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A Relational Model of Mortality at Older Ages in Low Mortality Countries*

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The level and shape of the age pattern of mortality at older ages has been an object of interest since the first life table was produced. Some of this interest reflects practical concerns. Demographers require information on age patterns of mortality in order to construct mortality indices and to project the size and age structure of a population; actuaries need such information to calibrate annuities and insurance premiums. There are also theoretical reasons for concern with age patterns of mortality. For many years, it was hoped that mathematical representations of the shape of the mortality distribution would cast light on the nature of the underlying ageing process. However, this goal proved illusory as, for example, very different models of the ageing process were shown to produce the same mathematical expression for death rates as a function of age.[1] Other models which produced different predictions have proved to be indistinguishable in imperfect data that become very sparse at the oldest ages.

Even though the search for a 'law' of mortality has been frustrated, the empirical examination of age patterns has contributed to a basic understanding of the nature of the ageing process. First, the age-slope of the death rates is often used as an indicator of a population's rate of ageing. Thus, the fact that a species is found for which death rates in old age level off is taken to be an indicator that individual ageing may not be inevitable and inexorable.[2] Secondly, the rate of change of death rates in old age with time (rather than with age) bears on the degree to which very high mortality at very old ages is genetically programmed, or susceptible to important environmental influences. One school of thought holds that human life expectancy is unlikely to exceed 85 years without major interventions to alter the fundamental biology of ageing.[3] The implication

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[1] For example, Yashin *et al.* have shown that a model with fixed gamma-distributed frailty yields the same functional form as several models with changing frailty of the type suggested by Le Bras. A. I. Yashin, J. W. Vaupel and I. A. Iachine, 'Why life spans are different? Several concepts – the same model'. Duke University, April 1993.

[2] This is the interpretation given by the authors themselves and by many commentators to the dramatic findings that death rates stop increasing at advanced ages among Medflies. J. R. Carey, P. Liendo, D. Orozco and J. Vaupel, 'Slowing the mortality rates at older ages in large Medfly cohorts', *Science*, **258** (1992), pp. 457–461.

[3] This position is most closely associated with the name of James Fries, especially in 'Aging, natural death, and the compression of morbidity', *New England Journal of Medicine*, **303** (1980), pp. 130–136. The limit of 85 years was also endorsed in a prominent paper by demographers and gerontologists. S. J. Olshansky, B. A. Carnes and C. Cassell, 'In search of Methuselah. Estimating the upper limits of human longevity', *Science* **250** (1990), pp. 634–640. Although the concept of a fixed upper limit has been widely criticized it has been defended

that environmental influences have little effect on death rates at very high ages should be reflected in mortality levels that are nearly constant over time at those ages, or at least show smaller improvements than those observed at younger ages, as the limit of 85 years is approached.

In this paper we summarize data on mortality in old age in industrialized countries for the period 1950 to 1985. The focus is on the typical shape of the age pattern of mortality, although we also present results related to mortality trends. In particular, we construct a 'standard' age pattern of mortality by single years of age from age 45 to 99. We show that deviations of individual populations from the standard can be efficiently modelled by a two-parameter linear transformation of the logit of the standard death rates. The distinguishing feature of this study of old-age mortality is that we have constructed our standard age pattern of mortality by using a large amount of international data that have passed rigorous tests of quality. We have shown earlier that coverage and content error – especially age misreporting – bedevil attempts to establish age patterns of mortality at very old ages; but enough data sets survive our quality checks to make a reasonably firm foundation for identifying old age mortality patterns possible.[4] In addition, unlike most other international studies, we have analysed data by single year of age.

