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Aiming at a Moving Target: Period Fertility and Changing Reproductive Goals*

R.D. LEE†

I. INTRODUCTION

Following the United States baby boom, period fertility rates began to decline steeply several years before reproductive goals showed any downward change. Furthermore, while desired complete fertility varied by about one child over the course of the cycle, period total fertility varied by twice this amount. Demographers have appealed to coincidental changes in timing and contraceptive failure or questioned the causal ordering of attitude and behaviour in an attempt to reconcile these apparently discrepant series. But in fact, demographic science lacks a theory relating fertility targets and period rates, and is, therefore, unable to distinguish between consistent and discrepant changes. Similarly, if we were given foreknowledge of desired complete fertility for each age group at each future date, we would not know how to exploit it efficiently for forecasting purposes. In this paper, I attempt to provide the rudiments of a theory, and use it to show that the behaviour of rates and goals in the United States during recent decades has been fully consistent, and that the apparent anomalies actually illustrate intrinsic characteristics of the relation between changes in fertility rates and targets.

I begin with a lengthy non-technical discussion of the theory and its implications, illustrated with examples from the baby boom and bust in the United States. This discussion is followed by a formal development of the theory. It is hoped that this somewhat illogical ordering of topics will make the paper more readable to those uninterested in the mathematical detail.

II. BACKGROUND AND ASSUMPTIONS

To the extent that a theory relating reproductive goals to period fertility rates exist, it is as follows: Each couple formulate at marriage a desired completed family size (D), and pursue this relatively constant target throughout their reproductive life.[1] Averaging across couples, a value of D may be obtained for the cohort which should be constant over time, as variations in individual values of D tend to cancel. We can then construct a typically-shaped schedule of age (or duration) specific rates which, multiplied by the value of D for the cohort, would describe its normal progression toward its target. Deviations of actual cohort fertility from this path can be defined as variations in timing or spacing.[2] Changes over time in period total fertility (F_t) then arise from the different values of D for the successive cohorts of prime reproductive age. Such changes, caused by changes in the value of D for different cohorts may be reinforced or attenuated by coincidental changes in timing and spacing. While changes in the value of D within a cohort may also play some part,they

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[1] See N. Ryder, 'A Critique of the National Fertility Survey,' *Demography*, **10**, (4) (November 1973), pp. 495–506.

[2] See N. Ryder, 'Components of Temporal Variations in American Fertility,' in R.W.Hiorns (Ed). *Demographic Patterns in Developed Societies* London: Taylor & Francis, 1980

are an embarrassment to the theory rather than a central concern. This, I think, is a fair statement of the theory implicit or explicit in much demographic work. I will refer to it henceforth as the 'fixed-target' model.

The fixed-target model has resulted in much valuable fertility research at the aggregate and individual levels by providing a framework for data collection and analysis and a foundation for theory. However, its assumptions have been seriously challenged by recent figures which showed substantial change over time in the value of D for a given cohort,[3] and by theoretical recognition that sequential decision-making provides a more appealing framework.[4]

The theory to be developed here, which I will call the 'moving-target' model, stresses the demographic importance of changes over time in the values of D for individuals and cohorts. After all, a couple's desired family size at any point in their reproductive life can only be based on their forecast of the future socio-economic climate and their own future experiences; there is no reason to expect couples to be more successful in such forecasts than sociologists or economists, and fertility plans will frequently be revised with the benefit of new experience. To the extent that the same broad socio-economic forces influence all age groups at the same time, desired family sizes will all move together; but, of course, it is possible that values of D for members of individual age groups will be affected in different ways during a given period, particularly if incomes move differently in different groups.

In this paper I will adopt the extreme simplifying assumption that at any instant, values of D are *identical* in each age group, and that this common value varies systematically over time. This is diametrically opposed to the 'fixed-target' assumption, but I believe it reflects the actual patterns of change in the United States over the past several decades more closely. I want to stress that this assumption is not necessary; the analysis could be carried out by allowing each cohort its own pattern of change in D; however, the assumption seems to capture an essential feature of the baby boom/bust: across-all-ages synchronous variations in D and it greatly simplifies the analysis.

In Figure 1 various indicators of D by age and over time are shown for the United States. For the period after 1955, expected completed fertility is presented, chosen because of the length of the series, although for older women such expectations are increasingly contaminated by contraceptive failures and births no-longer-wanted.[5] Longitudinal data on changes over time, based on Deborah Freedman's study of a group of Detroit women are also shown; here it is the 'number of children wanted if life could be lived over again' that is plotted.[6] For the period before 1955, I have plotted an averaged version of Judith Blake's ideal family size data.[7]

Taken together, these figures suggest that the value of D rose over the post-war period, peaked between 1960 and 1965, and declined rapidly thereafter, apparently stabilizing in the later 1970s. These changes appear roughly synchronous for all age groups, and the change over time

[3] For example, see C. Westoff and N. Ryder, 'The Predictive Validity of Reproductive Intentions,' *Demography*, **14**, (4)(November 1977), pp. 431–453. Also, early results from Deborah Freedman's 1977 re-interviews of Detroit women, first interviewed in 1962, show a decline between 1962 and 1977 of 0.75 children per woman wanted if life could be lived over again (personal communication from Ronald Freedman). A recent publication by M. Moore and M. O'Connell points out that U.S. women interviewed in 1960 predicted their actual cumulated fertility of 1976 almost exactly, but these authors were unable to take into account the substantial role that unwanted fertility must have played in bringing about this near-equality (*'Perspectives on American Fertility*,' U.S. Bureau of the Census, Special Studies Series P-23 No. 70, July 1978).

[4] See, for example, K. Namboodiri, 'Some Observations on the Economic Framework for Fertility Analysis,' *Population Studies*, **26** (1972), pp. 185–206.

[5] I suspect that the strong peaks for some ages in 1965 are artifacts due to a different form of the question asked in that year, but I am not sure of this.

[6] The sample of women interviewed all had a first, second or fourth birth in 1961. Their desired family sizes are higher than the national average for women at these parities because the proportion of Roman Catholics was higher.

[7] See Judith Blake, 'Ideal Family Size Among White Americans: A Quarter of a Century's Evidence,' *Demography*, (3) 1966, pp. 25–44, and 'Can We Believe Recent Data on Birth Expectations in the United States?' *Demography*, **11**, (1)(February 1974), pp. 25–44.

