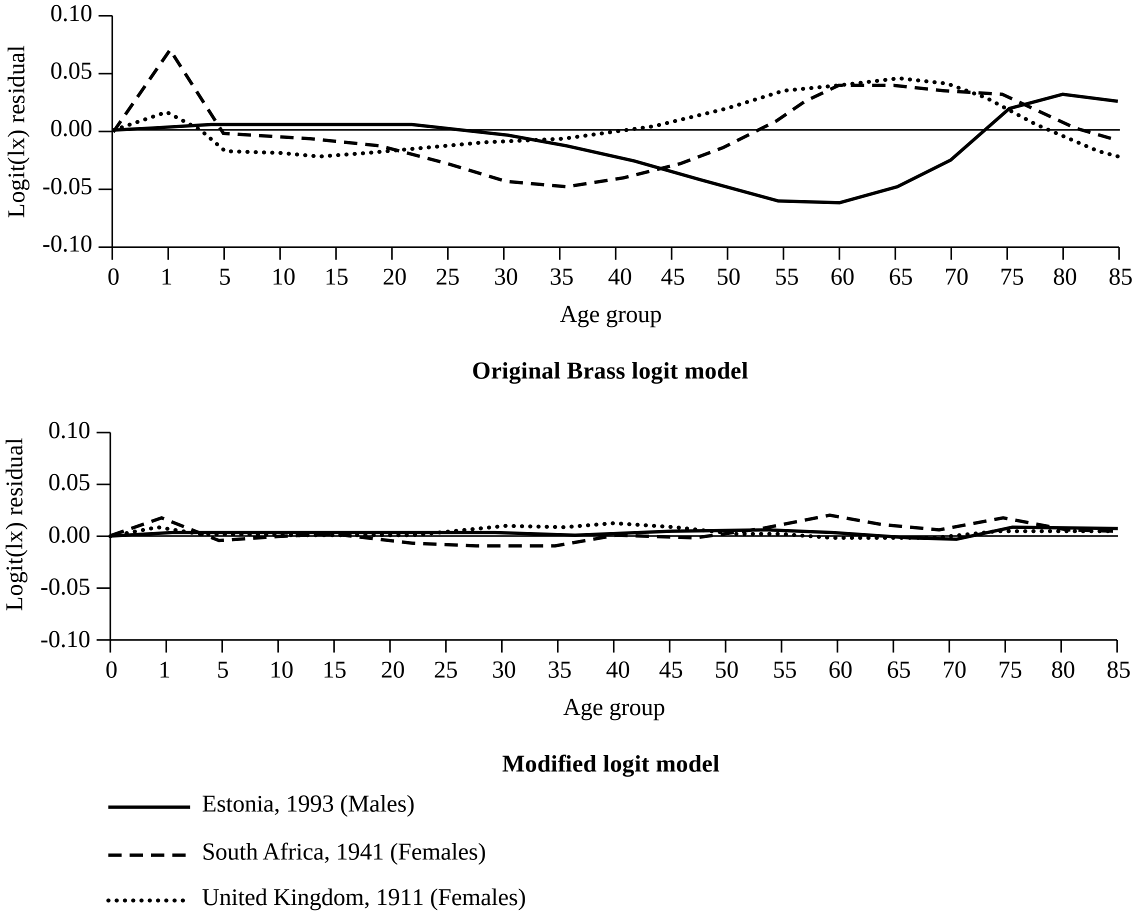

Figure 3 Deviations between observed and predicted logits by age, selected countries

linear with respect to age, a two-parameter fit on the transformed standard performs much better than the original simple logit transformation.

6.2. Developing model life tables

Having estimated θ_x and γ_x, we can proceed to developing model life tables using the modified transformation. It is important to note that γ_x and θ_x do not vary across countries or years. Because of this, each life table can still be uniquely defined with this transformation as a linear function of a standard using only two parameters. It is advantageous to use the life table functions l_5 and l_{60} as parameters to define a unique life table rather than α and β since the former values are more readily interpretable. Any pair of l_5 and l_{60} uniquely defines a life table because there is a one-to-one mapping between a pair of α_{ij} and β_{ij} values and a pair of l_5 and l_{60} values. It can be shown that

$$\alpha_{ij} = \frac{\mathrm{Logit}(l_5^{ij}) \cdot \mathrm{Logit}(l_{60}^{s}) - \mathrm{Logit}(l_5^{s}) \cdot \mathrm{Logit}(l_{60}^{ij})}{\mathrm{Logit}(l_{60}^{s}) - \mathrm{Logit}(l_5^{s})} \quad (5)$$

and

$$\beta_{ij} = \frac{\mathrm{Logit}(l_{60}^{ij}) - \mathrm{Logit}(l_5^{ij})}{\mathrm{Logit}(l_{60}^{s}) - \mathrm{Logit}(l_5^{s})}. \quad (6)$$

By sampling systematically from the range of l_5 and l_{60} values and discarding combinations that were logically impossible (e.g., $l_5 < l_{60}$), we have generated a large set of model life tables. Using this set, it is possible to visualize various life table functions, such as ${}_nq_x$ and e_x, as parameters in the two-dimensional space defined by l_5 and l_{60}. Figure 4a shows life-expectancy-at-birth isoclines corresponding to given values of l_5 and l_{60}. Each point on the isocline corresponds to a constant level of life expectancy generated by different age patterns of mortality. The same life expectancy is possible with low child mortality and high adult mortality or higher child and lower adult mortality. The isoclines demonstrate that the same life expectancy can occur with widely varying age patterns. This is illustrated more clearly in Figure 5, which shows the log of age-specific death rates for four model life tables selected from the isocline of male life expectancy equal to 65 years. The substantial variation in death rates illustrates the

