WHO System of Model Life Tables

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Introduction

The life table provides the most complete description of mortality in any population. The basic data input needed for its construction are the age-specific death rates calculated from information on deaths by age and sex (from vital registration) and population by age and sex (from census). In many developing countries, these basic data either do not exist due to lack of functioning vital registration systems, or are unusable because of incompleteness of coverage or errors in reporting. Where the issue is principally one of incompleteness, demographers have devised ingenious ways of deriving reasonably suitable life tables after the application of a variety of appropriate adjustment techniques. In cases of unusable or non-existent vital registration data, indirect techniques for obtaining mortality rates are employed. These techniques are predicated on the observed similarities in the age-patterns of mortality for different populations, and may range from the simple adoption of the mortality pattern of a neighbouring population with similar socio-biological characteristics, to the use of sophisticated demographic models.

The observed regularities in the age pattern of mortality is the prime motivation in the search for mathematical functions that *fully* capture the observed variations of mortality with age (Gompertz, 1825; Keyfitz, 1984). Failure to achieve this has led to the development of a number of empirical "universal" mortality models (or model life tables) of varying degrees of sophistication. The best known are (i) the UN Model Life Tables, (ii) The Coale-Demeny Model Life Tables, (iii) the UN Model Life Tables for Developing countries, (iv) the Ledermann System of Model Life Tables and (v) the Brass Logit System. The data underlying them vary in the range of human experience they encompass. As such, particular mortality models may be more or less suitable for specific geographic areas.

These models have contributed significantly to our understanding of levels and patterns of mortality over the last half century in areas of the world with very little demographic data. There are, however, substantial drawbacks to their continued use in many contemporary developing countries. Principally, the restricted nature of the original sample of life tables that underlie these models has always been a major disadvantage. It has become more so with the spread of HIV/AIDS whose effect on the age pattern of mortality has no corollary in recent history. Linked to these are the likely differences between the historical cause of death structure underlying these models and the cause of death structure prevailing in many developing countries today. Also, several of the models are essentially uni-parametric and therefore relatively inflexible. In this regard, the Brass logit system offers considerable advantages by being essentially independent of historical data. Such flexibility could be harnessed in extending its application to situations of extreme data poverty, e.g. in Africa and parts of SE Asia. The present paper presents a candidate method for achieving this, based on the relationship between

under-five mortality ($_5q_0$) and adult mortality ($_{45}q_{15}$) within a space bounded by the parameters of the logit system. The technique allows the derivation of complete life tables from a knowledge of the under-five mortality, the adult mortality ($_{45}q_{15}$) and a corresponding WHO regional standard life table. It forms the basis for a new WHO system of model life tables.

In the subsequent sections, we provide a historical perspective on the development, shortcomings and relative advantages of the model life table systems currently available. We assess, in particular, the performance of the Coale Demeny model life table system relative to the logit. We then rigorously evaluate and quantify the biases inherent in these models using both real data and hypothetical data from the four families of the Coale-Demeny model life tables. We finally evaluate the performance of the WHO system against real data. Detailed derivations of formulae are shown in the appendix.

Historical perspectives

The basic objective in the creation of any model life table is to construct a system that gives schedules of mortality by sex and age, defined by a small number of parameters that capture the level as well as the age pattern of mortality. If a particular model adequately represents reality, the characteristics of a given population can be summarized by the parameters of that model, thereby facilitating the study of variation among populations or within a population over time. Thus, model life tables are essential demographic tools for populations lacking accurate demographic data. The principles underlying each of the existing model life tables are discussed below.

UN model life tables (1955). The first set of model life tables was published by the UN in 1955. They were constructed using 158 life tables for each sex, using statistical techniques to relate mortality at one age to mortality at another age for a range of mortality levels. The model assumes that the value of $_n q_x$ for each age interval in a life table is a quadratic function of the rate in the preceding interval, namely $_5 q_{x-n}$ (except for the first two age groups, $_1q_0$ and $_4q_1$, all the other groups considered are 5 years in length). Thus, knowledge of only one mortality parameter (e.g., $_1q_0$ or equivalently the mortality level that indexes the $_1q_0$ values used) determines a complete life table. For this reason, the UN model life tables are collectively referred to as a *one-parameter system*. To each level of mortality there corresponds a model life table for males, females and both sexes combined.

The coefficients of the quadratic equations for each sex were estimated from the corresponding sample of 158 life tables. These were then used in generating the actual model life tables by first

choosing, arbitrarily, a convenient value of ${}_1q_0$. This value was then substituted in the equation relating ${}_1q_0$ to ${}_1q_4$ in order to obtain a value for ${}_1q_4$, which in turn was substituted in the equation relating ${}_5q_5$ to ${}_1q_4$ to obtain ${}_5q_5$, *etc., etc.* This "chaining" process continued until the model life table was completed.

The Coale and Demeny regional model life tables. These were first published in 1966. They were derived from a set of 192 life tables, by sex, from actual populations. This set included life tables from several time periods (39 from before 1900 and 69 from after the Second World War) and mostly from Western countries. Europe, North America, Australia and New Zealand contributed a total of 176 tables. Three were from Israel; 6 from Japan, 3 from Taiwan; and 4 from the white population of South Africa. All of the 192 selected life tables were derived from registration data, and were subjected to very stringent standards of accuracy.

Further analysis of the underlying relationships identified four typical age patterns of mortality, determined largely by the geographical location of the population, but also on the basis of their patterns of deviations from previously estimated regression equations. Those patterns were called: North, South, East, and West. Each had a characteristic pattern of child mortality. The *East* model comes mainly from the Eastern European countries, and is characterized by high child mortality in relation to infant mortality. The *North* model is based largely on the Nordic countries, and is characterized by comparatively low infant mortality, high child mortality and low old age mortality beyond age 50. The *South* model is based on life tables from the countries of Southern Europe (Spain, Portugal, and southern Italy), and has a mortality pattern characterized by (a) high child mortality in relation to infant mortality *at high overall mortality*, and (b) low child relative to infant mortality *at low overall mortality*. The *West* model is based on the residual tables not used in the other regional sets (i.e., countries of Western Europe and most of the non-European populations). It is characterized by a pattern intermediate between North and the East patterns. Because this model is derived from the largest number and broadest variety of cases, it is believed to represent the most general mortality pattern. In this system, any survivorship probability, whether from birth or conditional on having attained a certain age, uniquely determines a life table, once a family has been selected. Although technically a one parameter system, it could be argued that the choice of a family constitutes a separate dimension.

The Ledermann's system of model life tables (1959, 1969). This system is based on a factor analysis of some 157 empirical tables. The method of selection was less rigid than in the Coale-Demeny tables, but they represent more developing country experiences. Analysis of the tables disclosed five factors that apparently explained a large proportion of the variability among the life tables. The extracted factors related to (a) general level of mortality, (b) relation between childhood and adult mortality, (c) mortality at older ages, (d) mortality under age five, and (e)

male-female difference in mortality in the age range 5-70 years. Later, Ledermann developed a series of one- and two-parameter model life tables based on these result [i].

Brass logit system (1971). This system provides a greater degree of flexibility than the empirical models discussed above. It rests on the assumption that two distinct age-patterns of mortality can be related to each other by a linear transformation of the logit of their respective survivorship probabilities. Thus for any two observed series of survivorship values, l_x and l^s_x, where the latter is the standard, it is possible to find constants α and β such that

$$logit(l_x) = \alpha + \beta logit(l^s_x)$$

$$if\ logit(l_x) = 0.5 ln\left(\frac{(1.0 - l_x)}{l_x}\right)$$

Then

$$0.5 ln\left(\frac{(1.0 - l_x)}{l_x}\right) = \alpha + 0.5\beta ln\left(\frac{(1.0 - l^s_x}{l^s_x}\right)$$

for all age x between *1* and *T*. If the above equation holds for every pair of life tables, then any life table can be generated from a single standard life table by changing the pairs of ($\forall, \exists$) values used. In reality, the assumption of linearity is only approximately satisfied by pairs of actual life tables. However, the approximation is close enough to warrant the use of the model to study and fit observed mortality schedules. The parameter $\forall$ varies the mortality *level* of the standard, while $\exists$ varies the *slope* of the standard, i.e., it governs the relationship between the mortality in children and adults. Figure 1 shows the result of varying $\forall$ and $\exists$. As $\exists$ decreases, there is higher survival in the older ages relative to the standard, and vice versa. Higher values of $\forall$ at a fixed $\exists$ lead to lower survival relative to the standard.

The UN model life table for developing countries (1981). These were designed to address the needs of developing countries. The underlying data consisted of 36 life tables covering a wide range of mortality levels from developing countries, by sex. Sixteen pairs of life tables came from 10 countries in Latin America, 19 pairs from 11 countries in Asia, and one pair from Africa. Five families of models were identified, each with a set of tables ranging from a life expectancy of 35 to 75 years for each sex. Each family of models covers a geographical area: *Latin American, Chilean, South Asian, Far Eastern* and *a General*. The general model was constructed as an average of all the observations.

Shortcomings of the empirical MLTs

There are three major criticisms of the UN model life tables. First, the fact that they are one-parameter systems makes them relatively inflexible. Such a single parameter model cannot adequately describe the complex mortality patterns available. In some cases, they have failed to describe adequately life tables that were known to be accurate (Menken, 1977). Second, because the estimate of mortality in each age-group is ultimately linked to the infant mortality rate through the chaining process, measurement errors are easily accentuated. The third criticism concerns the poverty of developing country life tables in the original design of the model. Additionally, some of the empirical tables included were of dubious quality (UN, 1983). The UN model life tables for developing countries also suffer from some of these limitations. Hence, both systems of UN model life tables have a selective if not a limited range of application.

The Coale-Demeny model life tables had much higher standards of accuracy for the empirical tables. This demand, however, limited the number of non-European countries represented. As such, the Coale-Demeny tables may not cover patterns of mortality existing in the contemporary developing world. In fact, there are examples of well documented mortality patterns that lie outside the range of the Coale-Demeny tables. In particular, Demeny and Shorter found no table within the family that adequately refleceted the Turkish mortality experience (Demeny & Shorter, 1968). Although the North, South, East and West classification provides an added dimension, the uni-parametric nature of each family still limits its flexibility.

The Ledermann system is criticized primarily for its relative complexity which essentially precludes its use in most developing countries. Even though it does provide some flexibility through a wider variety of entry values, in practice most of these values are not easily estimated for most developing countries. This drawback reduces its relative advantages over the UN and the Coale-Demeny models. A second major limitation is that the independent variables used in deriving the model refer, with only one exception, to parameters obtained from data on both sexes combined. The user is, therefore, forced to accept the relationships between male and female mortality embodied in the model even when there is evidence to the contrary. For instance, it is near impossible to estimate a Ledermann model life table in which the male expectation of life exceeds that of females (UN, 1977).

Another shortcoming common to all three empirical models is their dependence on the type of data that generated them. The databases upon which they were built exclude a significant proportion of possible mortality schedules. Although the UN set of model life tables attempted

to address this issue, there were serious flaws in the selection of life tables as well as the criteria of acceptance.

It is clear, therefore, that there are serious technical issues that complicate the use of existing empirical models in describing mortality patterns in contemporary developing countries. These are further compounded by the emergence of HIV/AIDS as a major cause of death in Africa and parts of Asia. For these and other reasons WHO is proposing a new three-parameter system of model life tables anchored on the logit system. The choice of the logit system was based on a careful comparative evaluation of the logit and the Coale-Demeny systems. This evaluation process is presented in the next section.