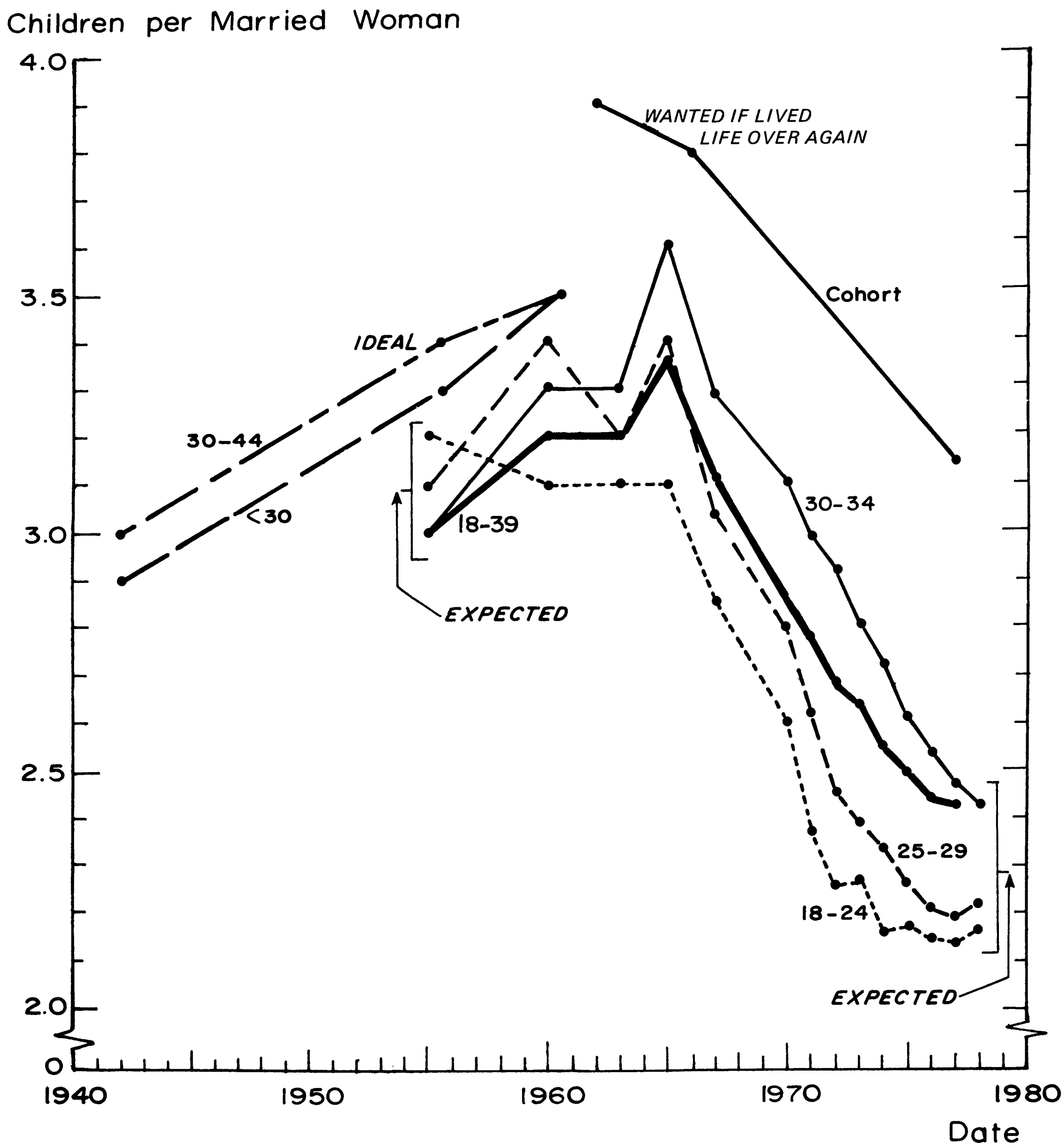

Figure 1. Measures of Fertility Goals for U.S. Women

Sources: The ideal family size series is calculated from data in Judith Blake, 'Ideal Family Size Among White Americans: A Quarter of a Century's Evidence.' *Demography,* 3 1966, pp. 25–44. The points are averages, weighted by sample size, for the 1930s and 1940s, the 1950s, and 1960–61. The longitudinal figures for Detroit women are, for 1977, preliminary results from Deborah Freedman's re-interview study, communicated to me, along with the figures for 1962 and 1966, by Ronald Freedman. The expectations data come from R. Freedman and L. Bumpass, 'Fertility Expectations in the U.S.: 1962–64,' *Population Index*, 32, (2)(April, 1966), pp. 181–197, for 1955 to 1962–64; from N. Ryder and C. Westoff, *Reproduction in the United States 1965*, (Princeton: Princeton University Press, 1971) for 1965; from U.S. Bureau of the Census, *Current Population Reports*, Series P-20, No. 325, 'Fertility of American Women: June 1977,' U.S.G.P.O., Washington D.C., 1978 for 1967 to 1977, and from no. 330 of the same series for 1978. The figures for 1955 relate to white women only.

in a group of Detroit women roughly matches the inter-cohort change in the expectations of young women.

The central behavioural assumption of the moving-target theory is that the period fertility rate of a cohort of women is closely related to their average *additional* desired fertility, that is to the gap between their desired family size and the number of their births to date. For the purposes of this paper, I shall assume a particularly simple form for this relationship: that at any age, *the annual birth rate is a fixed proportion of additional desired fertility.* In an earlier paper I presented empirical evidence that this was a good assumption for U.S. women aged 25 years and over. For younger women it is not quite correct; their birth rate is a linear function of additional desired fertility, but not a proportional one. However, it will capture the correct direction of change, which is what really matters in this context. It is possible to introduce more flexible assumptions, including changes in contraceptive failure rates, but it would then be necessary to resort to simulation to obtain results.[8]

Figure 2 is a scatter plot of aggregate marital fertility, corrected for unwanted births, against

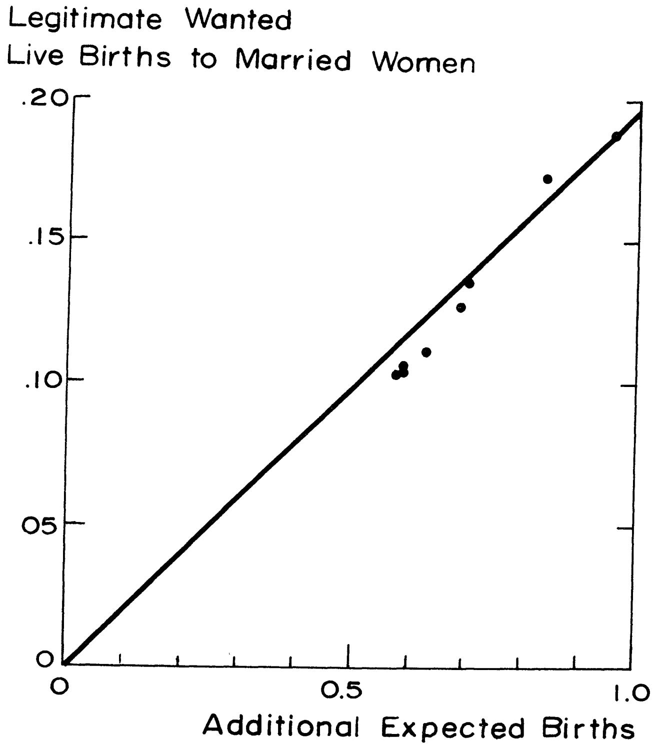

Figure 2. Period Marital Fertility and Additional Expected Births for U.S. Women, Selected Years, 1955–1975
Sources: The years are 1955, 1960, 1962–64, 1967, 1971, 1972, 1973, 1974 for Additional Expected Births, and one year later for marital fertility. The fertility expectation figures relate to women aged 18 to 39, and come from the sources listed for Figure 1. The 1955 figure is for white women only. Legitimate Wanted Live Births to Married Women were calculated in two steps. First, legitimate live births per married woman aged 15–44 were multiplied by 1.36 = (45–15)/(40–18) to obtain an approximation for the ages 18–39. Then, the contraceptive failure rate, calculated as the proportion of terminators aged 18–39 in each year multiplied by the failure rate given in N. Ryder, *loc. cit.* in footnote 9, p. 449, was subtracted.

[8] The more general model is developed in R. Lee, 'Target Fertility, Contraception and Aggregate Rates: Toward a Formal Synthesis,' *Demography*, **14**, (4)(November 1977). The notation in the cited paper will be found to differ from that used here.

additional expected births as surveyed in the preceding year.[9] The necessary data are available for eight of the years between 1955 and 1975. The points lie reasonably close to the straight line, corresponding to a fixed proportion of 0.18. That is, along that line aggregate annual marital fertility rates would equal 18 per cent of additional expected births. This is the constant of proportionality I shall use throughout the remainder of this paper. Note that this assumption implies that additional expected fertility will in an individual cohort with an unchanging target decline at an exponential rate of 0.18 per year (proportional to $e^{-0.18t}$), while cumulative fertility increases in proportion to $(1 - e^{-0.18t})$. This is a fairly accurate description of cohort fertility, although it overstates changes at earlier ages or marriage durations.[10]

In what follows, I make one additional assumption purely for convenience: I assume that every woman marries, and that all women marry at the same age, which is invariant over time. Under these circumstances period total fertility (F_t) and total marital fertility (F_{mt}) are equal, and I will use these terms interchangeably as summaries of period fertility rates.[11] Likewise, fertility measures by age and marriage duration are equivalent, and no distinction need be made between them. In fact, it has been shown that changes in marital status accounted for only 16 per cent of the decline in U.S. fertility from 1961 to 1975; changes in marital fertility accounted for 83 per cent of the decline; and changes in non-marital fertility had a negligible effect.[12]

III. IMPLICATIONS OF THE THEORY: NINE PROPOSITIONS

Given these two basic assumptions, a number of interesting propositions can be derived. These are listed below.

1. When D is constant, $F_t = D$. When D is rising, $F_t > D$. When D is falling, $F_t < D$.

Suppose D remains constant over a long period of time. Barring infecundity, extra-marital births and contraceptive failure, and assuming all women marry sufficiently young to achieve their desired family size, then $F_t = D$. Now suppose that D suddenly increases by one child per woman for women of all ages. If F_t were also to increase by one child, then older couples would never attain D because their earlier childbearing had been geared to a lower target. Their rate of childbearing must increase at every age to overcome this cumulated deficiency and, therefore, for a time $F_t > D$, although as new cohorts replace the old, it will subside to equality once again. More generally, when D is rising, members of each cohort will be behind schedule in the sense that a smaller proportion of D will have been attained at each age than if the current value of D had always been the target. Similarly, when D is falling, couples will find themselves *ahead* of schedule, with more than the usual proportion of D attained at each age.

This point may be illustrated by a comparison of fertility in 1955/56 with 1967/68, shown in Table 1. The interpretation of these numbers is muddied by contraceptive failure and births no longer wanted (a concept discussed later in this paper), but they are, nonetheless, suggestive. Comparing 1955 and 1967, we observe that total expected fertility was actually slightly higher at

[9] The correction for contraceptive failure was made by multiplying the proportion of women aged 18 to 39 who had terminated childbearing by the unwanted birth rate given in N. Ryder, 'A Model of Fertility by Planning Status,' *Demography*, 15 (4) November 1978, p. 449. This was subtracted from the number of legitimate live births per married women aged 15 to 44, multiplied by 1.36 = (45–15)/(40–18).

[10] See Lee, *loc. cit.* in footnote 8, pp. 464–465. For marriage durations between 5 and 15 years, the implied pattern of cumulative fertility is virtually indistinguishable from a Gompertz curve, which is often used to fit cumulative marital fertility schedules.

[11] Neither is satisfactory as an empirical representation of the period fertility flows I am modelling; F_t is unadjusted for change in marital status, and F_{mt} is unadjusted for the numbers of married women at different ages. Ideally, I would want to use a measure which aggregated marital fertility over durations.

[12] See C. Gibson, 'The U.S. Fertility Decline, 1961–1975: The Contribution of Changes in Marital Status and Marital Fertility,' *Family Planning Perspectives*, 8, (5)(Sept./Oct., 1976), pp. 249–252.

Table 1. *Comparative Fertility for U.S. White Married Women in 1955/56 and 1967/68*

	Period Total Fertility	Total Expected Births	Additional Expected Births	Additional Proportion Remaining
1956	3.504	3.0	0.96	0.32
1968	2.341	3.1	0.72	0.23

Sources: Period Total Fertility is taken from U.S. Public Health Service, National Center for Health Statistics, *Vital Statistics of the United States 1975 vol. 1,* United States Government Printing Office, 1978. Total and Additional Expected Births are taken from the sources for Figure 1, and relate to the *preceding* year. The Additional Proportion Remaining is Additional Expected divided by Total Expected.

the later date, but that in 1955, when desired family size had been rising, couples found themselves much farther behind schedule, with 32 per cent of their childbearing remaining, compared with only 23 per cent in 1967 when D had dropped slightly from a high of 3.2. This factor alone could explain a decline of 25 per cent in F_t between the two periods; in fact the value fell by 33 per cent, while marital fertility of women aged 15 to 44 fell by 29 per cent, or by 31 per cent if we correct for contraceptive failure.[13] Note also that in 1956 F_t was substantially above D, while in 1968 it was substantially below it.