CONSTRUCTING A RELATIONAL MODEL OF OLD-AGE MORTALITY

Demographers typically describe the age pattern of mortality in one of three ways. First, mathematical representations express age-specific death rates (or some transformation thereof) of a single population as an algebraic function of age. Gompertz, Makeham, Perks, Beard, and Heligman and Pollard have contributed to a long history of attempts to represent the age pattern of mortality mathematically.[5] Secondly, model life table systems summarize the experience of many populations by displaying standard age-specific mortality patterns at different levels of mortality, usually indexed by the expectation of life at birth. The major model life table systems have been produced by Coale and Demeny, the United Nations, and Coale and Guo.[6]

Finally, relational models combine features of both of these. The basic relation between mortality and age is expressed empirically in a tabulated set of values, as in model life tables, while mortality in an actual individual population is modelled through a mathematical transformation of that standard. Relational models are parsimonious; unusual features of the age pattern of mortality are captured in the empirical standard,

by biologists who have pointed out that functional decline with age affects nearly all physiological systems and suggest that the specific disease processes that 'cause' death are basically irrelevant to the degraded capacities associated with 'normal ageing'. L. Hayflick, 'The human life span', in G. Lesnoff-Caravaglia (ed.), *Realistic Expectations for Longer Life* (Human Sciences Press, New York), pp. 17–34.

[4] G. A. Condran, C. L. Himes and S. H. Preston, 'Old age mortality patterns in low-mortality countries: An evaluation of population and death data at advanced ages, 1950 to the present', *Population Bulletin of the United Nations*, No. 30 (1991), pp. 23–61.

[5] B. Gompertz, 'On the nature of the function of the law of human mortality and on a new mode of determining the value of life contingencies'. *Philosophical Transactions of the Royal Society*, **115** (1825), pp. 513–585; W. M. Makeham, 'On the law of mortality', *Journal of the Institute of Actuaries*, **13** (1860), pp. 325–358; W. Perks, 'On some experiments in the graduation of mortality statistics', *Journal of the Institute of Actuaries*, **63** (1932), pp. 12–57; R. E. Beard, 'A theory of mortality based on actuarial, biological, and medical considerations'. In IUSSP, *Proceedings of the International Population Conference, New York, 1961*. Vol. 1 (Liège, 1963), pp. 611–625; L. Heligman and J. Pollard, 'The age patterns of mortality', *Journal of the Institute of Actuaries*, **107** (1980), pp. 49–80.

[6] A. J. Coale and P. Demeny, *Regional Model Life Tables and Stable Populations* (Princeton University Press, Princeton, 1966); A. J. Coale, P. Demeny and B. Vaughan, *Regional Model Life Tables and Stable Populations*, 2nd Ed. (Academic Press, New York, 1983); United Nations, *Model Life Tables for Developing Countries* (Population Study No. 77, New York, 1982); A. J. Coale and G. Guo, 'Revised regional model life tables at very low levels of mortality', *Population Index*, **55** (1989), pp. 613–643.

rather than in a large number of parameters in a mathematical equation. In addition, a relational model requires only one tabulated function, rather than the long series demanded by model life table systems. Brass introduced relational models to the analysis of mortality.[7] More recently, Ewbank, deLeon and Stoto have produced relational models of mortality[8] and Lee and Carter[9] have demonstrated their value in summarizing mortality change in the United States during the twentieth century.

The relational model of old-age mortality that we have constructed summarizes the annual deaths and enumerated populations from 16 industrialized countries and covers the period from 1950 to 1985. For most countries and periods, the population data we have used come from census enumerations that were independent of death registration. However, in the few cases where there were no census enumerations, we used population estimates from population registers to check the quality of the data and to calculate mortality rates. Table 1 lists the countries and periods included in the final data set, along with the sources of population estimates. Only deaths and population figures recorded by single year of age were included in the data base. Moreover, the quality check that we performed restricted inclusion to periods bounded by enumerations. France, East and West Germany, and the United States are notable absentees, because the data for a complete intercensal period were either not available, or not in a format that could be used in the quality checks.