Contrasting the logit with the Coale -Demeny MLTs

Since the Coale-Demeny is the most extensively used of the empirical models, discussions will be limited to it and the logit system. In contrast to the Coale-Demeny models, the logit system is not dependent on any fixed empirical data. It allows the choice of a locally-applicable standard life table. Once a suitable standard life table or equivalently a standard series of l_x values has been selected, the problem reduces to finding appropriate set of values of $\forall$ and $\exists$. A life table can then be generated using the logit equations presented earlier. It is, therefore, possible to generate a wide range of mortality schedules that provide a reasonably accurate representation of most observbed mortality patterns.

Implications of the choice of standard for mortality predictions

Any life table can potentially be used as a standard in the logit system. Table 1 shows the standard l^s_x chosen by Brass. The fits obtained using this standard compares reasonably well with those obtained from the Coale-Demeny regional model life tables. Occasionally, however, relatively large discrepancies occur between fitted and observed data, especially at the extremes of the age distribution , i.e., under age 5 and above age 70 (Zaba, 1979). Such discrepancies can be avoided by a judicious choice of a reference standard. It is only when the mortality pattern of the chosen standard resembles that of the comparison l_x function will the linear relationship on the logit scale be evident. There are, however, no clear guidlines on the criteria for selecting a suitable standard. As a general rule, if the comparison life table does conform to the logit system generated by the chosen standard, then the corresponding plotted points logit(l_x) against logit(l^s_x) should form a fairly straight line with intercept $\forall$ and slope $\exists$. When, however, observed deviations from linearity are systematic, e.g., when the deviations become larger as age increases or as age decreases, or when the plot is decidedly curvilinear, a different standard should be used

(UN, 1983). In appropriate cases, regression technmiques may be used to obtain the parameters of the best fitting line.

Choosing standards for the WHO system of MLT

In selecting a standard for the WHO system of model life tables, the choices were between (i) country-specific standard life table, (ii) a global standard life table, and (iii) a standard life table based on WHO mortality strata (see Appendix A). For the country-specific standard, the life table for the latest available year for each country was chosen. The global standard was based on UN data from the 1998 revision of the World Population Prospect. The standards based on the WHO mortality strata are from historical series and rely on several age patterns of mortality for different kinds of populations. In order to objectively compare each of these standards, the mean absolute deviation (or error) between the observed (real data) and predicted ${}_nq_x$, for a given standard, were computed over the entire age range, for time series of data for France, Argentina, Greece and the Netherlands.

In assessing the mean error using the country-specific standard, the following index was used:

$$\textit{Mean absolute deviation} = \frac{\sum \left| \left(1 - \frac{{}_n\hat{q}_x}{{}_nq_x} \right) \right|}{n}$$

where n is the number of age groups in each life table. A plot of the mean error over time for France, Argentina, Greece and the Netherlands are shown in Figure 2. The results show that, within the time series for any given country, the mean error for a prediction becomes smaller the closer the time location of the standard relative to the predicted life table. The plots also show marked variation in the size of the mean prediction error between countries. Hence, for the purposes of international comparison, the use of country-specific standards is likley to produce incomparable results depending on the age pattern of mortality in each country relative to the standard. A "neutral" standard that is not based on the experience of any particular country will, therefore, be preferable.

The choice is then between a single global standard and a set of region-specific standards based on the WHO mortality strata. There are of course comparability issues surrounding the use of regional standards, but these are no different from those concerning the use of different families of the Coale-Demeny model life tables. The results of this comparison are shown in Figure 3 for Argentina (1977, 1985 and 1996) using *Amr B* standard, France (1970, 1985, 1996) and Greece

(1970, 1985 and 1977) using *Eur A* standard, and for Poland(1970, 1985 and 1996) Using *Eur B* standard. In general, the global standard tends to produce over-estimates of the age-specific mortality rates at the extremes of the age range. This is particularly true in France, Greece and Argentina, probably reflecting the younger age distribution of the standard relative to the distributions in these populations. In contrast, the WHO regional mortality standards tend to yield smaller mean errors. In the case of Argentina and Greece, the regional standards tend to produce higher age-specific mortality rate between age 15 and 40 years. In the case of Poland, the age pattern produced by the WHO regional standard closely approximates that of the global standard. Thus relative to the global standard, the WHO regional standards tend to produce smaller age-specific bias.

In summary, country-specific standards are ideal for country-specific comparisons. However, for international comparison, a neutral standard has obvious advantages. While a global standard would have been preferable, it appears to be strongly influenced by the relative age pattern of mortality at the extremes of the age range. In contrast, the regional standards tend to yield smaller biases although, issues of comparability across regions must be taken into consideration in interpreting results. After considering these issues, the WHO regional mortality standards were chosen for subsequent analysis.

Quantification of the bias in the Coale-Demeny and the Logit systems

Having chosen appropriate regional standards for the WHO system of model life tables, the next step involved an assessment of the relative quality of the predictions of the logit and Coale-Demeny systems. To do this, data for three countries and for different time points were selected: France, 1970, 1985 and 1996; Hungary, 1970 and 1997; and Argentina, 1977 and 1996.

For the Brass logit, the appropriate WHO regional mortality standard life table was selected: *Eur A* for France, *Eur C* for Hungary, and *Amr B* for Argentina. Using these standards and country-specific data, predicted life tables were produced for each country and for each time point. These estimates were then compared with the corresponding real data. The deviation between the observed (${}_nq_x$) and predicted (${}_nq_x$)values were calculated using a fit index defined as:

$$Fit\ index = \left(\frac{{}_n\hat{q}_x}{{}_nq_x} \right)$$

In each country and for each time point, a Coale-Demeny model life table corresponding to the observed life expectancy was selected from each of the four families. Each identified life table was compared with the life table based on real data. The fit index defined above was then used to assess the deviation between observed and predicted. Figures 4-6 show comparison plots of the age-specific devations due to the logit system and those due to the Coale-Demeny models for each country and for each time point. The West and East families of the Coale-Demeny model produce better overall fit to the data than the North and South, especially at the younger ages, 0-15 years. The logit system systematically performs better than the Coale-demeny except in the case of Argentina, where both systems do relatively poorly. In particular, the logit system tends to over-estimate mortality while the Coale-Demeny under-estimates mortality. In France, the fit produced by the logit system are practically similar to those from the East and West models, except at the very youngest ages. In Argentina, the Coale-Demeny East and West models significantly over-estimate mortality below age fifteen years.

In summary, the Coale-Demeny model life tables tend to exaggerate mortality at the younger ages, especially in the case of the North and South models. In contrast, the Brass logit system tends to produce better fit except in the case of Argentina, where both the Coale-Demeny and the logit systems perform poorly. The logit system tends to fit poorly at the extremes of the age range. This is probably a consequence of the age pattern of mortality implied by the standard. However, the results vary significantly, with the time location of the observed data relative to the standard. The more recent the data, the worse the predictions of both the logit and the Coale-Demeny models, further emphasizing the caution needed in using these models in predicting contemporary mortality schedules.

WHO system of model life tables

A desirable property of any new model life table will be a capacity to adequately reflect the age patterns of mortality found in contemporary populations without being constrained to represent exclusively the patterns in the data used to construct it. This is a specific advantage of the Brass logit system, which makes it possible to construct logit model life tables with enough parameters to provide greater accuracy in describing observed patterns of mortality (Brass, 1977; Zaba, 1979; Ewbank *et al*., 1983). An important question though, is whether it is possible to construct models with fewer parameters than the four or five parameter dimensions needed for greater accuracy? Is it possible to identify parsimonius models whose relatively few parameters can be selected on the bases of knowledge of auxilliary variables?

In attempting to answer these questions, we explored the relationship between under-five mortality ($_5q_0$) and adult mortality ($_{45}q_{15}$) using the attractive properties of the logit scale. These variables are known life table functions that are easily estimated, and many recent surveys include questions designed to collect the necessary data for their calculation. An underlying principle for the WHO system of model life tables is that, knowledge of the values of these two auxilliary variables ($_5q_0$ and $_{45}q_{15}$) and an appropriate regional standard life table should uniquely define the set of $\forall$ and $\exists$ parameters for a unique life table. One can then generate life tables for any contemporary population.

Construction of the WHO system of MLTs

The WHO system of model life tables is a graphical extension of the Brass logit system anchored on the relationship between under-five mortality and adult mortality, within a space whose coordinates are defined by the coefficients of the logit equation, i.e., the $\forall$ and $\exists$ parameters corresponding to a given standard. The *x*-axis and *y*-axis reprsent the $\forall$ and the $\exists$ values, respectively. For each standard mortality schedule, e.g., the *Eur A* standard, isobars of $_5q_0$ and $_{45}q_{15}$ are plotted within the logit space according to the following equations:

$$\text{For } {}_5q_0 \text{ the general equation for the isobar is}$$
$$\alpha = logit(l_5) - \beta\, logit(l_5^{EurA})$$
$$\text{For } {}_{45}q_{15} \text{ the general equation for the isobar is}$$
$$\alpha = 0.5\, ln\left[\frac{k}{(1-k)e^{\beta c_{60}} - e^{\beta c_{15}}}\right]$$

where $k={}_{45}q_{15}$, $c_{15}=logit(l_{15}^{Eur\,A})$, $c_{60}=logit(l_{60}^{Eur\,A})$, l_5 and l_5^{EurA} are constants. Also l_5 and l_5^{EurA} are the probabilities of surviving to age 5 years in the life table of interest and the *Europe A* standard life table, respectively. By definition, the isobars for the under-five mortality are linear while those for the adult mortality are curvilinear. The detailed algebraic derivations of the above equations are presented in the appendices B and C. In constructing the isobars for any given value of under-five mortality, hypothetical values of β (e.g., from 0.2 to 1.4) are substituted in the first equation to obtain corresponding values of α on the given standard (Table 2). Similarly, to construct an isobar for a given value of adult mortality (k), different hypothetical values of $\exists$ are substituted in the second equation to obtain correspoding vales of $\forall$ (Table 3).

The paired ($\forall$,$\exists$) values at any given level of under-five or adult mortality are plotted on the $\forall$,$\exists$ grid. Points corresponding to the same level of mortality are then joined to form the isobars (Figure 7). Figure 7 shows a detailed plot of the output grid from this model for the WHO *Amr A*

standard life table. Isobars of constant child mortality (lines slanting from bottom left to upper-right corner on the figure) and constant adult mortality (curves with convexity to the left) are shown in fine gradations for values of ${}_5q_0$ ranging from 0.005 to 0.065, and for values of ${}_{45}q_{15}$ ranging from 0.1 to 0.66. Depending on need, the finer gradations may or may not be plotted (see Figure 9).

The ∀,∃ Grid

The point of intersection between an isobar for child mortaity and one for adult mortality uniquely defines a life table. Thus, to estimate a life table for the population under study, the point on the grid defined by values of ${}_5q_0$ and ${}_{45}q_{15}$ for a population is located, and the corresponding values of ∀ and ∃ are then read off corresponding axes. The ∀,∃ pair corresponding to this point and the age-specific logit values corresponding to the appropriate WHO regional standard mortality schedule are substituted in the logit equation for that region to generate a complete life table. As an example, suppose that values of ${}_5q_o$ and ${}_{45}q_{15}$ are available for a country from a demographic survey or from intercensal survival analysis. If these were estimated at 20/1000 and 170/1000 respectively, then locating this point (marked with an *) on Figure 7, and reading across and down to the axes, suggests a value of ∀ = 0.11 and ∃ = 0.89. Applying these values in the logit equation with the specified regional standard yields the schedule of l_x values at all ages.

The plot also shows that as adult mortality levels decline, adult mortality isobars shift to the left along a line at a positive angle to the x-axis (∀-axis). In other words, both ∀ and ∃ decrease in value with decline in adult mortality. In contrast, ∀ decreases but ∃ increases in value as child mortality declines, albeit slowly. The child mortality isobars move to the left along a line at a negative angle to the x-axis (∀-axis).