2. When D fluctuates, F_t will fluctuate with greater amplitude.

This follows immediately from the first proposition, since if F_t is above a rising D, and below a falling one, it must fluctuate more widely. This clearly happened in the past in the United States when D varied between approximately 2.1 and 3.1 (as measured by the total expected fertility of the youngest age group), while F_t reached a peak of 3.7 in 1957, and a trough of 1.8 in the mid to late 1970s.

3. F_t responds more sensitively to a rising D than to a falling one, because of the irreversibility of fertility.

When D is rising, fecund couples who had previously expected to have ended childbearing can change their minds and proceed to have an additional birth. When D is falling, only those who have not yet completed childbearing can reduce their fertility; those who had already ceased childbearing may merely revise their D downward, but this only leads to an accumulation of children that are in a sense no longer wanted, or which would not have been wanted when they were born if their parents had had perfect foresight.[14] John Knodel has pointed out to me that the increasing use of sterilization as contraception for terminators will make fertility irreversible on the upswing as well as the downswing in D.

4. When D fluctuates, turning points in F_t may precede those in D by as much as five years.

This is the most striking finding which runs counter to intuition, but it is quite robust, and does

[13] This is based on the rates for 1956 and 1968, since there is a lag of about one year between the decision to have a birth and its occurrence. This is similar to the explanation for fertility decline in the 1960s given by R. Freedman and L. Bumpass, 'Fertility Expectations in the U.S.: 1962–64,' *Population Index*, **32**, (2)(April, 1966), pp. 181–197.

[14] This phenomenon, which I have analyzed elsewhere (R. Lee, 'Fluctuations in U.S. Fertility, Age Structure and Income,' Contract Report to NICHD HD-72857, 1977), is due to the irreversibility of fertility, and is clearly evident in Deborah Freedman's statistics on Detroit women. For example, in 1962, ten per cent of the women who had borne four children would have liked to have had fewer; by 1977, 25 per cent of these same women would have liked to have had fewer, a net increase of 15 per cent. For women with two children in 1962, the net increase was ten per cent. My work showed that the cumulation of births no longer wanted by the 1970s was comparable in size to the cumulation of unwanted births caused by contraceptive failure.

not depend on any detailed aspect of the assumptions. The crucial point is that F_t depends on the difference between D and the number of children women have already borne. If we follow an imaginary age group of women over time, we observe that the cumulated fertility of women at a given age is increasing at the rate at which D had been rising in recent years. If D begins to rise more slowly, cumulated fertility will continue to rise rapidly, and *additional* desired fertility will begin to decline, causing a decline in F_t. For this reason, F_t begins to fall when the rise in D begins to slow down; therefore, turning points in F_t precede those in D.

In Figure 3 values of F_t are plotted for the years 1934 to 1978, and a measure of D (total expected fertility) for 1955 to 1978. It shows clearly that F_t began to decline a number of years before D, although the precise dating of turning points is impossible in either series.[15] It also shows clearly the relative magnitude of the swings in D and F_t, and illustrates once again that F_t lies above D when D is rising and below D when D is falling.

In Figure 4 measures of wanted marital fertility, total expected fertility and additional expected fertility are shown for eight years between 1955 and 1975 for which information was available. While D peaked some time between 1960 and 1965, additional desired fertility declined over the entire period, and corresponded closely to decling marital fertility lagged by one year.

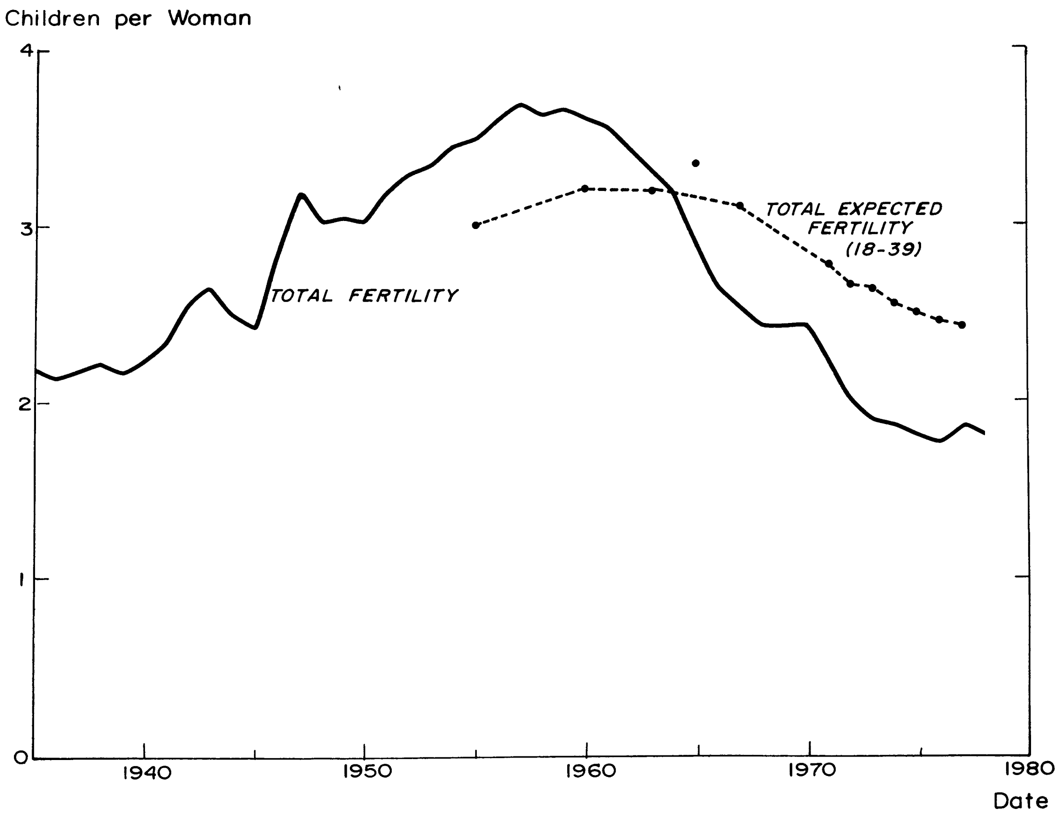

Figure 3. Expected Completed Fertility and Period Total Fertility for U.S. Women

Sources: Expected Completed Fertility is taken from the sources listed for Figure 1, and is for married women aged 18 to 39; the 1955 figure is for white women only. The point for 1965 is shown, but the line is not drawn through it due to suspected non-compatibility. The Period Total Fertility up to 1973 was taken from R. Heuser, *Fertility Tables for Birth Cohorts by Color; United States, 1917–1973* (Department of Health, Education and Welfare Publication no. HRA 76-1152, 1976), and updated from various government publications. It is for all women, regardless of marital status.

[15] Using five-year moving averages, the peak of F_t occurs in 1958, rather than 1957.

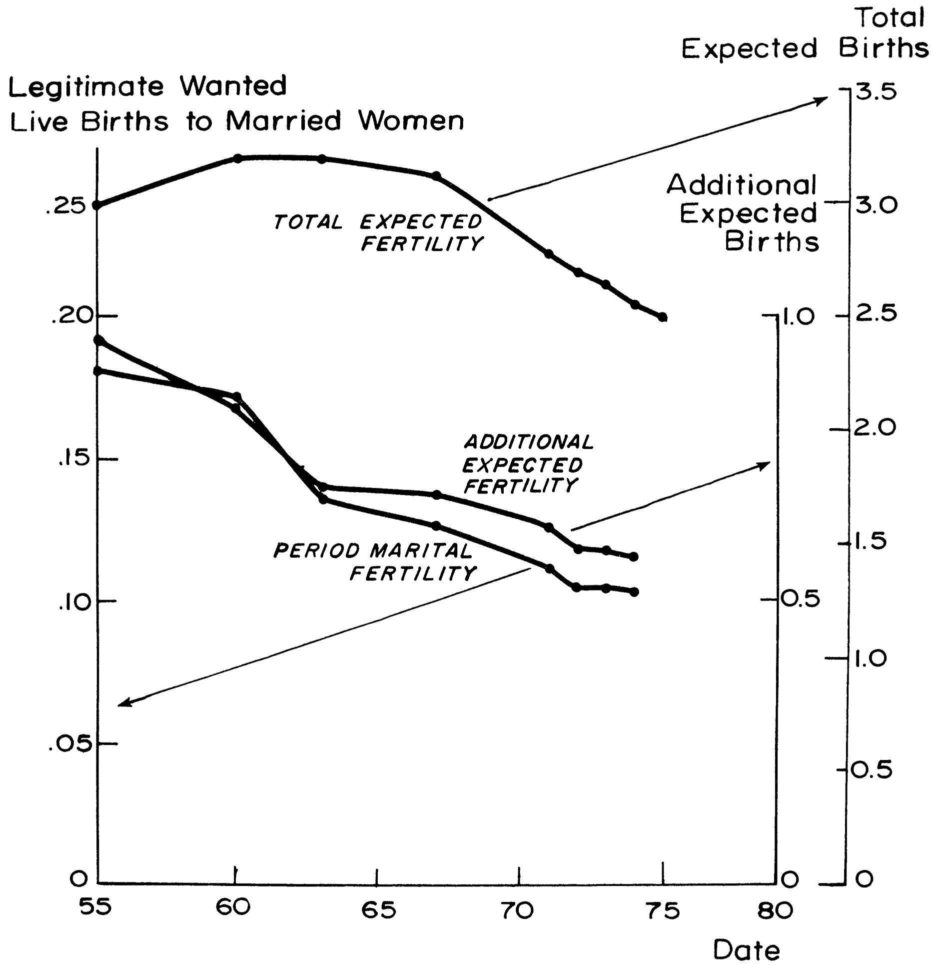

Figure 4. Wanted Marital Fertility, Additional Expected Fertility, and Total Expected Fertility for U.S. women 18 to 39

Sources: Same as for Figure 2.