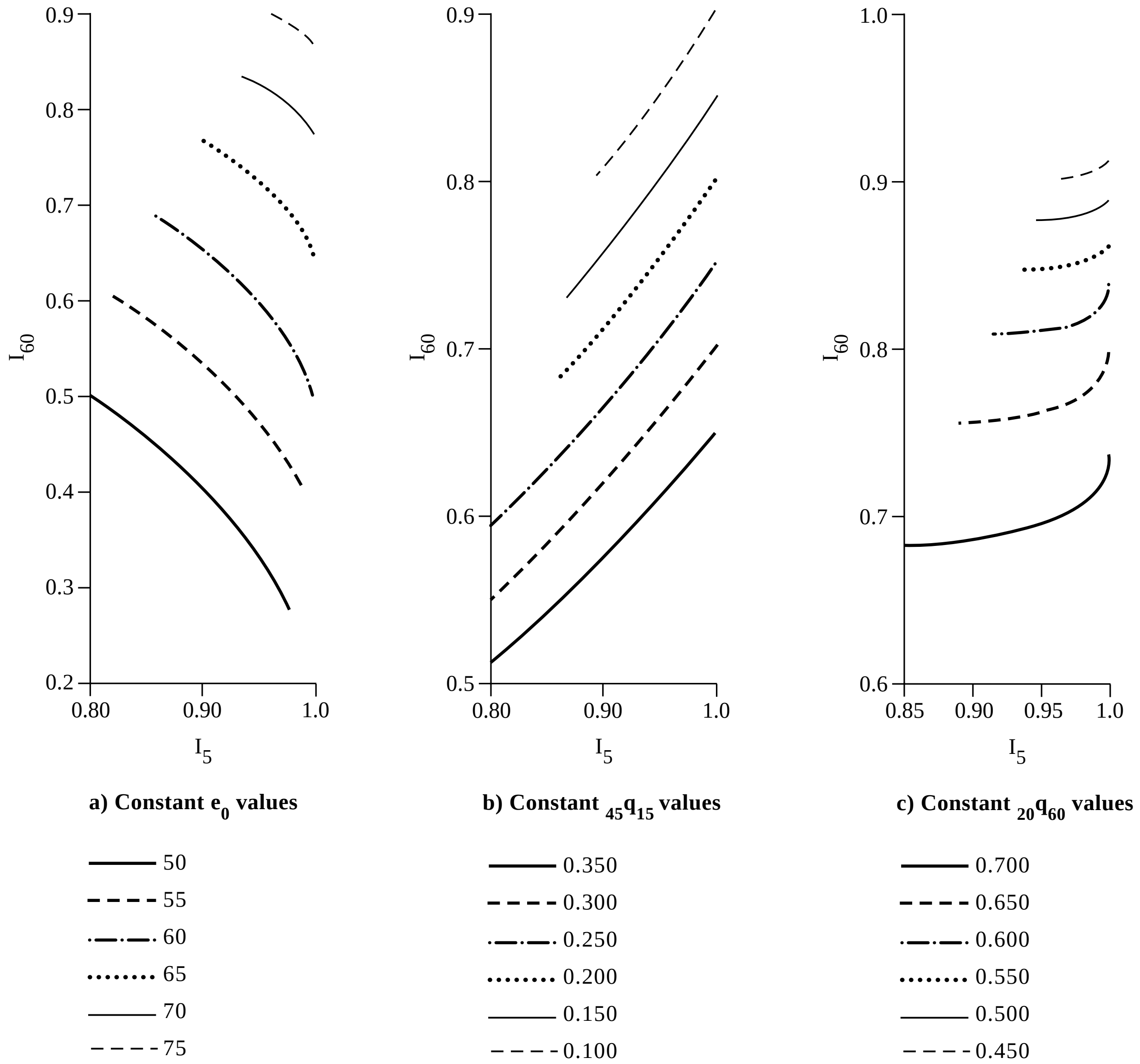

Figure 4 Isoclines of e_0, ${}_{45}q_{15}$, and ${}_{20}q_{60}$, selected values, males

heterogeneity of mortality patterns that should be captured by any life table system. With regard to this issue, analysis will confirm that the Brass system and the modified logit system exhibit a considerably greater degree of flexibility than the Coale–Demeny system.

Figures 4b and c show, respectively, how adult mortality (${}_{45}q_{15}$) and mortality among the elderly (${}_{20}q_{60}$) vary according to the two parameters, l_5 and l_{60}, in the set of model life tables. At a given level of child mortality, the slope of successive isoclines remains relatively constant. Figure 4c, on the other hand, indicates that the impact on older age mortality of declining levels of child mortality is much less apparent. Levels of ${}_{20}q_{60}$ are much more strongly determined by levels of adult mortality.