For any given population and standard, a time series of points defined by ∀,∃ pairs represent the mortality trajectory of that population over time. Figure 8 shows the trajectories for Canada, Cuba and USA using *Amr A* standard life table. The USA and Canada show a sustained decline in both adult and child mortality, with later slowdown in child mortality (at very low levels) in the most recent period. Cuba demonstrates a substantial decline in child mortality with minimal change in adult mortality. Figure 9 shows similar plots for Finland, Sweden, Spain and Israel. Spain shows a historical pattern of decline in both adult and child mortality followed by contined decline in child mortality but relatively little change in adult mortality. The plot for Sweden shows two phases: an earlier phase of substantial decline in adult mortality with relatively little change in child mortality, followed by a second phase characterized by decline in both adult and

child mortrality. Finland shows a substantial decline in adult mortality with only marginal change in child mortality at very low levels.

To summarize, in circumstances where an historical sequence of life tables are available it is possible to generate a time series of α,β pairs using either a country-specific standard or a WHO regional mortality standard. A plot of α and β, separately, against time should produce a trajectory of points (see Lopez *et al*., 2000). If the plot of points for each parameter fall along a fairly straight line, that line could theoretically be projected forward to forecast estimates for any time in the future. These α,β estimates can then be substituted into the appropriate logit equations to obtain the corresponding life tables. Alternatively, it is possible to plot the α,β pairs of points on the α,β grid corresponding to a particular WHO reference standard (see Figures 8 and 9). The trajectory of such points could then be projected forward to obtain estimates of α and β. The disadvantage of this approach is that, it is not easy to assign a time location to the life table generated.

In the absence of an historical trajectory, a life table may also be defined by first estimating ${}_5q_0$ and ${}_{45}q_{15}$ in the year of interest and then locating the point of intersection between the isobars corresponding to these values of ${}_5q_0$ and ${}_{45}q_{15}$. The finer the gradations of the isobars the better the predictions. Wide gradations lead to greater uncertainty. Using this method, the level of uncertainty in the estimates of ${}_5q_0$ and ${}_{45}q_{15}$ may be translated into uncertainty around the life table. For example, probability distributions around ${}_5q_0$ and ${}_{45}q_{15}$ may be defined, and multiple life tables generated using Monte Carlo simulation methods. The range of life tables may then represent the probability distribution of predicted age-specific mortality patterns given uncertain summary measures of child and adult mortality. For more details refer to Salomon and Murray (2000).

Discussion

The logit system for developing life tables has a number of intuitively appealing characteristics. To begin with, the system is not dependent on the existence of a large empirical database of age-specific mortality rates which are required to effectively model age-patterns of mortality. This is particularly relevant for regions where reliable estimates of age-specific death rates may be available for only a few countries, and then for only a few periods. The model does not assume any *a priori* knowledge about the form of the relationship between age-specific death rates. Rather, the statistical property of linearity in logits is atheoretical and invariant across mortality patterns.

The higher degree of parametization in the WHO system (3 parameters: $\forall, \exists$ and the choice of standard) compared with the empirical systems of model life tables can also be expected to provide a more robust basis for estimating mortality patterns and levels. The parameters $\forall$ and $\exists$ are also readily interpretable. $\forall$ varies around a central value of 0, with values greater than zero indicating progressively higher mortality overall relative to the standard, and values below zero the opposite. For $\exists$, a reasonable range of values appear to be from about 0.6 to 1.4 (Newell, 1988), depending on the standard. Low values of $\exists$ suggest high infant and child mortality relative to the standard, whereas high values imply the reverse (i.e. lower child, higher adult mortality relative to the standard). (Note that choosing a value of $\exists$ = 1.0 reduces to a one-parameter ($\forall$) system, similar to the level parameter of conventional Coale-Demeny model life table systems).

As overall population health improves, one would expect values of $\forall$ to decrease. The trends in $\exists$ are more difficult to predict and depends very much on the standard. Thus in the case of a standard with relatively high mortality at younger ages, the age at which half the births exposed to the standard mortality rates will survive is relatively low. Below this age, declining mortality would result in β increasing, but decreasing above this age. As the standard migrates towards a lower mortality set of l_x's, the median age of survival rises, often to age 80 or higher. As a result, β tends to increase as mortality declines, as has been observed in the more developed countries with low mortality standards over the past few decades.

Although data on child mortality are becoming increasingly available, reliable estimates of adult mortality are much less common. Considerable uncertainty remains as to current adult mortality levels, particularly in populations with high prevalence of HIV/AIDS. Using the methods described in this paper, we have developed a new set of life tables that take into account the best available data on child and adult mortality, while at the same time reflecting the different levels of uncertainty around each of these inputs, and across different countries. Ranges of uncertainty around the life tables were derived using simulation techniques, which allow the level of uncertainty around ${}_5q_0$ and ${}_{45}q_{15}$ to be translated easily into uncertainty around the ultimate quantities of interest. In the World Health Report 2000, we report both the point estimates and the uncertainty intervals for a variety of measures computed from the life tables, including life expectancy at birth and disability-adjusted life expectancy, as described elsewhere. We believe strongly that communicating this uncertainty is as critical as communicating point estimates. Examining the level of uncertainty in each country helps to highlight the major challenges for demographic estimation and to identify priorites for accelerating survey programmes and sentinel surveillance in developing countries.

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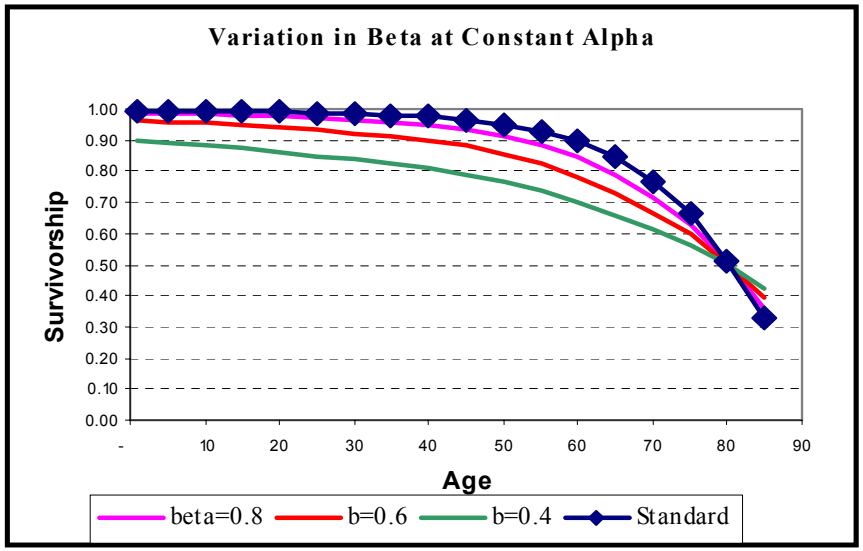
Table 1. Logit Values of the Brass General Standard Life Table

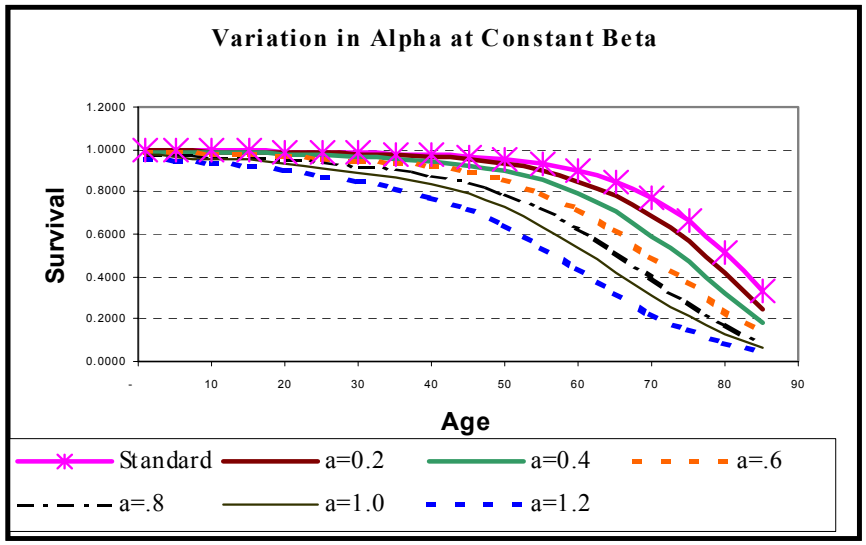
Age (x)	l_x	Logit Value	Age (x)	l_x	Logit Value	Age (x)	l_x	Logit Value	Age (x)	l_x	Logit Value	Age (x)	l_x	Logit Value
0	1.000													
1	0.850	-0.867	**21**	0.707	-0.440	**41**	0.583	-0.167	**61**	0.383	0.2394	**81**	0.065	1.330
2	0.807	-0.715	**22**	0.700	-0.425	**42**	0.576	-0.153	**62**	0.368	0.2701	**82**	0.054	1.428
3	0.788	-0.655	**23**	0.694	-0.410	**43**	0.569	-0.138	**63**	0.345	0.3204	**83**	0.044	1.535
4	0.776	-0.622	**24**	0.688	-0.396	**44**	0.561	-0.123	**64**	0.338	0.3364	**84**	0.036	1.649
5	0.769	-0.602	**25**	0.683	-0.383	**45**	0.553	-0.107	**65**	0.322	0.3721	**85**	0.028	1.772
6	0.764	-0.588	**26**	0.676	-0.369	**46**	0.545	-0.091	**66**	0.306	0.4097	**86**	0.022	1.905
7	0.760	-0.577	**27**	0.670	-0.355	**47**	0.537	-0.075	**67**	0.289	0.4494	**87**	0.016	2.049
8	0.756	-0.567	**28**	0.664	-0.341	**48**	0.529	-0.057	**68**	0.272	0.4912	**88**	0.012	2.205
9	0.753	-0.558	**29**	0.658	-0.328	**49**	0.520	-0.040	**69**	0.255	0.5353	**89**	0.009	2.374
10	0.750	-0.550	**30**	0.652	-0.315	**50**	0.511	-0.021	**70**	0.238	0.5818	**90**	0.006	2.557
11	0.748	-0.543	**31**	0.647	-0.302	**51**	0.501	-0.002	**71**	0.221	0.6311	**91**	0.004	2.756
12	0.745	-0.537	**32**	0.641	-0.289	**52**	0.491	0.018	**72**	0.203	0.6832	**92**	0.003	2.973
13	0.743	-0.530	**33**	0.635	-0.276	**53**	0.481	0.038	**73**	0.186	0.7385	**93**	0.002	3.208
14	0.740	-0.522	**34**	0.628	-0.263	**54**	0.470	0.060	**74**	0.169	0.7971	**94**	0.001	3.464
15	0.736	-0.513	**35**	0.622	-0.250	**55**	0.459	0.082	**75**	0.152	0.8593	**95**	0.001	3.742
16	0.733	-0.504	**36**	0.616	-0.236	**56**	0.447	0.106	**76**	0.136	0.9255	**96**	0.000	4.046
17	0.729	-0.494	**37**	0.610	-0.223	**57**	0.435	0.130	**77**	0.120	0.9960	**97**	0.000	4.376
18	0.724	-0.482	**38**	0.603	-0.209	**58**	0.423	0.155	**78**	0.105	1.0712	**98**	0.000	4.735
19	0.719	-0.469	**39**	0.597	-0.196	**59**	0.410	0.182	**79**	0.091	1.1516	**99**	0.000	5.127
20	0.713	-0.455	**40**	0.590	-0.182	**60**	0.397	0.210	**80**	0.078	1.2375			