The reason why additional desired fertility declined while D continued to rise is, of course, that cumulated births per woman rose even faster.

5. When D fluctuates, turning points in average cumulated fertility follow turning points in D by up to five years.

This follows from the argument given for the fourth proposition. It is precisely because changes in average cumulated fertility lag behind changes in D that cumulated fertility still rises rapidly when D begins to rise more slowly, and, therefore, additional desired fertility begins to decline, thus casuing F_t to fall. In the United States, cumulated births per woman reached a peak in 1965; cumulated births per ever-married woman, a more appropriate measure, are not unfortunately available.

So far, all the propositions have dealt with aggregate period measures, but it is also helpful to consider individual ages (or marriage durations). The next three propositions give the main results:

6. When D fluctuates, the fertility of older women will fluctuate proportionately most, that of young women least.

The reason is obvious; young women have their entire reproductive life ahead of them and can adjust to a changing value of *D*; older women must try to squeeze their entire adjustment into a few years. There are, however, two offsetting influences. First, old women have cumulated more unwanted births resulting from contraceptive failure than younger women. Therefore, when *D* rises, many older women already have an 'extra' child that can be retroactively transferred to the 'wanted' category, obviating the need to bear an additional child. Secondly, since most older women will have ceased childbearing, the fertility of older age groups will not fall as sharply as is predicted by the model when *D* declines, since they cannot reduce their future fertility below zero.

In Table 2 we show proportionate changes in marital age-specific fertility rates over the upswing and the downswing of the baby boom/bust. It bears out this proposition on the downswing in *D*, but not on the upswing.

Table 2. *Relative Increases and Declines in U.S. Age-Specific Marital Fertility, 1950 to 1975*

Ratios of Age-Specific Marital Fertility Rates	Age Group 15–19	20–24	25–29	30–34	35–39	40–44
1957/1950	1.18	1.26	1.17	1.11	1.11	1.06
1975/1957	0.65	0.51	0.59	0.46	0.32	0.27

Source: Calculated from data in U.S. Public Health Service, 1978, *op cit.* in note to Table 1.

7. When D fluctuates, fertility rates at the youngest ages have turning points at the same time as D, while at the older ages turning points precede those in D.

The reason is again obvious. At the youngest ages, cumulated fertility is zero, and additional desired fertility equals *D*. Therefore, fertility flows are synchronous with *D*. At the older ages the relative role of cumulated fertility is greater, and the argument given for the fourth proposition holds more strongly.

In order to reduce the influence of minor fluctuations in period fertility, I have used five-year moving averages of the annual marital fertility rates to search for peaks. In Table 3 I show the peak years by age according to this criterion. We see that older age groups experienced a downturn about five years before the youngest age group. Note, however, that the downturn for women aged 15–19 or 20–24 would imply a peak value of *D* by 1959 or 1960, too early to be consistent with the survey figures. This observation is consistent with other evidence in this paper

Table 3. *Dates of Peaks in Five-Year Moving Averages of Age Specific Marital Fertility*

Age	Date of Peak
15–19	1961
20–24	1959
25–29	1959
30–34	1956
35–39	1956
40–44	1955

Source: Calculated from statistics in U.S. Public Health Service, 1978, *op. cit.* in Note to Table 1 suggesting that the value of D for the younger age groups may have turned down somewhat earlier than that of the older ones, contrary to the assumption I have made.

8. When D fluctuates, cumulative fertility at the youngest ages has turning points at the same time as D, while at the older ages turning points occur well after those in D.

This is clearly true for the youngest ages since these women have no reproductive history to complicate the picture. But the older the woman, the greater the extent to which cumulative fertility reflects the influence of desired family size in previous years, and, therefore, the greater the lag of cumulative fertility behind the *current* value of D.

In Table 4 the date of peak cumulated fertility for each age group is shown. It provides strong confirmation for the last proposition, since the older the group, the later does cumulative fertility reach its peak.

Table 4. *Date of Maximum Cumulated Fertility of U.S. Women*

Age	Date of Peak Cumulated Fertility
15–19	1958
20–24	1961
25–29	1963
30–34	1966
35–39	1970

Source: Calculated from Heuser, *op. cit.* in note to Figure 3

9. During the transition from high to low values of D, F_t will fall more rapidly than D until D stops falling, then F_t will rise to the new stable level of D, creating a 'ski jump' effect as the transition is completed.

This follows immediately from the first proposition. $F_t = D$ initially, and once D begins to decline, F_t will have to decline more rapidly in order to be below D as implied by Proposition 1. But when D stabilizes, F_t will have to rise in order to equal it once again.

I have stated these propositions as if they held completely generally, but in fact they hold only for some broad classes of change in D which I believe to be fairly typical. For example, all the propositions about the timing of turning points rely on a deceleration of D before it turns up or down, and I will prove them only for the case of a sinusoidal variation in D. If the time path of D, instead, looked like a broken stick or V, with no deceleration, then the turning point of F_t would not precede that of D.

IV. FORMAL DEVELOPMENT OF THE THEORY AND PROOF OF THE PROPOSITIONS

These propositions can be demonstrated more formally with the help of a mathematical model. Recall that all women are assumed to marry at the same age. I will refer to the time elapsed since marriage as the 'age' of a cohort, although this could more correctly be called its marriage duration. For a single cohort, we need not distinguish notationally between age and time; I will let x represent both. Let $D(x)$ be the value of D for the cohort at age x, let $C(x)$ be its cumulated fertility up to age x, and let $A(x)$ be its additional desired fertility at age x. Assuming there are no unwanted births,

$$A(x) = D(x) - C(x). \tag{1}$$

If $g(x)$ is the birth rate at age x then:

$$C'(x) = g(x) \tag{2}$$

i.e. cumulated fertility increases due to new births (the prime over the C indicates a derivative with respect to age or time).

The fundamental behavioural assumption on which the entire model rests is that

$$g(x) = \lambda A(x), \tag{3}$$

or the flow of births is proportional to the additional desired fertility. This was empirically demonstrated in an earlier article for age groups 25–29, 30–34 and 35–39, but it is not a very good assumption for the younger ages. Nonetheless, it will be employed here, and some justification may be found in Figure 2 above, which suggested a value 0.18 for λ. Note that Equation (3) abstracts from the lag of about one year between a planned and realized birth.

Equation (1) may be differentiated to find:

$$A'(x) = D'(x) - C'(x), \tag{4}$$

and substitution from (2) and (3) into (4) gives

$$A'(x) = D'(x) - \lambda A(x). \tag{5}$$

This is the basic differential equation of the model. Its general solution is:

$$A(x) = e^{-\lambda x}\left[\int_0^x e^{\lambda u} D'(u)du + k\right] \tag{6}$$

where $x = 0$ corresponds to the age at marriage of the cohort (or duration 0), and k is a constant chosen to satisfy the initial condition that at duration 0 no children have been born, and so

$$A(0) = D(0). \tag{7}$$

Multiplying Equation (6) by λ gives $g(x)$, the duration-specific marital fertility rate, when the fertility target is changing over time.

Given some explicit time path of $D(x)$, which describes the way a particular cohort revises its planned completed fertility as it ages and moves through time, Equation (6) may be solved explicitly for the time path of marital fertility, $g(x)$.

The simplest case to consider is a linear increase in desired completed fertility, D, so that:

$$D(t) = a + bt \tag{8}$$

$$D'(t) = b. \tag{9}$$

If the cohort is aged x at time t, then at marriage the value of D was $a + b(t-x)$, or:

$$D(0) = a + b(t-x). \tag{10}$$

Substituting from (9) into (6) and solving, we find:

$$A(x) = (b/\lambda)(1 - e^{-\lambda x}) + e^{-\lambda x}k. \tag{11}$$

The constant k must be chosen so that $A(0) = D(0)$, so:

$$A(0) = a + b(t-x). \tag{12}$$

Combining (11) and (12) gives:

$$k = a + b(t-x). \tag{13}$$

Finally, substituting from (13) into (11) gives:

$$A(x) = (b/\lambda)(1 - e^{-\lambda x}) + e^{-\lambda x}[a + b(t - x)] \tag{14}$$

Note that if $b = 0$, D is fixed, and the right hand side of (14) vanishes except $e^{-\lambda x}a$; in this case, additional desired fertility and the marital fertility rate $g(x)$ decline exponentially toward 0 as the couple move through reproductive life. When $b > 0$, fertility declines less rapidly and tends towards b rather than 0. It can easily be seen that the mean age of childbearing will be higher when $b > 0$.