The fact that the isoclines in Figure 4 are monotonically increasing or decreasing in the l_5 and l_{60} parameter space implies that it is sufficiently correct to say there is one combination of l_5 and l_{60} that will correspond with any two life table mortality functions. This is not an algebraic relationship but an empirical one that follows from the monotonic isoclines. It means that we can find a matching life table in the modified logit system for most combinations of two life table functions. If two life table indices, such as ${}_5q_0$ and e_0, are known, a unique life table is defined in this system at the point where the different contours intersect. For example, referring to Figure 4a, if we know ${}_5q_0$ is 100 per 1,000 and life expectancy at birth is 60 years, the unique life table is defined by an l_5 of 0.900 and an l_{60} of 0.652.

The actual contour lines are a function of the global standard survivorship function as well as of equation (4) so that they cannot easily be defined analytically. To help in the practical use of this system, we have developed a simple computer program,

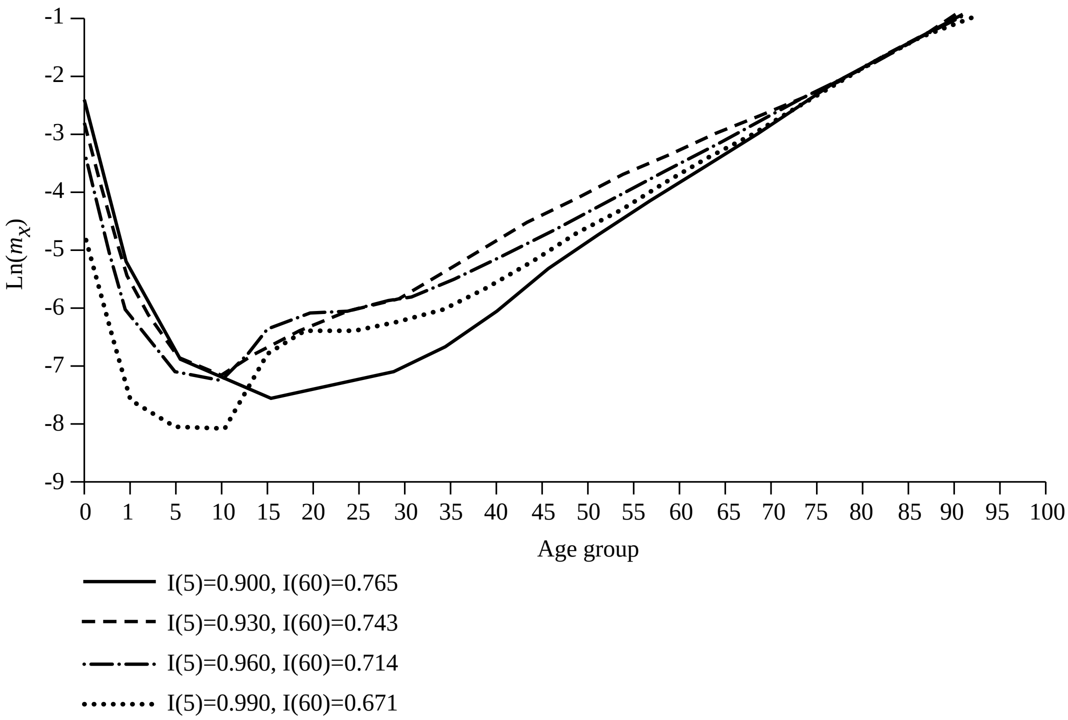

Figure 5 Ln(m_x) for four populations with e_0 of males = 65 years

ModMatch, which identifies a modified logit life table on the basis of any two life table functions as parameters (Ferguson 2002). Given the values of two life table functions, such as e_0 and ${}_5q_0$, the program interactively searches in l_5–l_{60} space to identify a combination of l_5 and l_{60} that yields a life table matching the given input values with a sufficient degree of precision. This program simplifies the matching of model life tables to selected empirical life table functions.

7. Predictive validity of the Coale–Demeny, Brass logit, and modified logit systems

7.1. Predictive validity of the Coale–Demeny vs. the modified logit system

A key use of a model life table system is that of creating a full life table given information on only two life table indices, such as life expectancy and child mortality or, more probably, adult mortality and child mortality. A strong test of this predictive use of a model life table system is to take an empirical life table, select a model life table using two aggregates from the empirical table, and then compare the age-specific death rates from the model life table with the observed rates. We have conducted two such tests: choosing model life tables on the basis of ${}_5q_0$ and e_0, and ${}_5q_0$ and ${}_{45}q_{15}$.

How well do these model life table systems capture the observed range of mortality experience? As noted above, one important criterion for a model life table system is that it adequately represents the known range of mortality experience across countries. Figures 6–8 make three types of comparisons: ${}_5q_0$ and e_0, ${}_5q_0$ and ${}_{45}q_{15}$, and ${}_{45}q_{15}$ and ${}_{20}q_{60}$, respectively. In each figure the observed points from the underlying data-set are shown and compared with the Coale–Demeny model life table values. It is clear that the range of mortality experience captured in the Coale–Demeny system is much smaller than the range observed in the empirical life tables, particularly at medium levels of mortality.

The limited range of mortality patterns captured in the Coale–Demeny model life table systems can be explained, in part, by the relatively recent emergence of the high adult mortality and low child mortality pattern now observed in parts of Eastern Europe and the Newly Independent States. The Coale–Demeny system was developed when there was little evidence of this pattern. Even excluding these countries, however, the range captured in this system is much smaller than the real variation seen worldwide. In contrast, the modified logit system can capture the entire range of mortality patterns illustrated in Figures 6–8, as demonstrated in the contour figures shown earlier. On this criterion, the modified logit system is clearly better able to capture the diverse array of mortality patterns now seen.