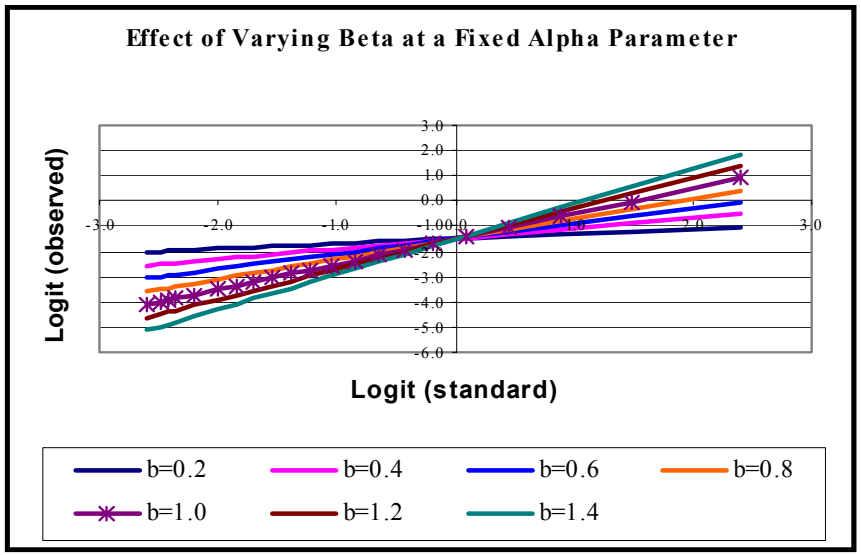
Table 2. Values of Beta for Various Values of Under-five Mortality Rate (5Q0)

	Alpha																
5q0	-0.3	-0.25	-0.2	-0.15	-0.1	-0.05	0.00	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.50
0.005	1.651	1.686	1.721	1.756	1.791	1.827	1.862	1.897	1.932	1.967	2.002	2.038	2.073	2.108	2.143	2.178	2.214
0.010	1.405	1.440	1.476	1.511	1.546	1.581	1.616	1.651	1.687	1.722	1.757	1.792	1.827	1.862	1.898	1.933	1.968
0.015	1.261	1.296	1.331	1.366	1.401	1.437	1.472	1.507	1.542	1.577	1.613	1.648	1.683	1.718	1.753	1.788	1.824
0.020	1.158	1.193	1.228	1.263	1.299	1.334	1.369	1.404	1.439	1.474	1.510	1.545	1.580	1.615	1.650	1.685	1.721
0.025	1.078	1.113	1.148	1.183	1.218	1.253	1.289	1.324	1.359	1.394	1.429	1.464	1.500	1.535	1.570	1.605	1.640
0.030	1.012	1.047	1.082	1.117	1.152	1.187	1.223	1.258	1.293	1.328	1.363	1.399	1.434	1.469	1.504	1.539	1.574
0.035	0.956	0.991	1.026	1.061	1.096	1.131	1.167	1.202	1.237	1.272	1.307	1.342	1.378	1.413	1.448	1.483	1.518
0.040	0.907	0.942	0.977	1.012	1.047	1.083	1.118	1.153	1.188	1.223	1.258	1.294	1.329	1.364	1.399	1.434	1.470
0.045	0.864	0.899	0.934	0.969	1.004	1.039	1.075	1.110	1.145	1.180	1.215	1.250	1.286	1.321	1.356	1.391	1.426
0.050	0.825	0.860	0.895	0.930	0.965	1.000	1.036	1.071	1.106	1.141	1.176	1.212	1.247	1.282	1.317	1.352	1.387
0.055	0.789	0.824	0.860	0.895	0.930	0.965	1.000	1.035	1.071	1.106	1.141	1.176	1.211	1.246	1.282	1.317	1.352
0.060	0.757	0.792	0.827	0.862	0.897	0.933	0.968	1.003	1.038	1.073	1.108	1.144	1.179	1.214	1.249	1.284	1.320
0.065	0.727	0.762	0.797	0.832	0.867	0.903	0.938	0.973	1.008	1.043	1.078	1.114	1.149	1.184	1.219	1.254	1.289

Table 3: Values of *Alpha* for corresponding *Beta* and $_{45}q_{15}$								
	45q15							
Beta	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
0.1	0.356							
0.2	-0.203	0.532						
0.3	-0.385	0.169	0.730					
0.4	-0.469	0.026	0.441	0.962				
0.5	-0.509	-0.043	0.319	0.696	1.261			
0.6	-0.524	-0.074	0.260	0.581	0.971	1.741		
0.7	-0.522	-0.082	0.234	0.526	0.848	1.310		
0.8	-0.508	-0.076	0.229	0.502	0.789	1.151	1.866	
0.9	-0.486	-0.060	0.238	0.498	0.763	1.076	1.562	
1.0	-0.458	-0.036	0.256	0.508	0.758	1.041	1.435	2.575
1.1	-0.426	-0.007	0.281	0.526	0.766	1.030	1.374	2.049
1.2	-0.389	0.027	0.311	0.552	0.784	1.035	1.347	1.873
1.3	-0.350	0.064	0.346	0.583	0.809	1.050	1.341	1.790
1.4	-0.309	0.104	0.383	0.618	0.840	1.073	1.349	1.751
1.5	-0.266	0.146	0.424	0.656	0.874	1.102	1.366	1.738
1.6	-0.221	0.190	0.466	0.696	0.912	1.135	1.391	1.740
1.7	-0.175	0.235	0.510	0.739	0.952	1.172	1.421	1.754
1.8	-0.127	0.282	0.556	0.783	0.995	1.211	1.456	1.776
1.9	-0.079	0.329	0.602	0.829	1.039	1.253	1.493	1.804







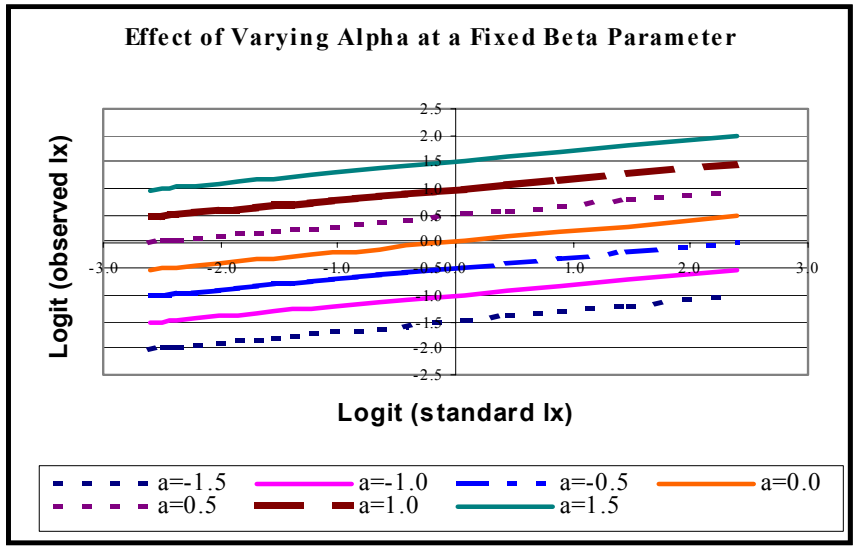


Figure 1. Effect of Changing Alpha and Beta on Pattern of Observed Mortality Relative to Standard

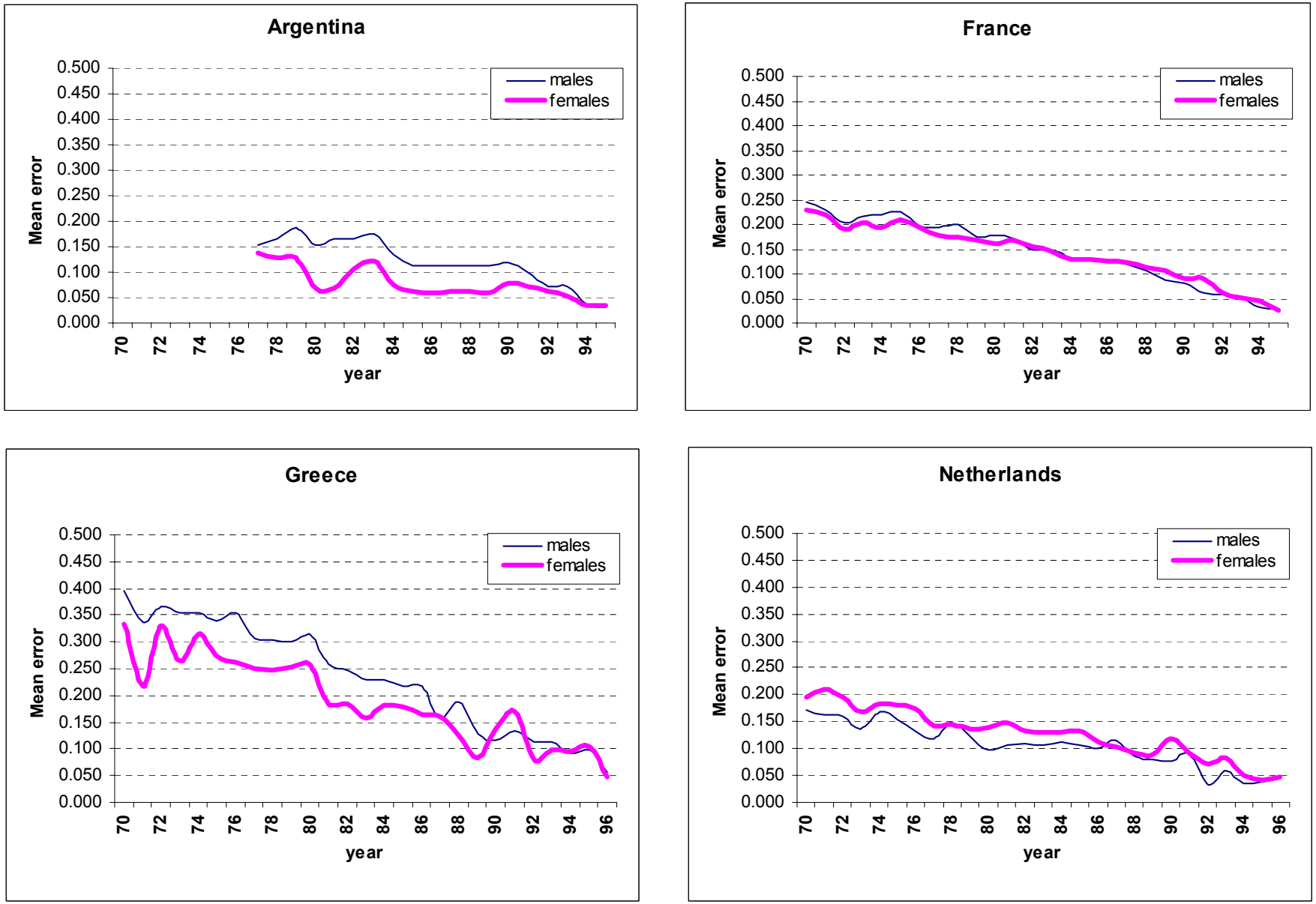
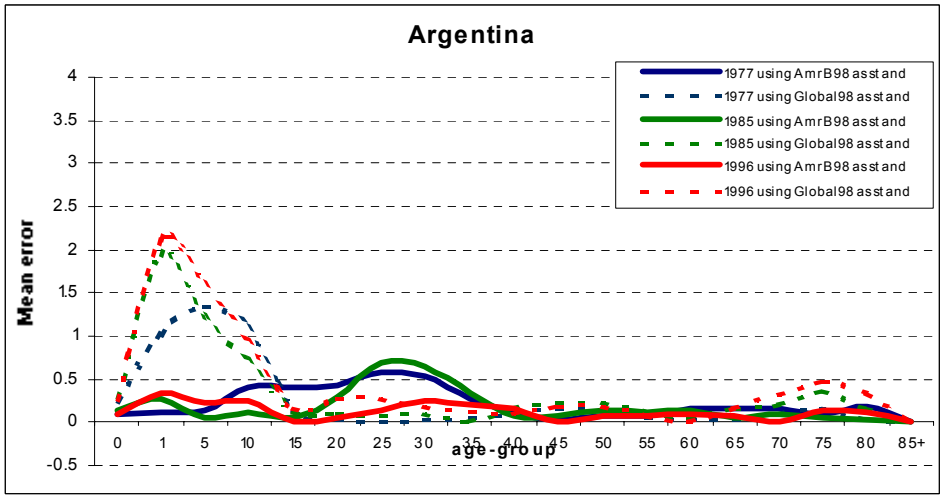
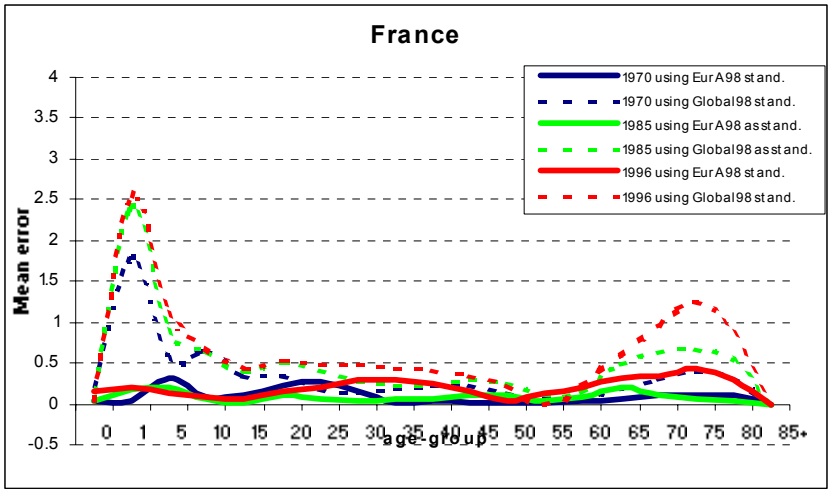
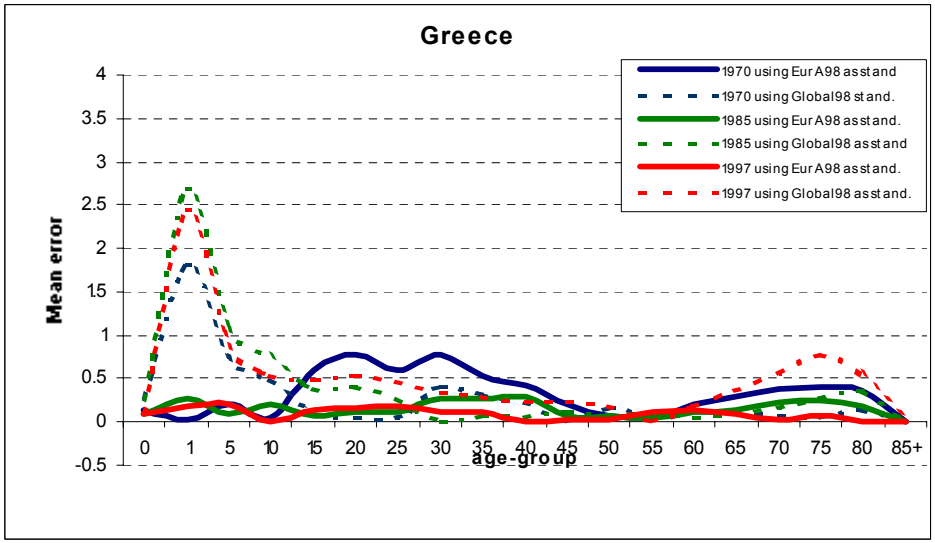