Now I will turn to the behaviour of global period fertility, as measured by F_t. This is calculated by aggregating over all age groups in a given year. For this purpose it will be convenient to alter the notation slightly. Recalling that the preceding equations were derived for a cohort aged x at time t, I will index all variables by both age and time, so that $A(x)$ becomes $A(x, t)$, for example. With this notation, F_t at time t is defined as:

$$F_t(t) = \int_0^\beta g(x, t)dx \tag{15}$$

where β is the upper limit of the reproductive period. I will generally take $\beta = 20$, implying that sterility occurs 20 years after marriage. Recalling that $g(x,t) = \lambda A(x,t)$, (15) may be rewritten as:

$$F_t(t) = \lambda\Gamma(t) \tag{16}$$

where

$$\Gamma(t) = \int_0^\beta A(x, t)dx. \tag{17}$$

Γ is an aggregate measure of additional desired fertility, and Γ/β is the average additional desired fertility for women all reproductive ages.

For linearly increasing D we have:

$$\Gamma(t) = \int_0^\beta (b/\lambda)(1 - e^{-\lambda x}) + e^{-\lambda x}[a + b(t - x)]\, dx. \tag{18}$$

Integrating and multiplying by λ gives:

$$F_t(t) = D(t)(1 - e^{-\lambda\beta}) + b\beta(1 + e^{-\lambda\beta}) - (2b/\lambda)(1 - e^{-\lambda\beta}). \tag{19}$$

The term $e^{-\lambda\beta}$ represents the final proportion of D which remains unattained at the onset of sterility; with $\lambda = 0.18$ and $\beta = 20$, this is only 0.027. If we take this to be zero, then, to a good approximation the expression can be rewritten:

$$F_t(t) \doteq D(t) + b(\beta - 2/\lambda). \tag{20}$$

This is the final result, which is fortunately quite simple. It tells us that when desired completed fertility is constant so that $b = 0$, $F_t = 0$. When D changes linearly, then $b \neq 0$, and F_t differs from $D(t)$ by $b(\beta - 2/\lambda) = b(20 - 2/0.18)$ or by about $9b$. Thus, if desired completed fertility is rising by one child per decade, $b = 0.1$, and $F_t - D = 0.9$. This establishes the first proposition, at least for linearly rising or falling D.[16]

With additional calculations, not shown here, we could derive the time path of F_t if D suddenly ceased its linear movement and remained constant. F_t would then move fairly rapidly toward the now stable D, but would not equal it until β years had passed. These results are illustrated in Figure 5.

[16] The proposition is not perfectly general; it would be possible to construct examples for which it did not hold.

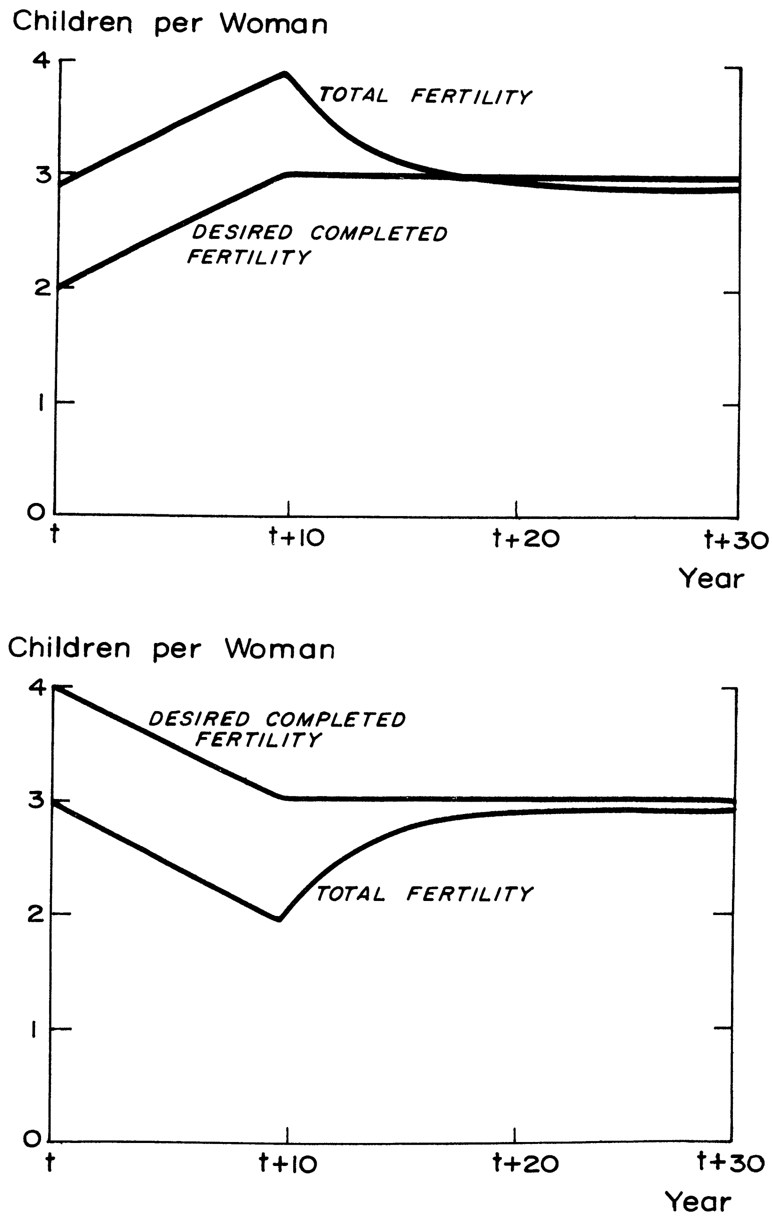

Figure 5. Period total fertility when desired completed fertility changes linearly, then is constant.
Sources: Calculated from Equation 19 with $\lambda = 0.18, \beta = 20$, and $b = \pm\, 0.1$.

It is probably more realistic to consider the case in which D changes smoothly over time, rather than as a series of linear segments. A sine wave is a reasonable caricature of the kind of change I want to portray, and despite our relative ignorance about pre-1955 changes in D, a sine wave with a period of about 50 years, a mean of 2.6 children, and a peak-to-trough amplitude of one child, would describe a swing in D from 2.1 to 3.1 and back to 2.1 children, which might not be a bad representation of changes in D since the 1930s.

Suppose, then, that D follows a sinusoidal pattern:

$$D(t) = \bar{D} - a \cos \frac{t}{w} \tag{21}$$

$$D'(t) = (a/w)\sin\frac{t}{w}. \tag{22}$$

Since the mean value of the cosine in zero, $\bar{D}$ is the mean value of $D(t)$. Since the maximum values of the cosine are $+1$ and -1, respectively, the extreme values of $D(t)$ will be $\bar{D} \pm a$. The constant w determines the periodicity of $D(t)$, which has a cycle length of $2\pi w$.

Consider a cohort 'aged' x at time t. This cohort married at $t - x$, so at age u the value of D for this cohort is given by $D(t - x + u)$. Using equations (6) and (22) this allows us to write the expression for $A(x, t)$:

$$A(x,t) = e^{-\lambda x}(a/w)\int_0^x e^{\lambda u}\sin\frac{t-x+u}{w}\,du + k(t-x). \tag{23}$$

Note that k is constant for each cohort, but will differ across cohorts; for this reason I have written it as a function of the birth date of the cohort, $t - x$.

After solving this integral and using the initial condition $A(0, t-x) = D(t-x)$ to find $k(t-x)$, we obtain the following expression for $A(x, t)$:

$$A(x,t) = [a/(\lambda^2 w^2+1)]\left[\lambda w \sin\left(\frac{t}{w}\right) - \cos\left(\frac{t}{w}\right)\right]$$

$$+ e^{-\lambda x}\left\{\bar{D} - [a/(\lambda^2 w^2+1)]\left[\lambda^2 w^2 \cos\left(\frac{t-x}{w}\right) + \lambda w \sin\left(\frac{t-x}{w}\right)\right]\right\}. \tag{24}$$

This is not exactly a marvel of simplicity and worse is ahead, since this must be integrated over x to find $\Gamma(t)$. The integration is quite complicated and would take too much space to report here, so I will give only the result.

$$F_t(t) = (1 - e^{-\lambda\beta})\bar{D} + \frac{a\beta\lambda}{z}\left(y\sin\left(\frac{t}{w}\right) - \cos\left(\frac{t}{w}\right)\right)$$

$$-\frac{a(y^4 - y^2)}{z^2}\left[e^{-\lambda\beta}\cos\left(\frac{t-\beta}{w}\right) - \cos\left(\frac{t}{w}\right)\right]$$

$$-\frac{2ay^3}{z^2}\left[e^{-\lambda\beta}\sin\left(\frac{t-\beta}{w}\right) - \sin\left(\frac{t}{w}\right)\right] \tag{25}$$

where

$$y = \lambda w \tag{26}$$

$$z = 1 + y^2. \tag{27}$$

Note that the $F_t(t)$ is just λ times the integral, up to age β, of $A(x, t)$. This makes it easy to check the result (25), since regarding F_t as a function of β, we should have:

$$\partial F_t(t,\beta)/\partial\beta = A(\beta, t)\lambda \tag{28}$$

and

$$F_t(t, 0) = 0. \tag{29}$$

To find the peak (or trough) in $F_t(t)$ we set the derivative with respect to t equal to zero, and solve for the t^* which satisfies the equation. It is given by:

$$t^* = w \tan^{-1}\left\{\frac{\beta y\lambda z - 2y^3 + e^{-\lambda\beta}\left[(y^4 - y^2)\sin\left(\frac{\beta}{w}\right) + 2y\cos\left(\frac{\beta}{w}\right)\right]}{-z\lambda\beta - (y^4 - y^2) + e^{-\lambda\beta}\left[(y^4 - y^2)\cos\left(\frac{\beta}{w}\right) - 2y\sin\left(\frac{\beta}{w}\right)\right]}\right\} \quad (30)$$

The peak and trough values of $F_t(t)$ can then be obtained by calculating $F_t(t^*)$.