Using the 30 per cent of the original data-set of life

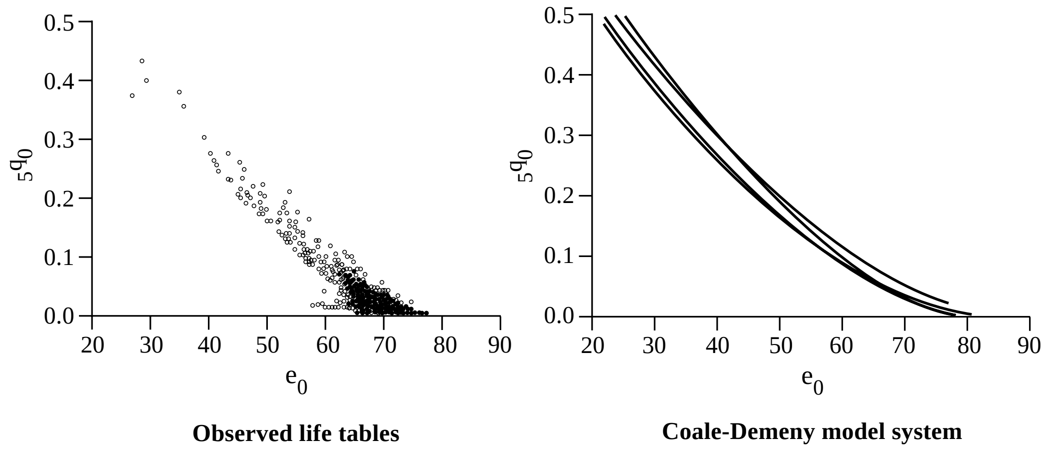

Figure 6 Comparison of observed patterns of $_5q_0$ and e_0 with Coale–Demeny model life table values, males

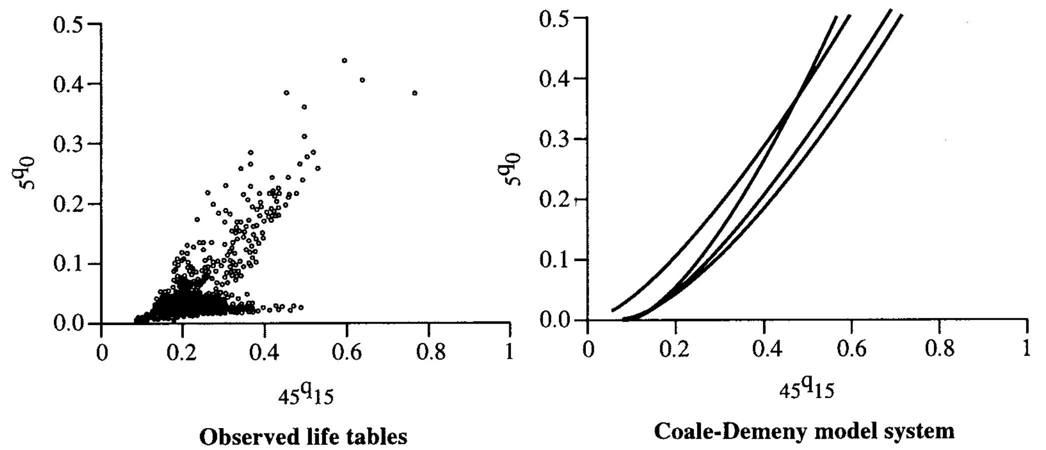

Figure 7 Comparison of observed patterns of $_5q_0$ and $_{45}q_{15}$ with Coale–Demeny model life table values, males

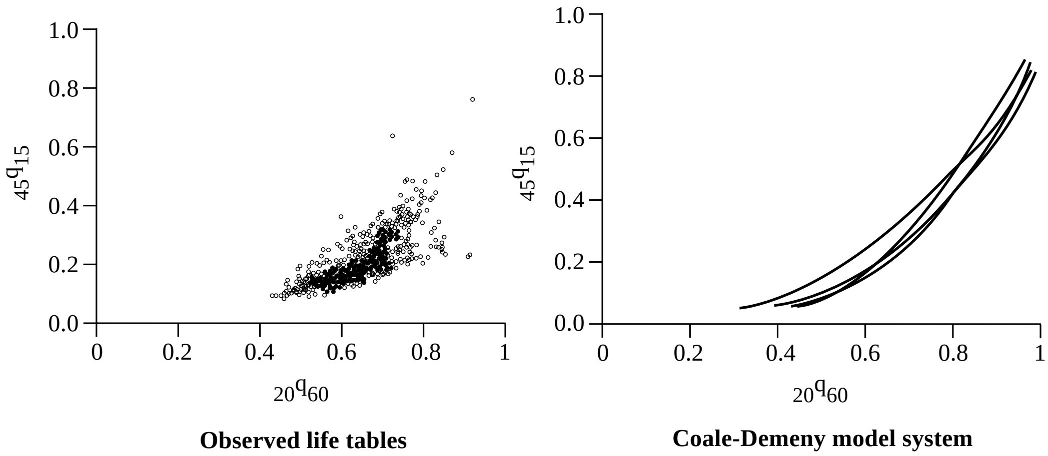

Figure 8 Comparison of observed patterns of $_{45}q_{15}$ and $_{20}q_{60}$ with Coale–Demeny model life table values, males

tables (541 life tables) reserved for the validation test, we have applied the Coale–Demeny and modified logit systems to select a model life table on the basis of $_5q_0$ and e_0. The Coale–Demeny model has been selected by first matching each e_0 on all families and then selecting the family with the closest $_5q_0$. The life table from the modified logit system has been selected using the iterative matching algorithm described earlier. After repeating this procedure for each of the 541 life tables, the fit between predicted and observed mortality rates has been summarized using the RMSE in the logarithm of the death rates, since the logarithm of the death rates allows a more meaningful comparison across age groups.