Figure 2. Brass Logit System: Mean Absolute Prediction Error Using Country-Specific Standards







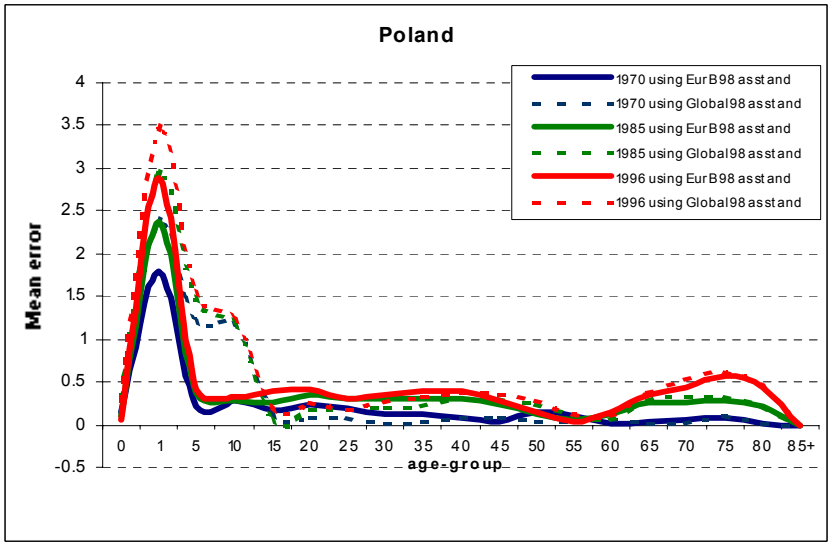
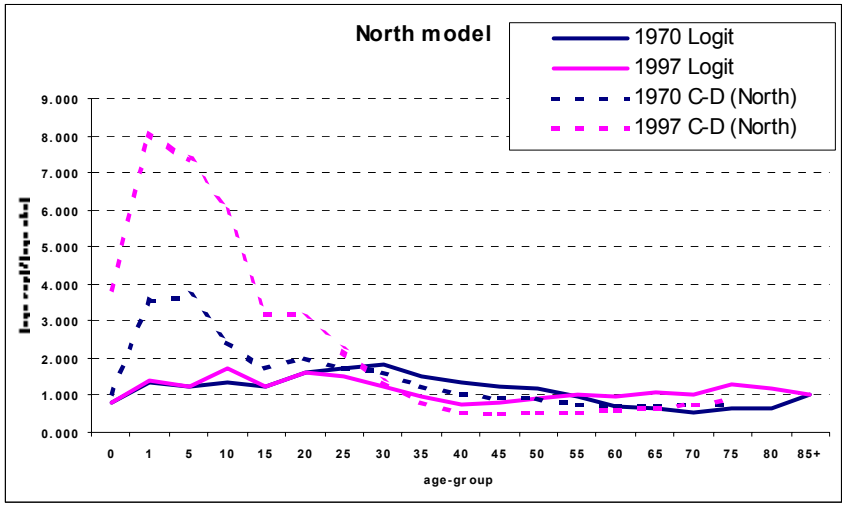
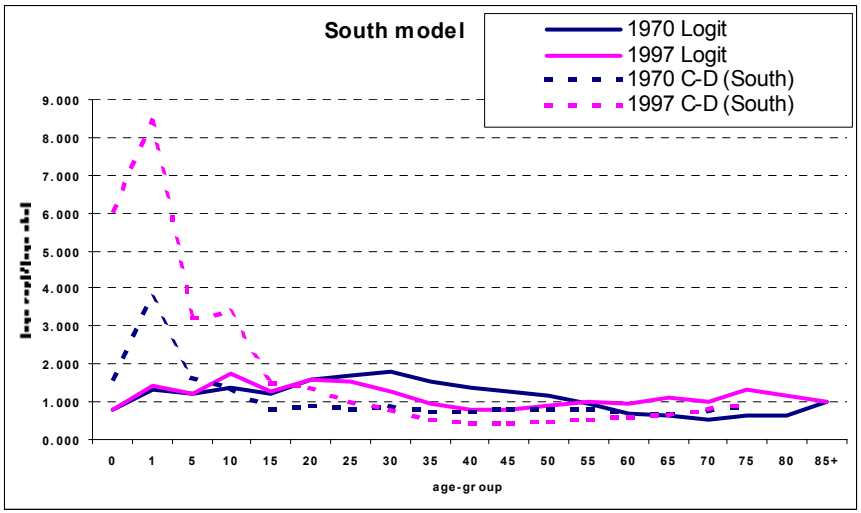
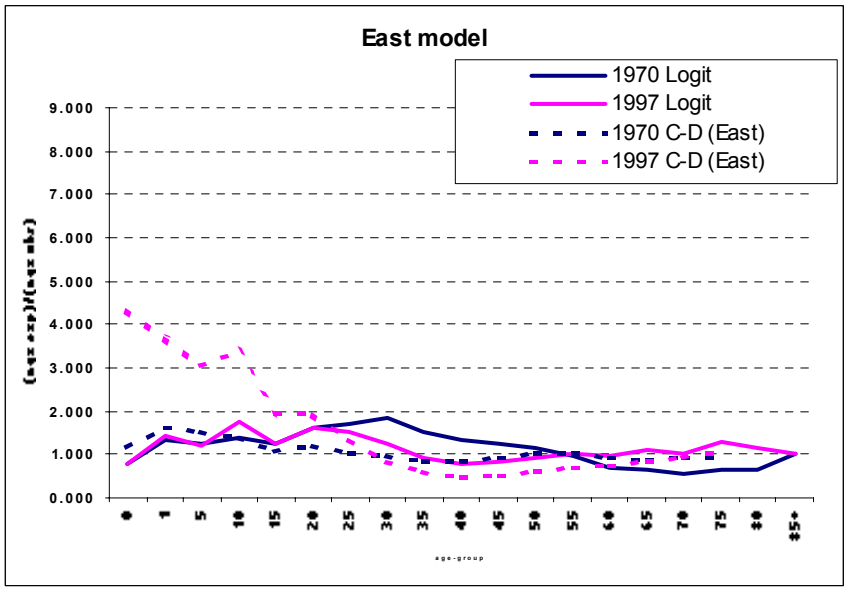


Figure 3. Brass Logit System: Mean Prediction Error in nqx - WHO Regional Standard Versus a Global Standard