For each country and period, our basic data-quality test examined the consistency between the deaths registered during an intercensal period and the two population estimates that bound that period. These were derived from census enumerations, or from population registers. We focused on the ratio of the actual population enumerated in a cohort at the second census to the population in that cohort predicted from the earlier census, and the intervening deaths.[10] A value of exactly 1.00 indicates perfect consistency between the sources. For most countries, the deviations of the ratios from 1.00, and hence the inconsistency between sources, increased with age. Data were not included in the set used to construct the standard beyond the age at which a five-year moving average of single-year ratios centred on the age deviated from 1.00 by ± 0.05. As a result, the number of ages included in the analysis of death rates varies from country to country and from period to period. The data for three countries – Czechoslovakia, Ireland, and Northern Ireland – were sufficiently inconsistent throughout the age range for us to exclude them entirely from the final data set. For females, only four country/period combinations met the consistency requirement up to age 99; for males, only three met it, and for either sex none had consistent data above age 99. Table 1 gives the last age for which death rates were included for each country and period.

The data for each country in the final data set included the enumerated population at each census date, an estimate of the mid-year population for each intercensal year, and estimates of the number of deaths for each calendar year, each by single year of age. Using these data, we calculated death rates by single year of age for five-year periods between censuses. When censuses were more than five years apart, intercensal periods could contribute more than one set of rates and/or could contain years that were not used in any of the estimated death rates. Deaths which occurred during the fragments

[7] W. Brass, 'On the scale of mortality'. In W. Brass (ed.), *Biological Aspects of Demography* (Taylor & Francis, London, 1971).

[8] D. C. Ewbank, J. G. Gomez de Leon and M. A. Stoto, 'A reducible four-parameter system of model life tables', *Population Studies*, **37** (1983), pp. 105–128.

[9] R. Lee and L. Carter, 'Modeling and forecasting U.S. mortality', *Journal of the American Statistical Association*, **87** (1992), pp. 659–670.

[10] A detailed discussion of the data-quality tests is presented in Condran, Himes and Preston, *loc. cit.* in fn. 4.

Table 1. *Countries, time periods, and oldest age*[1] *used to construct the standard mortality pattern*

Country	First year of period[2]	Oldest age included: Females	Oldest age included: Males	Country	First year of period[2]	Oldest age included: Females	Oldest age included: Males
Australia	1961	81	81	Hungary	1960	85	85
	1966	81	81		1965	85	85
	1971	81	81		1970	85	85
	1976	91	89		1975	85	85
Austria	1961	87	78	Italy	1951	82	85
	1966	87	78		1956	82	85
	1971	86	78		1961	85	84
	1976	86	78		1971	87	83
Belgium	1948	88	79	Japan	1955	86	83
	1953	88	79		1960	88	88
	1957	88	79		1965	87	88
	1962	88	87		1970	89	85
	1966	88	87		1975	88	86
	1971	88	83		1980	87	87
	1976	88	83				
				Netherlands	1956 (R)	91	90
Canada	1951	82	82		1961 (R)	96	99
	1956	80	81		1966 (R)	94	96
	1961	81	83		1971 (R)	90	90
	1966	80	81		1976 (R)	91	90
	1971	88	88				
	1976	90	90	New Zealand	1951	81	81
					1956	81	81
Denmark	1950	86	87		1961	81	81
	1955	86	87		1966	81	82
	1960	92	91		1971	81	81
	1965	87	87		1976	90	91
	1970 (R)	87	87				
	1976 (R)	89	91	Norway	1960	88	87
	1981	89	97		1965	88	87
					1970	92	91
England and Wales	1951	81	79		1975	92	91
	1956	81	79				
	1961	81	79	Scotland	1951	80	79
	1966	81	79		1961	80	82
	1971	87	86		1971	92	92
	1976	87	86				
				Spain	1960	77	75
Finland	1951	80	83		1965	77	75
	1956	80	83				
	1961	81	82	Sweden	1951 (R)	94	94
	1966	81	82		1956 (R)	94	91
	1971	89	87		1961 (R)	89	88
	1976 (R)	95	95		1966 (R)	90	90
	1981 (R)	99	95		1971 (R)	99	98
					1976 (R)	99	99
					1981 (R)	99	99

[1] Age at which a five-year moving average of the ratios of expected to actual populations deviated from 1.00 by 0.05 or more.