Table 4 Comparison of root mean square error of $\ln(m_x)$ of the Coale–Demeny, Brass, and modified logit systems, using 30 per cent of the full set of life tables

	Method		
Sex	Coale–Demeny	Brass logit	Modified logit
Males (e_0 and $_5q_0$)	0.3412	0.2594	0.2017
Females (e_0 and $_5q_0$)	0.3629	0.2544	0.2146
Males ($_{45}q_{15}$ and $_5q_0$)	0.4285	0.2741	0.1892
Females ($_{45}q_{15}$ and $_5q_0$)	0.2564	0.2820	0.1726

Table 4 summarizes the goodness-of-fit statistics from the two model life table systems. The top row gives the results for the first type of test described above where life tables were selected on the basis of $_5q_0$ and e_0. As Table 4 clearly demonstrates, the modified logit system gives much better predictions of age-specific death rates than the Coale–Demeny system for this set of 541 empirical life tables, particularly for males. Average RMSEs from the modified logit system are approximately 60–65 per cent of those from the Coale–Demeny system.

The second test that we have used to assess the predictive power of these systems is to select model life tables on the basis of $_5q_0$ and $_{45}q_{15}$, a situation that is more likely to be encountered. This is a more difficult test because the selection of the model life table is based on indices of mortality that cover a smaller age range than life expectancy at birth. For each observed life table in the test subset of 541, the Coale–Demeny model life table has been selected by matching on $_{45}q_{15}$ in all families and then choosing the family with the closest match to $_5q_0$. The matching procedure was repeated by first matching on $_5q_0$ and then choosing the family with the closest match on $_{45}q_{15}$. Using this approach, however, the magnitude of the RMSE was considerably greater than when matching on $_{45}q_{15}$ first. The life table from the modified logit system has been selected by matching on $_5q_0$ and $_{45}q_{15}$. The predicted age-specific death rates have again been assessed using the RMSE in the log death rates. Again, the modified logit system clearly outperforms the Coale–Demeny system, with average RMSEs being about 45 per cent of those from the Coale–Demeny system for males, and about one-third lower for females. This sex differential in relative performance of the two approaches relates to the fact that the variance in the mortality of adult males is greater than that for females.

Figures 9 and 10 show the relative performance of the two model life table systems in predicting the actual observed probability of adult death ($_{45}q_{15}$) (Figure 9) and life expectancy at birth (Figure 10) based on the subset of 541 life tables. If a system could exactly predict the true life table values, all sample points would lie on a straight line. As Figure 9 illustrates, the modified logit system more successfully predicts the true probability of adult death (for males) than the closest match from the Coale–Demeny system, selected on the basis of $_5q_0$ and e_0, as described earlier. In particular, the Coale–Demeny system performs relatively poorly for true levels of $_{45}q_{15}$ in excess of about 150 per 1,000, which would include much of the contemporary developing world. A similar pattern is apparent from Figure 10. This clearly shows the much closer fit between observed and predicted male e_0 for this sample of countries than is achieved using the Coale–Demeny

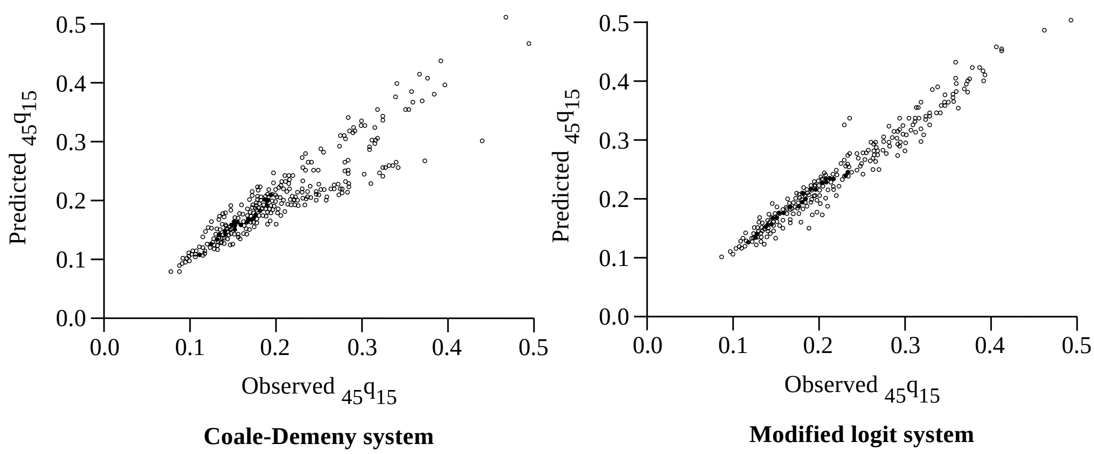

Figure 9 Observed vs. predicted $_{45}q_{15}$ of males, using the Coale–Demeny and modified logit systems, and selecting on the basis of $_5q_0$ and e_0 (n = 541)