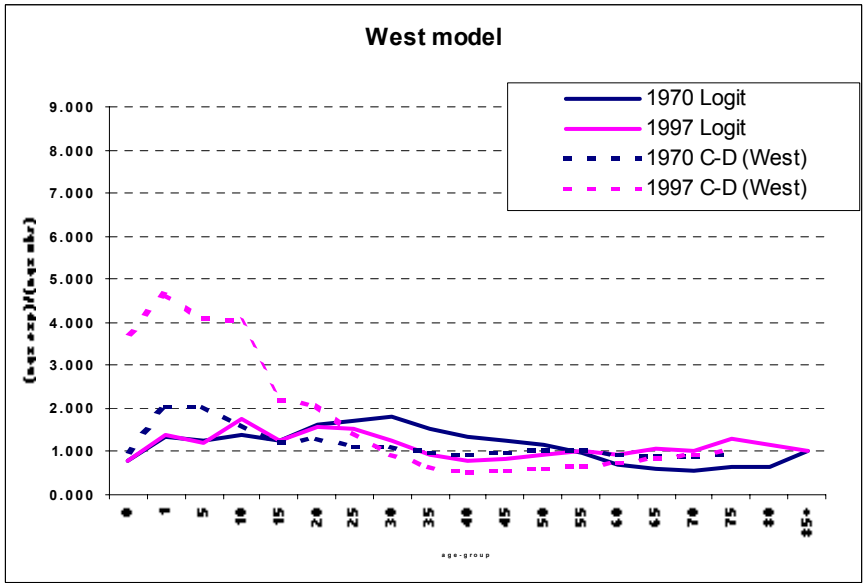
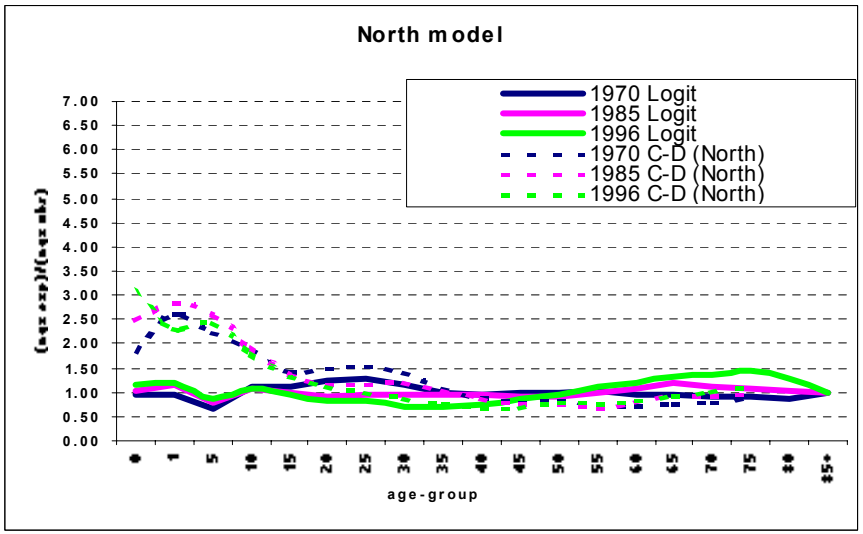
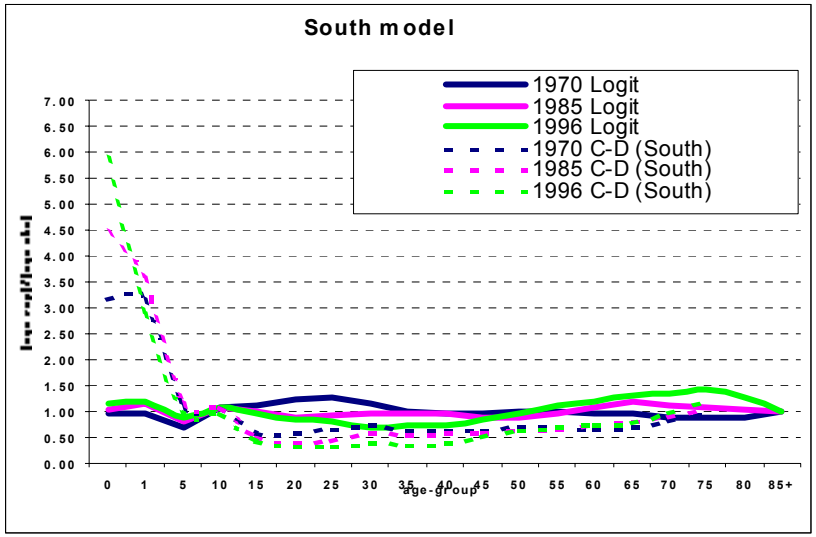
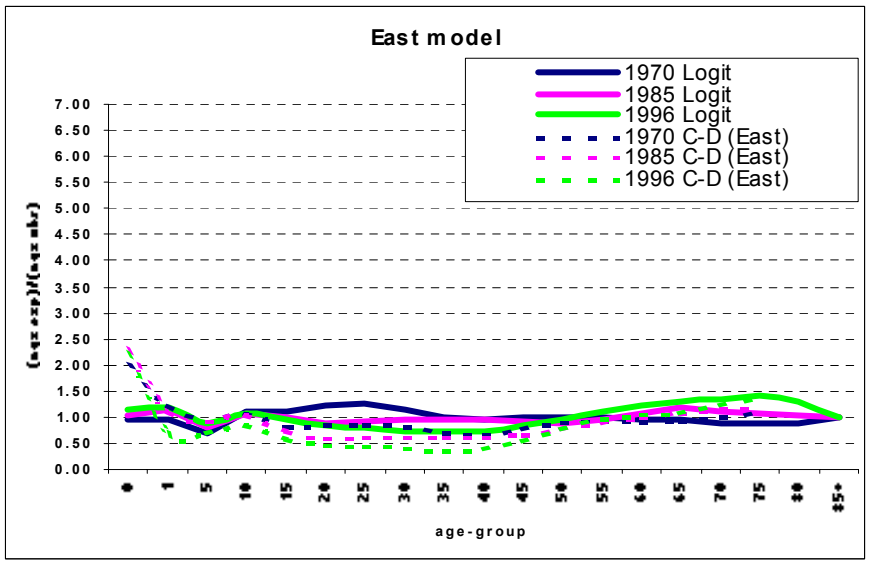


Figure 4. Index of Fit - Brass Logit and the Coale-Demeny Models - Hungary - Males, by Age and Time Period - (Eur C Standard)







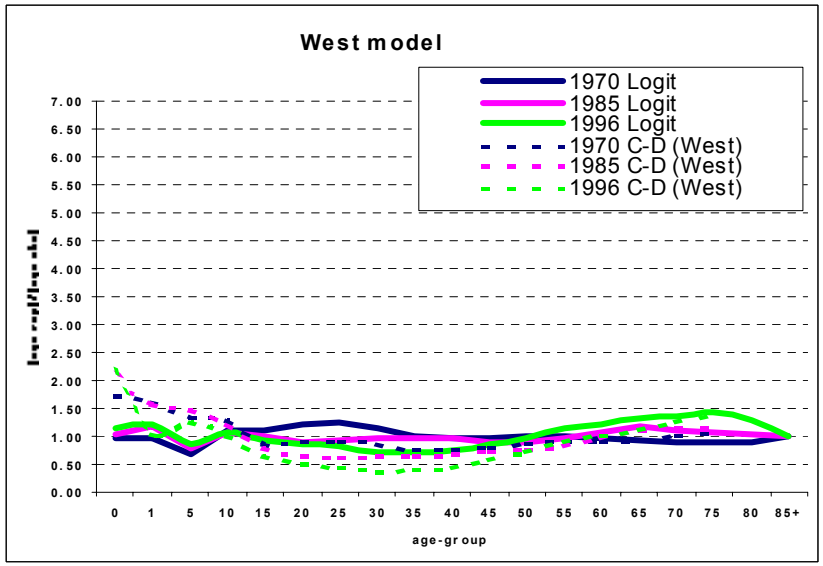
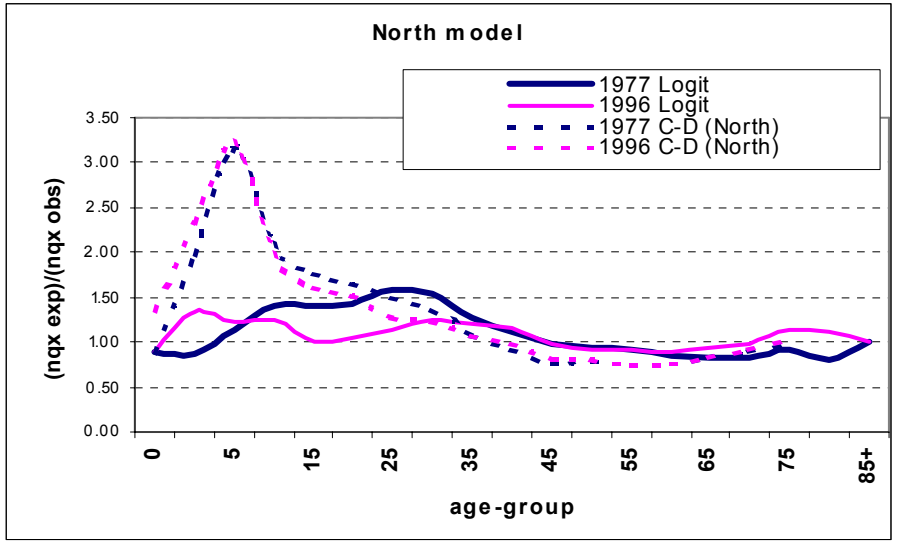
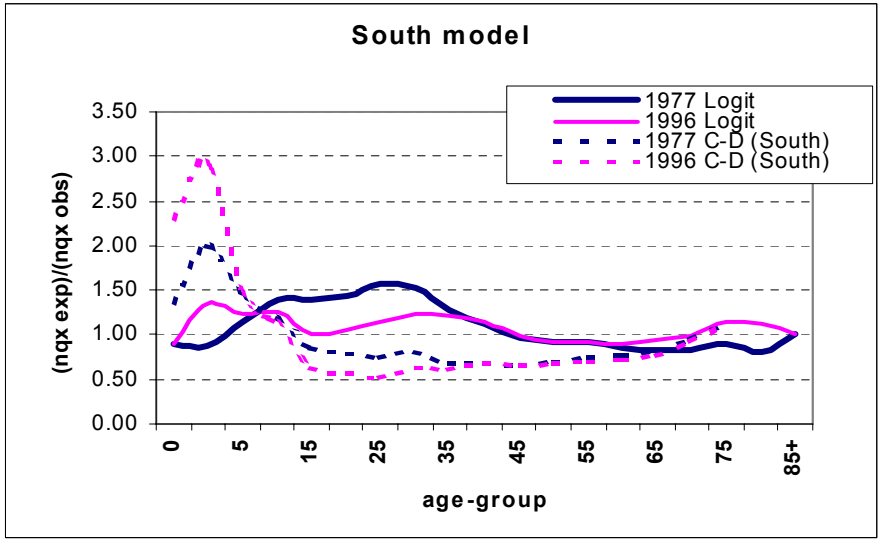
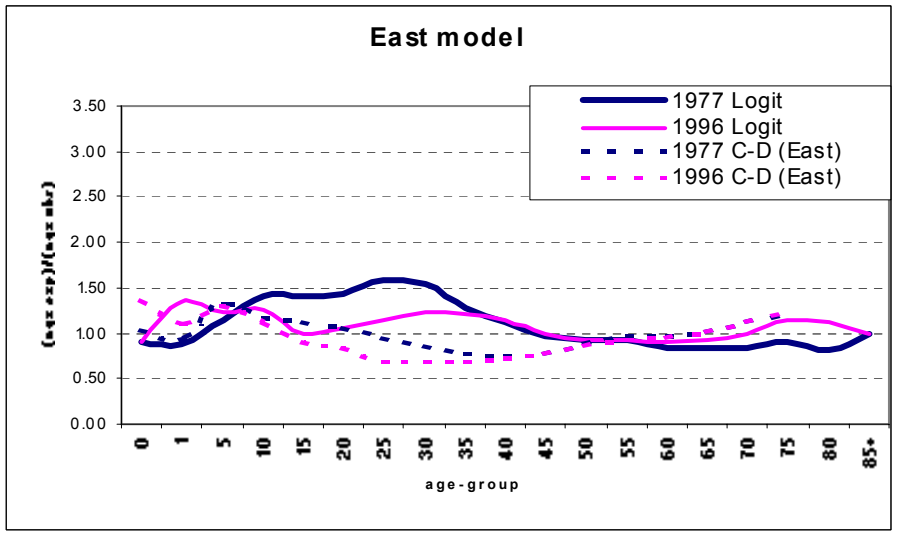


Figure 5. Index od Fit - Brass Logit and the Coale-Demeny Models - France - Males, by Age and Time Period - (Eur A Standard)