The sort of swing in D which occurred over the baby boom/bust may be roughly represented as a cosine with a mean of 2.6 children ($\bar{D}$), a peak-to-trough amplitude of one child ($a = 0.5$), and a cycle length of 50 years ($w = 8$; $2\pi w = 50.3$). The corresponding time paths of F_t and D are shown in Figure 6. Note that $F_t > D$ when D is rising and $F_t < D$ when D is falling, illustrating the first proposition. F_t fluctuates between 1.63 and 3.48, or 1.85 times as widely as D, illustrating the second proposition, and conforming quite closely to the historical experience. The peak value of F_t occurs about 5.5 years before that of D, illustrating the fourth proposition, and again conforming to historical experience. By the time D has reached its peak, F_t has already declined by seven per cent; by the time D has fallen bv one-tenth of a child, F_t has fallen by 20 per cent.

More generally, we would like to know how F_t behaves for cycles of different lengths, and for different values of the rate of adjustment, λ. The lead/lag pattern is easily investigated by using the explicit solution for t^* given by Equation (30). The figures in Table 5 are calculated from this equation for various values of λ and w, and show the number of years by which F_t leads D. Plausible values of λ lie between 0.10 and 0.25; the lower end of this range represents the younger ages better, while the upper range represents the older ages. It is reassuring that, for a cycle length of 50 years, the lead of F_t is substantial for all values of λ within this range; in this sense, the result is 'robust'. It is also interesting to note that the lead increases more than proportionately with cycle length, and that for cycles of even ten or 20 years' length, the lead is almost negligible.[17]

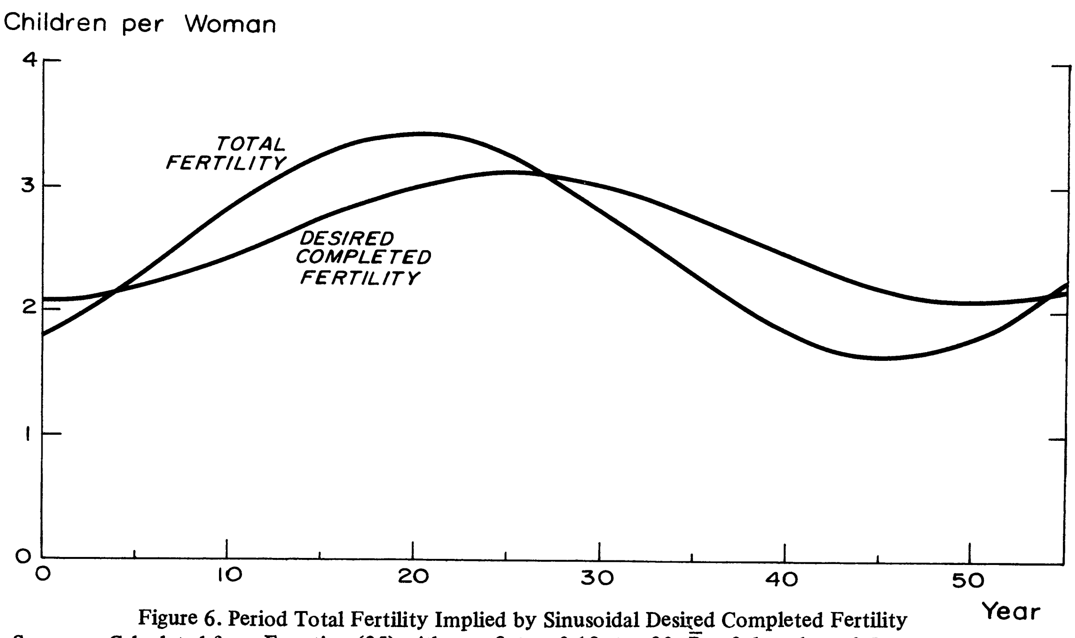

Figure 6. Period Total Fertility Implied by Sinusoidal Desired Completed Fertility
Sources: Calculated from Equation (25) with $w = 8$, $\lambda = 0.18$, $\beta = 20$, $\bar{D} = 2.6$, and $a = 0.5$.

[17] It is easily shown that as λ or $w \to 0$, so does t^*. Also $\lim_{\lambda\to\infty} t^* = w\tan^{-1}(\beta/w)$

Table 5. *Lead in Years of Peak Period Total Fertility Over Peak Desired Completed Family Size, When D Changes Sinusoidally*

Rate of Adjustment	Cycle Length in Years ($2\pi w$)						
λ	5	10	20	30	50	75	100
0.05	0.05	0.22	0.84	0.87	0.48	1.94	3.10
0.10	0.07	0.28	1.04	1.91	3.34	4.76	5.23
0.15	0.10	0.38	1.35	2.55	4.60	6.23	7.04
0.20	0.13	0.48	1.66	3.03	5.35	7.24	8.33
0.25	0.16	0.59	1.93	3.42	5.91	8.00	9.29
0.30	0.18	0.69	2.17	3.74	6.35	8.59	10.01
0.50	0.30	1.02	2.83	4.56	7.42	9.95	11.63
1.00	0.52	1.51	3.55	5.39	8.40	11.10	12.94

Source: Calculated from Equation (30) with $\beta = 20$.

So far, I have analyzed only the aggregate period flows as summarized by F_t. But the behaviour over time of fixed age or duration groups is also of considerable interest. Turning points need not be the same for each age group because the relative importance of the present and past changes in D varies with x. For very young age groups, intuition suggests that changes in D would dominate, while for older ones, changes in numbers of children ever born might do so. Similarly, we would expect that the relative magnitude of fluctuations is greater at older ages than at younger ones.

To study individual age groups when D varies sinusoidally, we can use the expression for $A(x, t)$ given in Equation (24). Let us compare the behaviour of fertility at the youngest age, $g(0, t)$, and at the oldest age, $g(\beta, t)$. The first is given by $\lambda A(0,t)$, which can be seen from Equation (24) to be:

$$g(0, t) = \lambda \left[\bar{D} - a \cos\left(\frac{t}{w}\right)\right]. \tag{31}$$

The expression in brackets is just $D(t)$, since cumulated births for this age group are 0. Therefore, the fertility of the youngest age group moves together with $D(t)$ with no lead or lag (more realistically, it would follow $D(t)$ by one year if we had allowed for conception and gestation lags).

The ratio of peak to mean value provides a straightforward measure of relative variability. The peak value of $g(0, t)$ occurs when $\cos(t/w) = -1$. The mean value of $g(0, t)$ is simply $\lambda\bar{D}$. From (31), we can infer that the ratio of the peak value to the mean value is $1 + a/\bar{D}$.

Now consider the fertility of the oldest age group, $g(\beta, t) = \lambda A(\beta, t)$. Ignoring trigonometric terms in $e^{-\lambda\beta}$, we have:

$$g(\beta, t) \doteq [\lambda a/(\lambda^2 w^2 + 1)] \left[\lambda w \sin\left(\frac{t}{w}\right) - \cos\left(\frac{t}{w}\right)\right] + \lambda e^{-\lambda w}\bar{D}. \tag{32}$$

Setting the derivative with respect to $u = t/w$ equal to zero and solving for u^* gives:

$$u^* = \tan^{-1}(-\lambda w). \tag{33}$$

The lead in years will be given by u^*w. For $\lambda = 0.18$ and $w = 8$, the case represented in Figure 6, this implies a lead of 7.7 years. This compares fairly well with the five-year difference between $g(0, t)$ and $g(20, t)$ shown in Table 3 above.

The average value of $g(\beta, t)$ is just $\lambda e^{-\lambda\beta}\bar{D}$. The peak value is attained at u^*, and after some calculation is found to be:

$$\max g(\beta, t) \doteq \lambda e^{-\lambda\beta}\bar{D} + \lambda a/\sqrt{\lambda^2 w^2 + 1}. \tag{34}$$

The ratio of the maximum to the mean is therefore

$$\frac{\max g(\beta, t)}{\text{mean } g(\beta, t)} \doteq 1 + (ae^{\lambda\beta}/\sqrt{\lambda^2 w^2 + 1})/\bar{D}. \tag{35}$$

From this we can see that the amplitude of the proportional fluctuation for the oldest age group is in the ratio $e^{\lambda\beta}/\sqrt{\lambda^2 w^2 + 1}$ to that of the youngest age group. For all fluctuations of interest, the proportional fluctuations will be greater for the older ages, and for a 50 year cycle they will be 21 times as great. This is far bigger than the ratios implied by Table 3 above, but some explanation is given in the discussion following Section VI.

These calculations establish the sixth and seventh propositions for the sinusoidal case.

Now consider cumulated fertility by age. It is obvious that at the youngest ages, cumulated fertility closely reflects current fertility, and, therefore, moves closely with D. For older ages, we can use:

$$C(x, t) = D(t) - A(x, t). \tag{36}$$

Using (32) for age β, we can derive:

$$C(\beta, t) \doteq \bar{D}(1 - e^{-\lambda\beta}) - [a\lambda w/(\lambda^2 w^2 + 1)] \left[\sin\left(\frac{t}{w}\right) + w \cos\left(\frac{t}{w}\right)\right]. \tag{37}$$

Setting the derivative with respect to $u = t/w$ equal to zero and solving for u^* gives:

$$u^* = \tan^{-1} [1/(\lambda w)]. \tag{38}$$

To determine whether u^* locates a minimum or a maximum it is necessary to evaluate the second derivative of Equation (37) with respect to u; this establishes that $C(\beta, t)$ reaches its maximum at $t = u^* w$ in the third quadrant, *after* $D(t)$. For a 50-year cycle, the implied lag of C behind D is 7.7 years. This confirms the eighth proposition.