[2] (R) indicates that population estimates come from a population register; in all other cases, population estimates come from censuses.

Table 2. *Standard schedule of mortality for males and females estimated from a logit transformation of m(x)*

	Females		Males	
Age	Logit of the death rate	Death rate based on logit	Logit of the death rate	Death rate based on logit
45	−5.94558	0.00261	−5.42940	0.00437
46	−5.86007	0.00284	−5.33531	0.00480
47	−5.76693	0.00312	−5.23099	0.00532
48	−5.66281	0.00346	−5.13399	0.00586
49	−5.57526	0.00378	−5.02300	0.00654
50	−5.49734	0.00408	−4.93471	0.00714
51	−5.42409	0.00439	−4.83709	0.00787
52	−5.31337	0.00490	−4.72272	0.00881
53	−5.25002	0.00522	−4.62753	0.00968
54	−5.15357	0.00575	−4.42406	0.01073
55	−5.08622	0.00614	−4.43314	0.01174
56	−4.98780	0.00677	−4.32837	0.01302
57	−4.90565	0.00735	−4.23117	0.01433
58	−4.79513	0.00820	−4.13023	0.01582
59	−4.71158	0.00891	−4.03053	0.01745
60	−4.61211	0.00983	−3.93937	0.01909
61	−4.53162	0.01065	−3.85509	0.02073
62	−4.40881	0.01202	−3.74178	0.02316
63	−4.30915	0.01327	−3.64221	0.02553
64	−4.20369	0.01472	−3.54836	0.02797
65	−4.09496	0.01638	−3.44985	0.03077
66	−4.00670	0.01787	−3.37510	0.03308
67	−3.88824	0.02007	−3.27458	0.03645
68	−3.77281	0.02247	−3.18171	0.03986
69	−3.66060	0.02507	−3.08497	0.04373
70	−3.54205	0.02814	−2.99089	0.04784
71	−3.43921	0.03109	−2.90956	0.05168
72	−3.30016	0.03557	−2.79584	0.05755
73	−3.18068	0.03990	−2.69967	0.06299
74	−3.05790	0.04488	−2.60260	0.06897
75	−2.93997	0.05021	−2.50570	0.07546
76	−2.81659	0.05643	−2.40211	0.08301
77	−2.70751	0.06253	−2.31442	0.08994
78	−2.57696	0.07064	−2.20445	0.09935
79	−2.45570	0.07902	−2.10086	0.10901
80	−2.33124	0.08857	−2.00393	0.11879
81	−2.22333	0.09767	−1.91595	0.12831
82	−2.07328	0.11172	−1.78485	0.14371
83	−1.94636	0.12495	−1.66780	0.15872
84	−1.82815	0.13846	−1.55382	0.17454
85	−1.71233	0.15286	−1.44786	0.19033
86	−1.59013	0.16937	−1.34584	0.20655
87	−1.47708	0.18587	−1.24805	0.22304
88	−1.35992	0.20425	−1.12869	0.24440
89	−1.23052	0.22609	−1.02338	0.26437
90	−1.12178	0.24568	−0.89735	0.28960
91	−0.98622	0.27166	−0.75228	0.32032
92	−0.84731	0.30000	−0.62639	0.34833
93	−0.69431	0.33307	−0.47906	0.38247
94	−0.59634	0.35518	−0.40893	0.39917
95	−0.51526	0.37396	−0.21081	0.44749
96	−0.35075	0.41320	−0.18447	0.45401
97	−0.24800	0.43832	−0.05900	0.48525
98	−0.10969	0.47260	0.00631	0.50158
99	−0.14514	0.46378	0.20148	0.55020

of calendar years on either end of the interval when censuses were not taken on January 1 were estimated by pro-rating the deaths for the calendar year based on the timing of the census. The population at risk of dying is the sum of the mid-year population estimates for each of the five years of data that were included in the death rate. The standard age pattern of mortality reported here was derived from a final data set that consists of 82 separate mortality schedules for each sex.