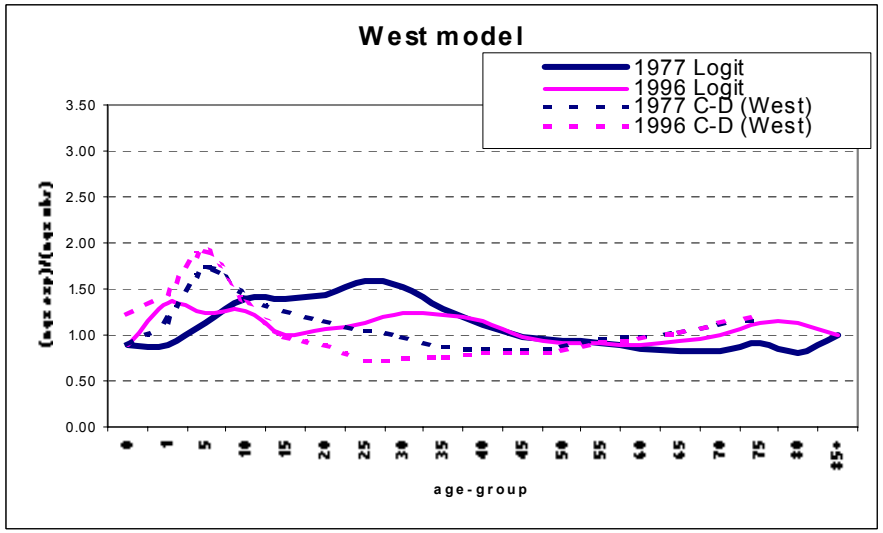


Figure 6. Index of Fit - Brass Logit and the Coale-Demeny Models - Argentina - Males, by Age and Time Period - (Amr B Standard)

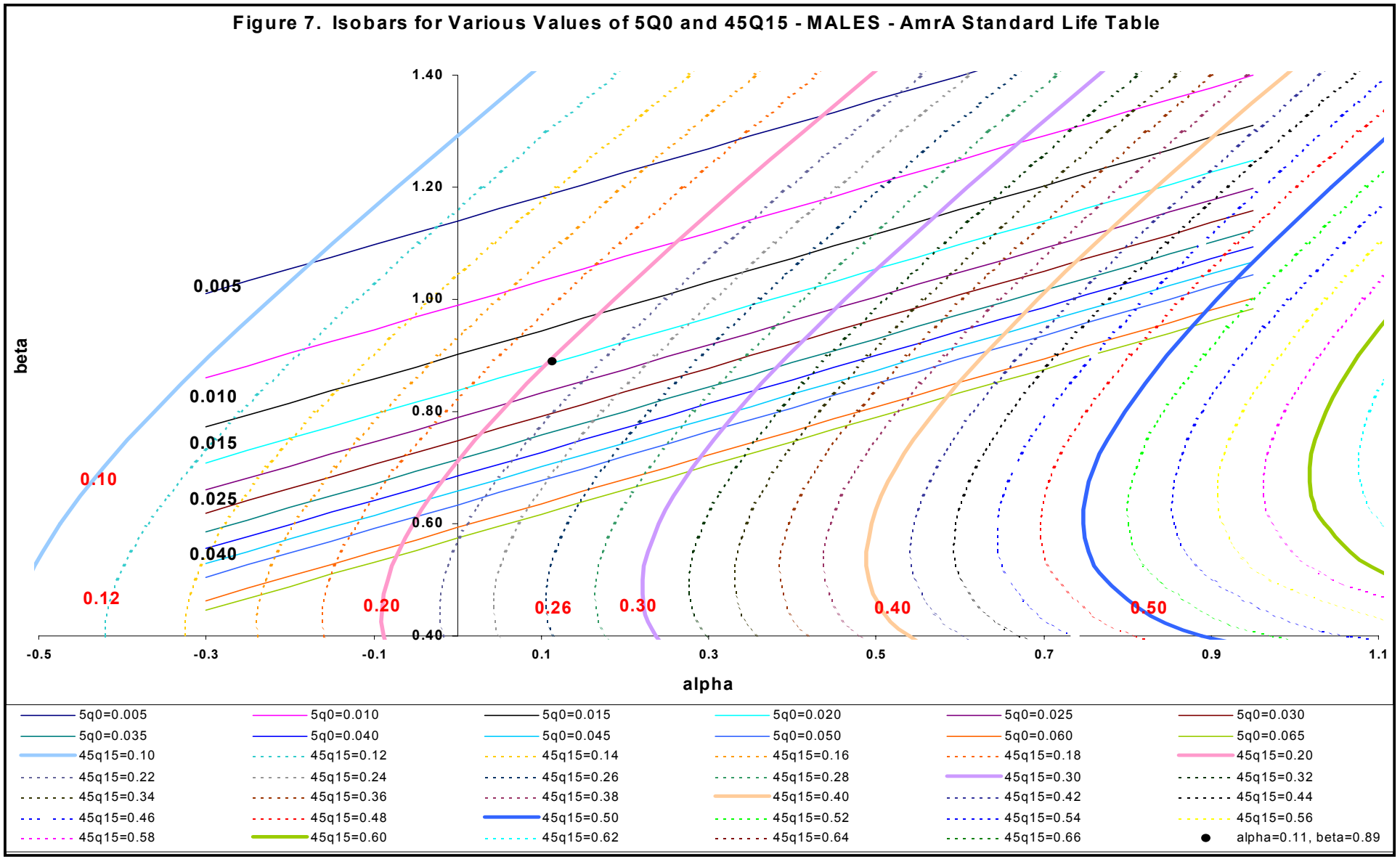


Figure 7. Alpha-Beta Grid

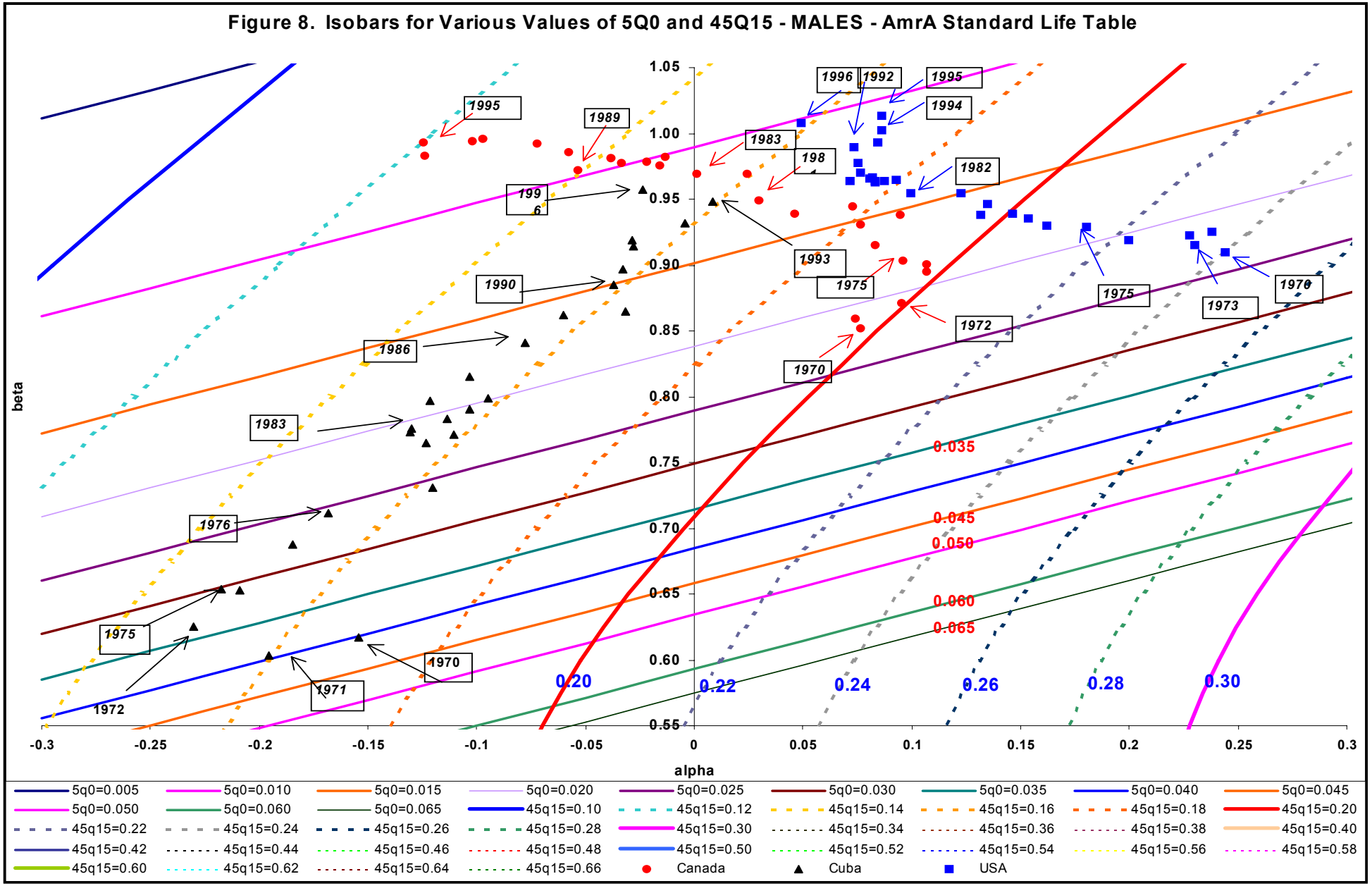


Figure 8. Alpha-Beta Grid Showing Trajectories of Selected Countries : Canada, Cuba and the USA.

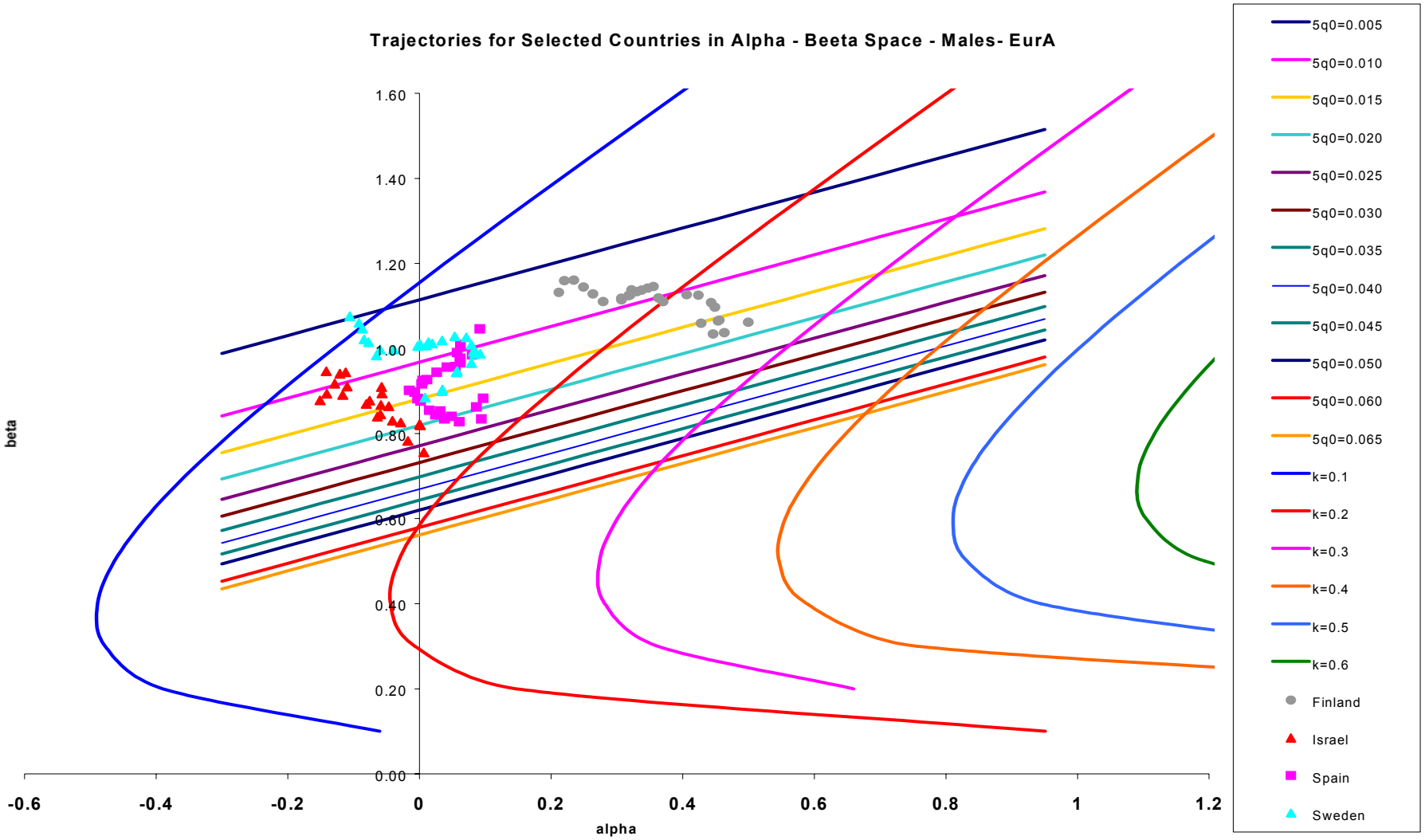


Figure 9. Alpha Beta Grid: Trajectories for Selected Countries Finland, Swden, Spain and Israel