The ninth proposition concerns a demographic transition, by which is meant a change in the D from a stable high value to a stable low value. In a general way we know that F_t must fall more rapidly than D, as women find their childbearing unexpectedly ahead of schedule; and because of this, toward the end of the transition, F_t must rise from its artificially low levels. To derive more detailed results, we must first specify an explicit form for the decline in D. Some possible candidates are a linear decline, a logistic decline, or a decline from the peak to the trough of a sine wave. The linear decline is implausibly jerky at the end-points, and the logistic is both computationally difficult and lacks a start and finish. Consequently I have used a sinusoidal decline. This is also computationally quite complicated, and I will not present either the derivation or the final equations for the various special cases. Instead, I present a plot of the results for a single case: a transition that takes 20 years to move from an initial D of six children to a final D of three children. The rate of adjustment, λ, is taken to be 0.10 rather than 0.18 or 0.20, since this is believed to be more representative of populations in less developed countries.[18] The lower value of λ makes the results less dramatic than they would otherwise have been, and it also implies that cohorts attain only $0.865 = 1 - e^{-20\,(0.1)}$ of their desired family size; therefore, the initial and terminal stable level of F_t moves from 5.19 to 2.60 rather than from six to three.

The results for these assumptions are plotted in Figure 7. Here the transition in D starts in year 100 and finishes in year 120, but F_t does not fully stabilize until year 140, although it changes only slightly after year 132. F_t reaches its lowest point 2.5 years before D finishes its

[18] See, for example, A. Jain and A. Hermalin, 'Fecundity Models for Estimating Fertility Over the Reproductive Period,' Paper presented at the 1969 meeting of IUSSP in London.

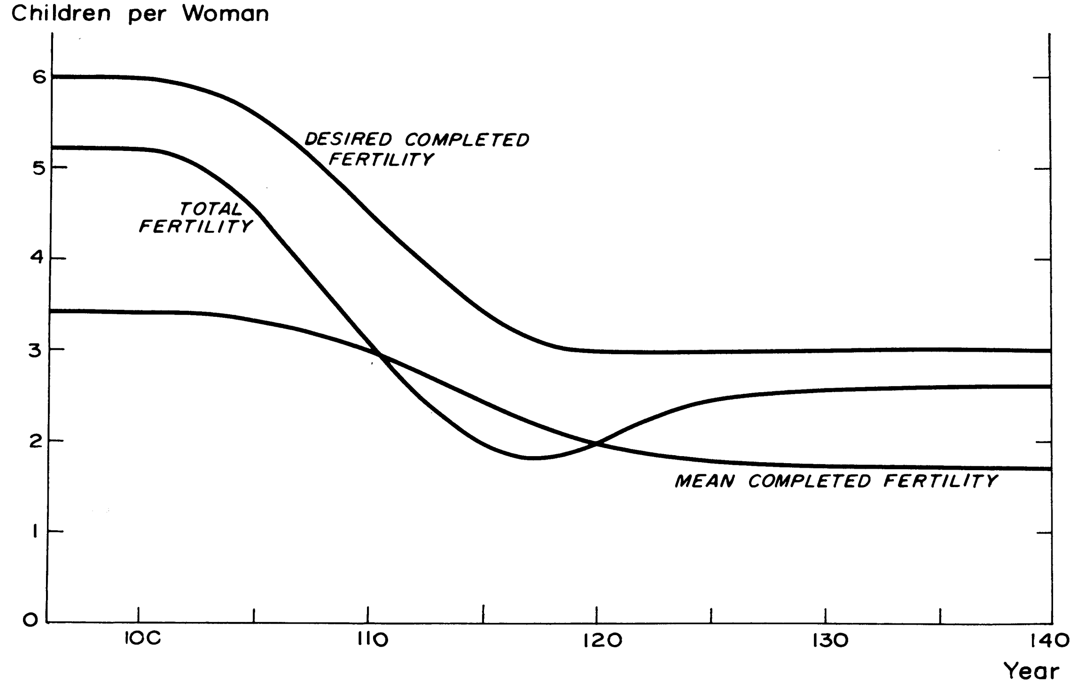

Figure 7. Period Total Fertility During A Demographic Transition

Sources: The transition is from a Desired Completed Fertility of 6.0 to one of 3.0 over a period of 20 years, with the pattern of decline being half a sine wave. Note that $\lambda = 0.1$, not 0.18; $\beta = 20$.

decline, at a level almost 0.8 below its final value. Between years 105 and 115, D declines by 2.12 children, while F_t declines by 2.55 children. The number of children per married woman, averaged over all ages, is also shown. This declines very slowly and monotonically, and like F_t does not stabilize completely until the year 140.

In actuality, pre-transition levels of F_t typically exceed those of D considerably, as a result of contraceptive failure and high mortality. Consequently, initial declines in F_t may often reflect changes in contraception and mortality rather than in D; this was apparently the case in Taiwan, for example.[19] A more complete analysis would have to take these factors explicitly into account, and deal with the irreversibility of fertility as well.

V. THE IRREVERSIBILITY OF FERTILITY

I have already mentioned that when D declines, couples who have previously completed child-bearing will find themselves with growing numbers of children that were wanted when conceived, but which would not have been wanted with more perfect foresight of coming social and economic conditions, and which, therefore, become, in a special sense, 'no-longer-wanted'. In the model as presented so far, these no-longer-wanted births are counted as part of the cumulated fertility of a cohort $C(x)$; but clearly they have no effect on aggregate period fertility. In the model analyzed above it was implicitly assumed that these no-longer-wanted children would lead to *negative* fertility rates for their parents, since when $C > D$, $A = D - C < 0$, and so is $g = \lambda A$. But fertility is irreversible, and the above treatment would make sense only if the parents of no-longer-wanted children put them up for adoption by fecund married women who substituted them for births.

[19] See T-H. Sun, H-S. Lin and R. Freedman, 'Trends in Fertility, Family Size Preferences, and Family Planning Practice: Taiwan 1961–76,' *Studies in Family Planning*, 9, (4)(April, 1978), pp. 54–70.

More realistically, the fertility of a cohort should be related to the value of D diminished by the cumulative fertility of those of its members who had not completed childbearing. The reason why similar problems do not occur when D is rising is that those who originally thought that their fertility was complete can change their minds and continue to bear children, if they revise their desires upward, assuming they have not been sterilized. I will not attempt to give a comprehensive treatment of the implications of irreversibility here, but shall, instead, indicate the way it can be approached, and present a numerical example for a cohort. Let the proportion of women in a group who had not completed childbearing be denoted by p, so that the proportion of those who had completed is $(1-p)$. In an earlier paper I showed that for women aged 25 and over, p is a constant fraction, $(1/\alpha)$, of A:[20]

$$p = A/\alpha, \tag{39}$$

where α varied somewhat with age, but was generally close to 1.5. Now let U be the average number of no-longer-wanted births per woman. The average number of no-longer-wanted births per *woman of completed fertility* will then be $U/(1-p)$.

Consider a cohort of women whose desired family size falls with time. Each decline in D will increase the no-longer-wanted births per woman of completed fertility by a corresponding amount, so we obtain the important equation:

$$U' = -(1-p)D' = (1-A/\alpha)D'. \tag{40}$$

If we let C^* be the remaining still wanted cumulated births per woman, then:

$$C = U + C^*. \tag{41}$$

The additional desired births, which may be denoted A^*, are given by:

$$A^* = D - C^* = D - C + U. \tag{42}$$

Marital fertility, g, is given by

$$g = \lambda A^* = \lambda(D - C^*). \tag{43}$$

Differentiating (42) gives:

$$A^{*\prime} = D' - C' + U' \tag{44}$$

and recalling that $C' = g$, and using (40) and (43) we have:

$$A^{*\prime} = D' - (1 - A^*/\alpha)D' - \lambda A^* \tag{45}$$

or

$$A^{*\prime}/A^* = D'/\alpha - \lambda. \tag{46}$$

This has solution:

$$A^*(t) = ke^{-\lambda t + D(t)/\alpha}. \tag{47}$$

Now consider a time t_0 before which D has been constant or rising, so that $U(t_0)$ is surely zero. Then $A^*(t_0) = A(t_0)$. For convenience let $t_0 = 0$. Then we can find k to be:

$$k = A(0)e^{D(0)/\alpha}, \tag{48}$$

which when substituted into (47) gives the final result for a cohort:

$$A^*(t) = A(0)e^{-\lambda t}e^{[D(t)-D(0)]/\alpha}. \tag{49}$$

This can be compared to Equation (6) which gives the basic solution when irreversibility is not considered; it is clear that (49) is actually simpler.

It is helpful to compare the time path of cohort fertility with and without reversibility for a linear decline in D. Suppose that D declines by k children per year. Then with irreversible fertility, (49) can be shown to imply that:

[20] See Lee, *loc. cit.* in footnote 8 and footnote 14.

$$g(t) = \lambda A(0)e^{-t(\lambda + k/\alpha)}. \tag{50}$$

Taking $\lambda = 0.18, A(0) = 1, \alpha = 1.5$ and $k = 0.1$, we obtain:[21]

$$g(t) = 0.18e^{-0.25t}. \tag{51}$$

For reversible fertility using (14), we have:

$$g(t) = 0.28e^{-0.18t} - 0.1. \tag{52}$$

With no change in D, we would have:

$$g(t) = 0.18e^{-0.18t} \tag{53}$$

with or without irreversibility.