Constructing a relational model does not require the assumption of any particular functional form (e.g. a logistic) to represent the age pattern of mortality. Instead, the data themselves determine the shape of the standard. However, there were several choices to be made in constructing the mortality variable to be analysed. We examined age-specific death rates (${}_1m_x$) rather than age-specific probabilities of dying or surviving (${}_1q_x$ or ${}_1p_x$) because our data related to age intervals, rather than to exact ages; constructing probabilities from rates would have required an additional assumption.[11]

We also faced a choice between a logarithmic and a logit transformation of the death rates, both of which have been widely used in mortality analysis to linearize the relation between death rates and age (beyond age 40 or so). In this regard, the logit transformation typically performs better than the logarithmic.[12] Our goal, however, was not to linearize relations between mortality and age, but to produce a linear relationship between the transformed death rates of a particular population and those in a 'standard' schedule derived from the data. In other words, we sought a transformation which made it possible for the age pattern of mortality in any population under study to be expressed, to a close approximation, as a linear function of the standard produced:

$$\Psi_j(x) = \alpha_j + \beta_j \Psi_s(x), \qquad (1)$$

where $\Psi_j(x)$ is some transformation of the death rate at age x in population j; α_j, β_j are parameters appropriate for population j; $\Psi_s(x)$ is some transformation of the death rate at age x in the standard.

However, the two issues are related, because, if each of two functions is linearly related to age, the two must be linearly related to one another. On this basis, we predicted – correctly, as described below – that the logit transformation would provide a better fit in Equation (1) than the logarithmic. Therefore, we constructed our standard by using a logit transformation of age-specific death rates.

Designating $Y_j(x)$ as the logit transformation of death rates at age x to $x+1$ in population j, we used ordinary least squares regression to estimate parameters of the following equation:

$$Y_j(x) = \delta + \Sigma \beta_x I_x + \Sigma \gamma_j J_j, \qquad (2)$$

where I_x is a dummy variable for age x ($=1$ if the observation relates to age x, 0 otherwise); J_j is a dummy variable for population j ($=1$ if the observation relates to population j, 0 otherwise); $\delta, \beta_x, \gamma_j$ are coefficients to be estimated.

There are 54 dummy variables to take account of the age (from 46 to 99) to which an observation (i.e. an age-specific mortality rate) relates. Age 45 was omitted as the reference category. Eighty-one dummy variables represent the populations (defined as country and period combinations) to which a particular age-specific death rate refers, with Sweden, 1950 to 1954, as the omitted population.

We used Equation (2) to develop the standard rather than, for example, a simple average of death rates at a particular age because the number of observations varied with age. Data that passed the quality test at age 45, for example, were available for all 82 populations but for females only four populations met the criteria for inclusion at age

[11] See C. L. Chiang, *Introduction to Stochastic Processes in Biostatistics* (John Wiley, New York, 1968).

[12] See Perks, *loc. cit.* in fn. 5; Beard, *loc. cit.* in fn. 5; Heligman & Pollard, *loc. cit.* in fn. 5; S. Horiuchi and A. J. Coale, 'Age patterns of mortality for older women. An analysis using the age-specific rate of mortality change with age', *Mathematical Population Studies*, **2** (1990), pp. 245–268.

99. By controlling (additively in logits) the population from which an observation was derived, Equation (2) avoids the bias that might result in a simple average from a systematic relation between the level and/or age pattern of mortality and the likelihood of contributing an observation at a particular age. Nevertheless, because the number of observations declines with age, the variability in the estimates of age patterns increases with age.