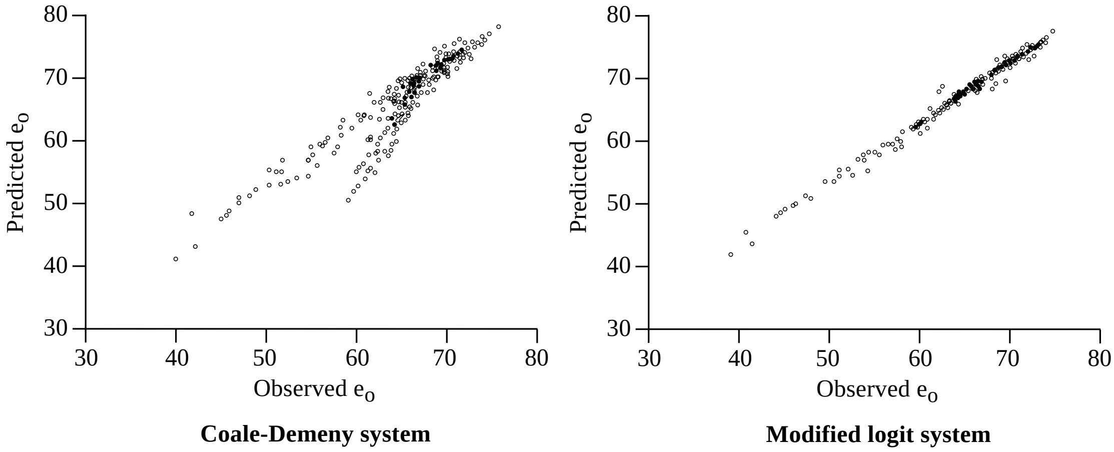

Figure 10 Observed vs. predicted e_0 of males, using the Coale–Demeny and modified logit systems, and selecting on the basis of ${}_5q_0$ and ${}_{45}q_{15}$ ($n = 541$)

system, selected on the basis of ${}_5q_0$ and ${}_{45}q_{15}$, irrespective of the level of true life expectancy.

In addition to assessing the overall fit between predicted age-specific death rates and those actually observed, we have tested for any systematic bias in the death rates at different ages. Table 5a summarizes the regression results of the observed and predicted values for various life table functions. If the modified logit system were able to predict perfectly the observed life table function (e.g., ${}_{45}q_{15}$ or ${}_{20}q_{60}$), the coefficient of the regression would equal one and the constant would be zero. As is clear from Table 5, this is very nearly the case for all tests conducted on the 541 life table subset, with the greatest departure from unity at ages 60–80 years for males. In exploring this bias further, we found that substituting the 25th percentile values for θ_x and γ_x at all ages 65 and over (obtained from their uncertainty distributions) leads to a reduction in the bias of predicted values of probability of death at higher ages while having little effect on the overall R^2 for e_0. As a result, we have used the 50th percentile of the distribution for males at all ages below 65 and the 25th percentile values for θ_x and γ_x at all ages 65 and over, while leaving the

Table 5a Results of regression of selected observed life table parameters on those predicted by the modified logit system ($n = 541$)

	Males				Females			
	α	β	R^2	RMSE	α	β	R^2	RMSE
e_0	–1.6905	1.0258	0.9882	0.6137	–1.1436	1.0155	0.9845	0.7742
${}_{45}q_{15}$	0.0006	0.9972	0.9995	0.0017	0.0008	0.9927	0.9993	0.0016
${}_{20}q_{60}$	–0.0360	1.0520	0.6069	0.0474	–0.0002	1.0047	0.7747	0.0438

Table 5b Results of regression of selected observed life table parameters on those predicted by the modified logit system, using 25th percentile values for males at ages 65+

	Males (25th percentile)[1]			
	α	β	R^2	RMSE
e_0	–0.8250	1.0127	0.9885	0.6068
${}_{45}q_{15}$	0.0006	0.9972	0.9995	0.0017
${}_{20}q_{60}$	0.0105	0.9799	0.6086	0.0473

[1] 25th percentile values are as follows: γ_{65-85+} = (–0.2466, –0.5744, –0.9952, –1.5372, –2.2597); θ_{65-85+} = (0.1148, 0.2544, 0.4099, 0.5862, 0.7939).

estimated values for females unchanged. Table 5b shows the effect of this modification on the comparison between observed and fitted life table functions for males.

7.2. Predictive validity of the modified logit system vs. the Brass logit system

The previous section demonstrated the much greater accuracy of the modified logit system over the Coale–Demeny system. Moreover, the modified system has far greater flexibility than the Coale–Demeny system since it is not dependent on a defined set of mortality schedules. This lack of flexibility is obviously not relevant to the original Brass system, but its predictive ability is. For the 30 per cent sample of life tables, Table 4 demonstrates that the modified logit system improves predictive validity by 15–40 per cent on average over the original Brass logit system.