These three cases are plotted in Figure 8. It is clear that taking irreversibility into account

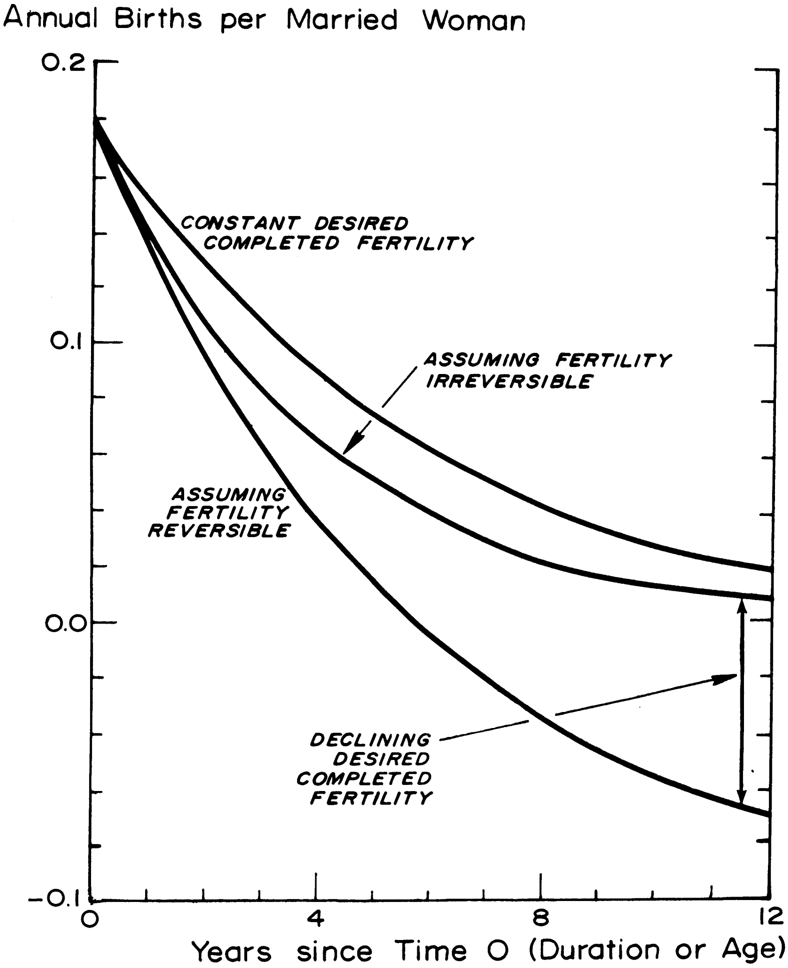

Figure 8. Cohort Marital Fertility by Marriage Duration or Age, for the Cases of Constant or Declining Desired Completed Fertility, With and Without Reversible Fertility

Sources: Duration or age 0 corresponds to 1.0 additional desired child, while D is assumed to decline linearly at $k = 0.1$ children per year. $\lambda = 0.18$. The equations for the three lines are: Constant D, Equation (53); Declining D, irreversible fertility, Equation (51); Declining D, reversible fertility, Equation (52).

[21] $A(0) = 1.0$ would be consistent with, for example, an initial value of D of 2.6 during the first 5.3 years of marriage since $2.6e^{-0.18(5.3)} = 1.0$.

makes a great deal of difference to the outcome, and that ignoring it leads to large errors, particularly, but not exclusively, at the older ages.

VI. DISCUSSION

The detailed conclusions reached in this paper have already been listed as nine propositions, and need not be recapitulated. Instead, I shall discuss in a more general way the implications of a moving target approach for the analysis of fertility change over time.

If the fixed target model is abandoned in favour of a moving-target model which explicitly incorporates changes in D within cohorts, certain procedures for analyzing fertility must also be discarded. Prominent among these are all those techniques that attach any significance to a cohort's actual completed fertility as a measure of its target throughout its reproductive years, as do some developed by Ryder, Butz and Ward and many others.[22] For example, if D were to vary over the reproductive life of a cohort, this may well induce changes in fertility that would, in the fixed target context, appear to be merely timing changes if viewed retrospectively. Furthermore, there are several ways in which actual completed fertility may misrepresent even the final D of a cohort when it moves out of the reproductive years. First, simple measures will include unwanted births. Secondly, if D has been rising, many members of the cohort may have been unable to attain their final desired family size. Thirdly, if D has been falling, then many couples who completed their fertility when their desired family size was high will be left with 'no-longer-wanted' children when they revise their desires downward. This cumulation of no-longer-wanted births must frequently be confounded with measures of contraceptive failure in retrospective questions on fertility surveys.

The moving-target model also underlines the danger of attempting to provide causal explanations for variations in period fertility without attention to changes in the more fundamental desired family size. For example, to account for the baby boom and bust, we should look for a set of positive influences which was increasing throughout the 1950s, but at a slackened pace in the late 1950s, and which perhaps turned down in the early to mid 1960s. On the other hand, there is some evidence that, contrary to the assumption made here, the desired family sizes of young people turned down earlier than those who were older. And, of course, in any case, attempts at direct explanation of the period rates of young people encounter the least difficulty, since their cumulated fertility is low.

The moving-target approach also lends support to certain strategies for collecting and using survey data. On one hand, it shows that survey measures of D do not merely reflect *ex post* rationalizations of previous reproductive behaviour, but rather appear to reflect the driving force behind period fertility rates. On the other hand, there are numerous pitfalls in the use of these data, aside from the problems of measurement *per se*. For example, it seems inappropriate to base forecasts directly on stated values of D since these can change rapidly over time for a given cohort. The appropriate procedure would be first to forecast D, based on such time series as are available, and then to base forecasts of period fertility rates on the difference between the forecast of D and births to date along the lines of the present analysis, but hopefully with somewhat more realistic stock adjustment assumptions for the younger ages. In such forecasts, due account should be taken of contraceptive failure rates and of the influence of irreversibility.[23]

The moving-target approach also underlines the need to survey the same cohort repeatedly over time in order to understand its reproductive behaviour, and is inconsistent with attempts to

[22] See Ryder, *loc. cit.* in footnote 2 and Butz and Ward, 'Completed Fertility and its Timing.' *Journal of Political Economy*. 1980 (in the press) Butz and Ward are well aware of the basic problem with their *'ex post'* measure of a cohort's target, and discuss it cogently in their paper.

[23] The appropriate approach for dealing with irreversibility and contraceptive failure is set out in Lee, *loc. cit.* in footnote 8 and footnote 14.

attach any particular 'true' value of D to a cohort. Similarly, this approach raises doubts about the feasibility of ascertaining the 'wantedness' status of births through surveys conducted long after they have occurred, since reproductive goals may have varied substantially since that time. For example, the number of Detroit women reporting the identical birth as unwanted when conceived increased by 25 per cent between 1962 and 1977, presumably reflecting the drop of 0.75 in D over this period for these women.[24] In short, the moving-target approach raises basic questions about such efforts as Ryder's reconstruction of U.S. fertility by planning status from 1950 to 1970, based on surveys in 1965 and 1970.[25]

Finally, the moving target approach directs attention to the neglected problem of no longer wanted births, which, like contraceptive failures, arise from imperfect foresight. These may rival contraceptive failures in prevalence and may have similar consequences for the parents, who in both cases find themselves with non-optimum numbers of children.[26]

An important conclusion from the moving target analysis is that minor fluctuations in D will typically generate major fluctuations in period fertility flows. Since it is the period flows that determine births, this augurs for continued large fluctuations in the numbers of births, and, therefore, in the age structures, of transitional and post-transitional populations.

The theory as I have outlined it in this paper is crude and oversimplified, and needs refining. Improvement of the model could proceed along two alternate lines. The simplest would be to replace the assumption of a fixed proportional rate of adjustment, λ, by a linear relationship between fertility flows and additional desired fertility, allowing the linear coefficients to vary with age. This would give a good description of the basic relationships, and serve as a basis for a more realistic simulation. However, it would provide no insight into the relation between individual and aggregate behaviour. The second approach, which would afford such insight, is to model the distribution of women by D, and changes in the distribution over time, rather than dealing only with the average value of D. Then women could be modelled to bear children at fixed or variable intervals until they had attained their desired family size.

More attention should also be paid to changes in desired birth intervals. Over the U.S. baby boom/bust, these were shorter on the upswing in D than on the downswing, and they have been shown to be inversely related to D for cross-sectional data. The model used in this paper implies constant birth intervals for women over 25 years of age or five years' marriage duration; for the lower ages or durations, however, the implied birth interval moves inversely with additional desired fertility.[27] This relationship, which is empirically plausible, has not been adequately considered in the present paper, or my earlier papers.

The model as it stands has the advantage of simplicity, and therefore analytic tractability. From two assumptions, it was possible to generate a surprisingly broad range of implications which appear consistent with many otherwise puzzling aspects of fertility change in the post-war United States.

[24] See R. Freedman's preliminary analysis of Deborah Freedman's survey of the Detroit women.

[25] See Ryder, *loc. cit.* in footnote 3.

[26] For calculations of prevalence, see Lee, *loc. cit.* in footnote 14.

[27] For evidence on changes in the desired birth interval, see Ryder, *loc. cit.* in footnote 9, Table 5, p. 447; for changes in the actual closed interval see Lee, 'Target,' *loc. cit.* in footnote 8, Table 3, p. 460. For older ages, empirical work (Lee, *ibid.*) has shown that the additional expected fertility per woman of incomplete fertility is relatively constant, as noted above in the discussion of irreversibility. This can be shown to imply that the closed birth interval and birth rate of such women must be constant, if the overall birth rate is to be proportional to additional desired fertility, as I have assumed. For the younger age groups, however, neither the additional desired fertility, or the birth rate per woman of incomplete fertility is constant; both vary systematically with additional desired fertility.