Equation (2) fits the data extremely well, with $R^2 = 0.9970$ for females and 0.9966 for males. The coefficients β_x from Equation (2) constitute the standard age pattern of mortality. As expected, these coefficients increase systematically with age for both sexes (with the exception of age 98–99 for females), an outcome that reflects neither our choice of a logit transformation nor any other assumptions made, but rather the underlying empirical mortality rates. Because the raw coefficients simply predict the death rates for the reference population (Sweden 1950–54), we used the mean of the coefficients of the dummy variables for population (J_j) to obtain a predicted mean value of $Y(x)$ at each age for the entire sample. Table 2 contains the standard mortality pattern expressed in logits and the age-specific death rates implied by these logit values.

We also calculated the standard by using a natural logarithmic rather than a logit transformation of the death rates to observe whether the logarithmic or the logit transformation would produce the more linear relation between sets of mortality rates in our data base and the standard mortality pattern. The answer lies in the fit (R^2) of Equation (1) to each of the 82 populations. When Equation (1) is fitted to standards expressed alternatively through logarithmic and logit transformations, the mean values of R^2 across all the populations are:

	Logit	Log
Males	0.99840	0.99841
Females	0.99909	0.99901

The results are very similar for the two transformations. The logit produces a slightly better fit than the logarithm for females; the mean values of R^2 are virtually identical for males. When the 32 individual country/sex combinations are examined, the value of R^2 (to six digits) averaged across time periods, and separately for males and females, was greater for the logit in 17 instances, for the logarithm in nine, and there was no difference in six cases. There is no guarantee that the logit version will provide better predictions of untransformed death rates, but we avoided this test because it gives, in effect, more weight to observations at very high ages when the variance in untransformed death rates is highest. On the basis of its very slight advantage, we elected to estimate the standard mortality pattern by using a logit transformation.

CHARACTERISTICS OF THE STANDARD PATTERNS

Both logit and untransformed values of the standard patterns of age-specific mortality are displayed in Figure 1. The logits of the standard for males are highly linear with age ($R^2 = 0.9983$), while for females the relationship between mortality level in logits and age is slightly curvilinear ($R^2 = 0.9959$). For both sexes, the age progression of death rates above age 92 in the standard is somewhat irregular, and probably reflects the declining number of observations as age advances. Rates are higher for males than for females at every age, with the greatest sex-difference (in logits) occurring between ages 55 and 65.

Horiuchi and Coale[13] in a study of mortality in 14 data sets from 10 countries have noted a slightly curvilinear relationship between the logarithm of mortality rates for females and age. They found that first differences in the natural logarithm of ${}_5m_x$ peaked

[13] *loc. cit.* in fn. 12.