The main limitation of the Brass logit system is that, as a population moves away from the standard pattern of mortality, the predictive validity of age-specific mortality rates becomes much worse. In this section, we investigate whether the modifications of the Brass logit system are sufficient to justify using a single global standard. To assess this, we divided the 30 per cent sample of life tables from our data-set into three categories, based on the extent to which the observed child and adult mortality levels corresponded to those of the standard. Specifically, we defined a sex-specific index to assess departures from the standard relationship between child and adult mortality as follows:

$$Z = \sqrt{\frac{\left(1 - \dfrac{\text{Logit}(l_5^{ij})}{\text{Logit}(l_5^{s})}\right)^2 + \left(1 - \dfrac{\text{Logit}(l_{60}^{ij})}{\text{Logit}(l_{60}^{s})}\right)^2}{2}}. \quad (7)$$

Next, the entire sample set of 541 life tables was grouped into three equal terciles based on the Z-score value of the life table. Taking males as an example, Tercile 1, with Z-scores ranging from 0.00 to 0.16, includes life tables with the child–adult mortality relationship that is most similar to the standard, with progressive departures grouped under Tercile 2 (Z-scores from 0.16 to 0.30) and Tercile 3 (Z-scores greater than 0.30). We then computed the average RMSE of the log of predicted age-specific death rates across all life tables in each tercile for both the Brass logit and the modified logit systems. We did this by means of two distinct matching procedures: one in which the closest life table in the modified logit system was chosen on the basis of the observed levels of ${}_5q_0$ and ${}_{45}q_{15}$, and one in which the closest life table was selected by means of matching the observed ${}_5q_0$ and e_0.

The results are summarized in Figure 11 for males and females separately. It is immediately obvious that the modified logit system is more accurate than the Brass system in predicting age-specific log death rates, particularly when child and adult mortality differ substantially from the standard. For example, when matching is done on the basis of ${}_5q_0$ and ${}_{45}q_{15}$, the modified logit system is about 25 per cent more accurate, on average, in predicting log death rates than the Brass system when deviations are less extreme (Tercile 1), about 35 per cent more accurate for life tables in Tercile 2, and roughly 40 per cent more accurate for Tercile 3 with more extreme deviations in mortality levels. Moreover, the relative invariance of the RMSE for the modified logit system across the three groups of life tables suggests that the system is much less affected than the Brass system by the choice of standard. The relative RMSE values for females using the ${}_5q_0/{}_{45}q_{15}$ matching procedure were similar, though slightly more extreme than for males. For both sexes, the relative invariance of the RMSE across terciles when using the modified logit system to predict log death rates is apparent, with only a marginally worse RMSE for Tercile 3.

The modified logit system also performs substantially better than the original Brass system when the closest life table is selected on the basis of ${}_5q_0$ and e_0, although the relative difference in average RMSE is less marked. Thus for Tercile 3, the modified logit system is about 25 per cent more accurate than the original Brass system in predicting log death rates for life tables, compared with 40 per cent when selected on the basis of ${}_5q_0$ and ${}_{45}q_{15}$. Overall, the new system is 25–45 per cent more accurate in predicting log death rates for life tables that depart substantially from the standard (Tercile 3). Moreover, given that the most common inputs into model life table systems would be estimated values of ${}_5q_0$ and ${}_{45}q_{15}$, rather than ${}_5q_0$ and e_0, the much greater accuracy of the modified logit system on this basis in predicting log death rates that differ substantially from the standard is arguably the more relevant comparison. It should also be kept in mind that the relative difference in actual death rates will be greater still, owing to the properties of the log transformation.