Appendix A: Definitions of WHO Mortality Strata and Regional Standards

The regional mortality standards for the various WHO Regions and sub-regions were obtained as follows: Firstly, the composition of WHO Regions is based primarily on socio-cultural and geographical factors and can therefore include countries with very different levels and patterns of mortality. For example, the WHO Region of the Americas groups countries of North America, with extremely low mortality levels, with Haiti, Peru and Bolivia, where death rates are much higher. In order to better differentiate countries within each WHO Region according to characteristics of their mortality patterns, countries were grouped on the basis of an indicator of child mortality (${}_5q_0$) and of adult mortality (${}_{45}q_{15}$). Quintiles of ${}_5$qo values were identified based on the analysis of Ahmad et al and three categories were identified from this distribution: countries in the bottom quintile (lowest ${}_5q_0$ values); countries in the 2nd and 3rd quintiles; and countries in the 4th and 5th quintiles. These were described as "very low child", "low child" and "high child" mortality respectively. For differentiating adult mortality, ${}_{45}q_{15}$ was first regressed on 5qo and in order to identify extreme values, countries falling more than one - standard deviation above the regression line were classified as either "high" or "very high" adult mortality, and countries below the line as "low" or "very low" adult mortality. These were labelled as follows:

Mortality stratum A	very low child, very low adult
Mortality stratum B	low child, low adult
Mortality stratum C	low child, high adult
Mortality stratum D	high child, high adult
Mortality stratum E	high child, very high adult

In this way, countries were cross-classified according to their mortality strata (A to E) and their WHO Region (Europe: *Eur*, the Americas: *Amr*, Africa: *Afr*, Western Pacific: *Wpr*, Eastern Mediterranean: *Emr* and SE Asia: *Sear*). Since not all strata are present in all Regions, the final number of analytical Regions is 14. The standards for these various WHO mortality strata are chosen as follows:

WHO African Region: mortality strata D and E. The life table for Sub-Saharan Africa for 1990 estimated for the World Development Report 1993 was used as the standard. This life table was largely uncontaminated by HIV/AIDS mortality and was based on fitting split-level Coale and Demeny models to estimates of child (and occasionally) adult mortality for countries in the Region.

WHO Western Pacific Region: mortality stratum E. The 1990 life table for China prepared for the WDR 1993 was used. Deaths reported in the previous 12 months from the 1990 census were adjusted for underreporting using standard demographic procedures.

WHO Eastern Mediterranean Region: mortality strata B and D. Mortality data from Egypt for 1991 and 1992 were averaged and adjusted for underreporting, particularly below age 5, based on a review of all demographic data sources on child mortality levels in Egypt.

WHO European Region: mortality stratum B : This standard was based on the unweighted aggregated mortality data for the following countries: Albania (1992-93), Armenia (1995-97), Azerbaijan (1995-97), Bulgaria (1996-98), Georgia (1988-90), Kyrgystan (1996-98), Poland (1994-96), Romania (1996-98), Slovakia (1993-95), Tajikistan (1990-92), Macedonia (1995-97), Turkmenistan (1992-94) and Uzbekistan (1991-93)

WHO South East Asia Region: mortality stratum B. Mortality data for Sri Lanka for 1991 and 1995 were averaged, and added to adjusted mortality data for Thailand (1988-90) and Malaysia (1974-76), using Growth-Balance to adjust for underreporting.

Mortality stratum D: Indian mortality data for 1995-97 from the Sample Registration Scheme was used for the standard, with adjustments above age 5 for underreporting (estimated at 13-14%, using the Bennet-Horiuchi technique).

WHO American Region: mortality stratum B. Based on average mortality rates for Argentina (1990-96), Bahamas (1993-95), Barbados (1993-95), Belize (1995), Brazil (1996) (with corrections for underreporting), Chile (1992-94), Columbia (1992-94) (corrected for underreporting), Costa Rica (1992-94), Jamaica (1983 and 1985), Mexico (1993-95), Panama (1985-87) (adjusted for underreporting), Trinidad and Tobago (1992-94), Uruguay (1988-90) and Venezuela (1992-94) (with adjustments for underreporting).

Mortality stratum D: historical data for Latin American countries at earlier (higher mortality) periods were used. These included: Antigua (1970-72),

Argentina (1969-70), Bahamas (1971-72), Barbados (1970-72), Chile (1970-72), Costa Rica (1970-72), El Salvador (1970-72), Dominican Rep. (1970-72), Mexico (1970-72), Panama (1970-72), Trinidad & Tobago (1970-72), Uruguay (1970-72), Venezuela (1970-72).

Appendix B - Derivation of the Equation for the Adult Mortality Isobars

$$Logit\ l_{15} = \alpha + \beta Logit\ l_{15}^{s} \ldots\ldots 1$$

$$Logit\ l_{60} = \alpha + \beta Logit\ l_{60}^{s} \ldots\ldots 2$$

$$k = {}_{45}q_{15} = \left(\frac{l_{15} - l_{60}}{l_{15}}\right) \ldots\ldots 3$$

$$\therefore\ l_{15}(1-k) = l_{60} \ldots\ldots 4$$

If $C_{15} = Logit\ l_{15}^{s}$ *and* $C_{60} = Logit\ l_{60}^{s}$, *then*

$$Logit\ l_{15} = \alpha + \beta C_{15} \ldots\ldots 5$$

$$Logit\ l_{60} = \alpha + \beta C_{60} \ldots\ldots 6$$

Substiituting equation 4 for l_{60} *in equation 6*

$$Logit\left(l_{15}(1-k)\right) = \alpha + \beta C_{60} \ldots\ldots 7$$

Expanding the Logits

$$0.5\quad ln\left(\frac{1-l_{15}}{l_{15}}\right) = \alpha + \beta C_{15} \ldots\ldots 8$$

$$0.5\quad ln\left[\frac{1-l_{15}(1-k)}{l_{15}(1-k)}\right] = \alpha + \beta C_{60} \ldots\ldots 9$$

$$ln\left(\frac{1-l_{15}}{l_{15}}\right) = 2\alpha + 2\beta C_{15} \ldots\ldots 10$$

$$ln\left[\frac{1-l_{15}(1-k)}{l_{15}(1-k)}\right] = 2\alpha + 2\beta C_{60} \ldots\ldots 11$$

Solving for l_{15}

$$\left(\frac{1-l_{15}}{l_{15}}\right) = e^{2\alpha+2\beta C_{15}} = e^{2\alpha} e2^{\beta C_{15}}$$

$$\therefore\ 1 - l_{15} = l_{15} e^{2\alpha} e^{2\beta C_{15}}$$

$$\therefore\ l_{15} = \frac{1}{\left(1 + e^{2\alpha} e^{2\beta C_{15}}\right)} \ldots\ldots 12$$

Substituting for l_{15} *inequation 11*

$$\left[\frac{1-l_{15}(1-k)}{l_{15}(1-k)}\right] = e^{2\alpha+2\beta C_{60}} = e^{2\alpha} e^{2\beta C_{60}}$$

$$\left[\frac{1-\dfrac{(1-k)}{\left(1+e^{2\alpha}e^{2\beta C_{15}}\right)}}{\dfrac{(1-k)}{\left(1+e^{2\alpha}e^{2\beta C_{15}}\right)}}\right] = \left[\frac{\dfrac{\left\{\left(1+e^{2\alpha}e^{2\beta C_{15}}\right)-(1-k)\right\}}{\left(1+e^{2\alpha}e^{2\beta C_{15}}\right)}}{\dfrac{(1-k)}{\left(1+e^{2\alpha}e^{2\beta C_{15}}\right)}}\right] = \left[\frac{\left\{\left(1+e^{2\alpha}e^{2\beta C_{15}}\right)-(1-k)\right\}}{(1-k)}\right] = e^{2\alpha}e^{2\beta C_{60}}$$

$$\therefore\ \left\{\left(1+e^{2\alpha}e^{2\beta C_{15}}\right)-(1-k)\right\} = (1-k)\left(e^{2\alpha}e^{2\beta C_{60}}\right)$$

$$\therefore\ \left(e^{2\alpha}e^{2\beta C_{15}}\right)+k = (1-k)\left(e^{2\alpha}e^{2\beta C_{60}}\right)$$

$$\therefore\ (1-k)\left(e^{2\alpha}e^{2\beta C_{60}}\right)-\left(e^{2\alpha}e^{2\beta C_{15}}\right) = k$$

$$\therefore\ e^{2\alpha}\left[(1-k)\left(e^{2\beta C_{60}}\right)-e^{2\beta C_{15}}\right] = k$$

$$\therefore\ \alpha = \frac{1}{2} ln\left[\frac{k}{(1-k)\left(e^{2\beta C_{60}}\right)-e^{2\beta C_{15}}}\right] \ldots\ldots 13$$

Appendix C: Derivation of the Equation for the Child Mortality Isobars.

$$logit(l_5) = \alpha + \beta logit(l_5^s)$$
$$\therefore \alpha = logit(l_5) - \beta logit(l_5^s)$$

Notes

[i] These tables were obtained by estimating the probability of dying between ages x and $x+5$ (${}_5q_x$), for males, females and both sexes combined, through a logarithmic regression equation of the following type:

$$\textit{for the one - parameter models:}$$
$$\ln({}_5q_x) = a_0(x) + a_1(x)\ln Q$$
$$\textit{for the two - parameter models:}$$
$$\ln({}_5q_x) = b_0(x) + b_1(x)\ln Q_1 + b_2(x)\ln Q_2$$

where Q, Q_1 and Q_2 are the independent variables used in each case, and $a_i(x)$ and $b_i(x)$ represent the estimated regression coefficients for the age group from x to T (Ledermann, 1969). In the single parameter models, seven independent variables were used, namely: ${}^{e^0}{}_0$, ${}_1q_0$, ${}_5q_0$, ${}_{15}q_0$, ${}_{20}q_{30}$, ${}_{20}q_{45}$ and m_{50+} (the central mortality rate for ages 50 and over). The two-parameter models were obtained by using the following pairs of independent variables: ${}_5q_0$ and ${}_{20}q_{45}$; ${}_{15}q_0$ and ${}_{20}q_{30}$; and ${}_{15}q_0$ and m_{50+}. Every parameter refers to both sexes combined, except for ${}_{20}q_{30}$, which refers only to females. The use of different independent variables to generate each set of model life tables makes it easier for the user to avoid the bias introduced when a model table is identified by way of an observed value that is not the independent variables used to generate the model. This type of bias affects the Coale-Demeny model life tables when a life table is identified on the basis of the observed l_2 value rather than on the basis of the observed e_0. A distinct feature of this system is the incorporation of a measure of the dispersion of the observed values around the estimated value (through the regression). This provides some indication of the possible magnitude of the discrepancies between estimated and actual values.