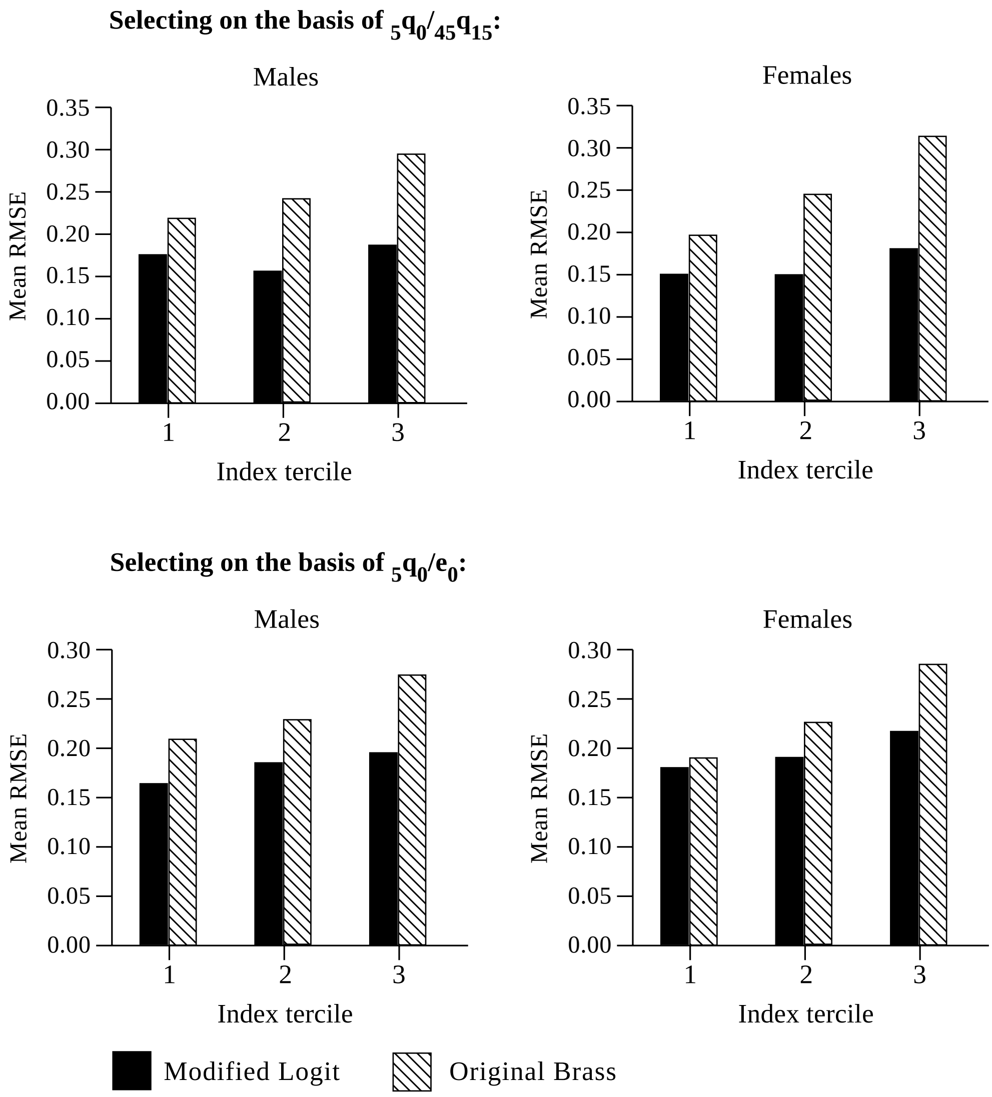

Figure 11 Mean root mean square error by index tercile for original Brass and modified logit systems, by sex

8. Discussion

In this paper, we have demonstrated that the modified logit life table system using a single global standard can represent the full range of mortality patterns seen across the high-quality life tables available internationally. The proposed system generates better predictions of age-specific mortality rates than the Coale–Demeny and original Brass systems and is indexed on two life table functions that are relatively easy to understand. While the modified logit system as presented here is indexed on l_5 and l_{60}, for practical use it is possible to identify approximately a unique life table with any two life table functions such as life expectancy at birth and child mortality.

The main limitation of this model life table system and of the tests of its predictive validity is that the sample of high-quality life tables is heavily weighted towards populations with life expectancies between ages 60 and 73 (for males) and ages 66 and 80 (for females). The addition of more high-quality and recent life tables for high-mortality populations might suggest alternative values of θ_x and γ_x that would minimize prediction error. Such analyses can be undertaken easily if new high-quality life tables become available. Based on the available set of life tables in our empirical data-set, however, the results appear to be quite robust to the selection of even small subsets of life tables. We have re-estimated the θ_x and γ_x for numerous random samples of 100 life tables selected from the overall database and have found that the parameter estimates are remarkably insensitive to the set of life tables on which they are estimated. This strengthens our view that the addition of new life tables will not alter substantially the estimates of θ_x and γ_x.

There is remaining uncertainty about how the model system would perform for populations with high levels of HIV. It is quite possible that in high-HIV settings, the age pattern of mortality projected from a sample by the model might not be accurate, although this possibility cannot be tested owing to the lack of high-quality life tables for these countries. As a very limited test, we have compared the estimates of age-specific mortality based on selecting a model life table in the absence of HIV, with HIV death rates added on *a posteriori* (Lopez et al. 2002) for Zimbabwe, South Africa, and Tanzania, with the model life table selected using values of l_5 and l_{60} that reflect the impact of HIV. Predicted life expectancy at birth was within 0.5 years of the value estimated from this two-stage procedure, with an even closer agreement for levels of adult mortality.

The use of the Coale–Demeny and UN systems is so widespread in demographic estimation that there are often circular arguments about levels and patterns of adult mortality. One set of analysts often use the results of other demographic analyses founded on these model life table systems without realizing that they substantially underestimate the variation in age-specific mortality patterns seen in the real world. The use of models is so deeply embedded in available international data-sets that it can be difficult to make genuine empirical tests of the models. We have tried to ensure that the observed life tables used in this analysis have not been modified using model life table systems, and hence that the modified system is based exclusively on observed data.

One implication of this analysis is that, for sub-Saharan Africa in particular, there is much more uncertainty about levels of adult mortality than is implied in currently available demographic estimates such as the UN Population Division life tables (UN 2001). Often, levels of adult mortality have been estimated by selecting a life table on the basis of estimated child mortality and an arbitrary choice of a model life table family (often West by default). This led to a tendency towards a one-to-one mapping of child mortality to adult mortality before the HIV epidemic. In reality, even the empirical record of countries outside Africa suggests that there can be much greater variation in levels of adult mortality than in child mortality than is captured in the Coale–Demeny and UN model life tables. We hope that the convenience of a simple model life table system parameterized using easily recognized aspects of a population's mortality experience and a single global standard will facilitate a wide use of the modified logit system.

A key issue in the application of this new system of model life tables will be the availability of the reliable estimates of child and adult mortality required to identify a fully specified life table. Decades of demographic interest in the measurement of child mortality have resulted in reasonably reliable estimates for almost all countries (Lopez et al. 2002), whereas the measurement of adult mortality has been largely neglected. Estimates of survival from ages 15 to 50 or 60 can be constructed from survey or census data on sibling survival, orphanhood, or recent household deaths, but require substantial adjustment for undercounting of deaths. A vast increase in data on adult mortality is urgently required, as is research into methods which can reliably correct the data for systematic under-reporting. The World Health Organization, through the World Health Surveys programme, has been at the forefront of international research efforts to address these issues.

Notes

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