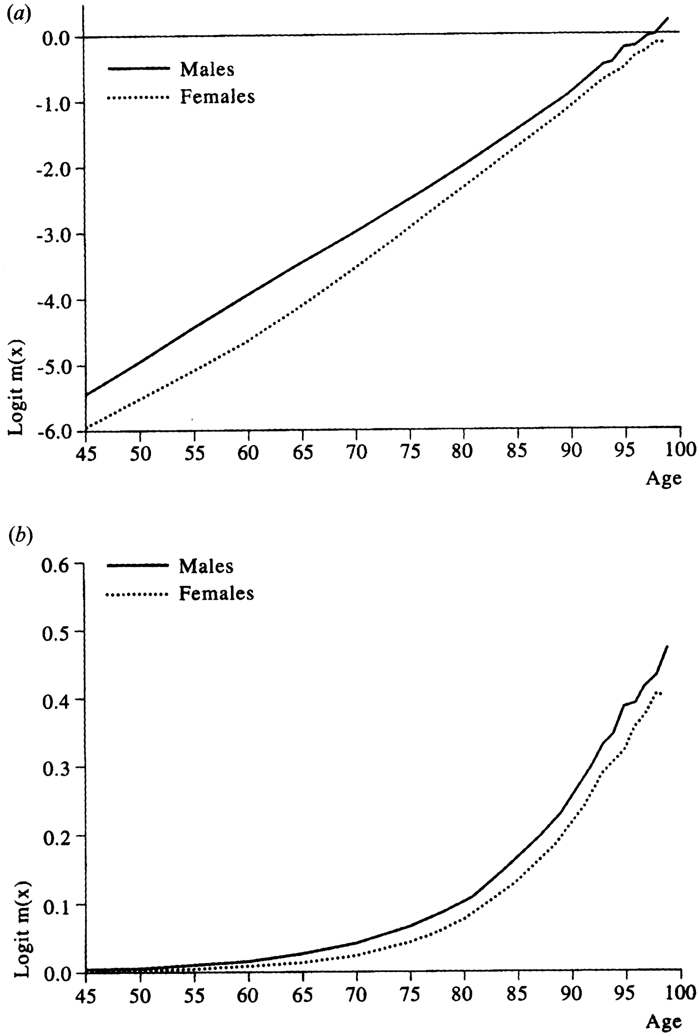


Figure 1. Standard mortality pattern for males and females. (*a*) Logits of the age-specific death rates; (*b*) Age-specific death rates.

at around age 75. In order to see whether a similar pattern occurs in our standard, we have converted the standard pattern (estimated from logits of death rates) into logarithms and taken first differences of the logarithms for ages ten years apart. Results are shown in Table 3. As suggested, the series of first differences for females peaks between ages 70 and 80. In both studies, the changes in the logarithms of the death rates for males differ from those for females; in our standard, they first decline with age, then reach a plateau between the ages of 65 and 85, and finally decline again. Horiuchi and Coale suggested that the pattern among males may reflect the influence of cohort factors related to wartime experience, since many of their sample populations had been combatants in World War I. However, our data, which exclude France and the two Germanies, are less heavily influenced by war participants and also suggest patterns of change in the logarithm of ${}_1m_x$ that differ for males and females. One possibility is that the data reflect other cohort influences on males, e.g. cigarette smoking.

The rates of change in the logits of the death rates are no more uniform or regular with age than those in logarithms (Table 3). For females, the pattern of change is similar,

Table 3. *Rates of change in the logarithm and logits of age-specific death rates in the standard mortality schedules*

Age interval	Change in logarithm of death rates between younger and older ages: Females	Males	Change in logit of death rates between younger and older ages: Females	Males
45–55	0.8555	0.9850	0.8594	0.9963
50–60	0.8793	0.9834	0.8852	0.9953
55–65	0.9812	0.9636	0.9913	0.9833
60–70	1.0517	0.9187	1.0701	0.9485
65–75	1.1202	0.8970	1.1550	0.9442
70–80	1.1466	0.9095	1.2108	0.9870
75–85	1.1133	0.9252	1.2276	1.0578
80–90	1.0203	0.8911	1.2095	1.1066
85–95	0.8946	0.8549	1.1971	1.2371
89–99	0.7185	0.7329	1.0854	1.2249

Source: Calculated from the logit standard shown in Table 2 and from standard death rates based on a logarithmic transformation not shown.

although the former is probably best described as rising to a plateau. Among males, the slow irregular decline in changes in the logarithms is replaced by a U-shaped age pattern of change in the logits. In short, the standard pattern shows systematic departures from linear relations with age whether expressed in logarithms or logits. These departures from linearity are especially visible for females. The standard pattern derived from our data, however, reflects these nuances.

The age-specific death rates in our standard, shown in Table 2, produce life expectancies at age 45 of 32.9 years for females and 27.0 years for males. These are roughly the values for whites in the United States in 1959–61 – 32.5 and 27.3 years respectively.[14] The life expectancies at age 65 implied by our standards are 15.8 for females and 13.0 for males and at age 85, 4.7 and 4.1 respectively.

COMPARISON OF THE RELATIONAL MODEL WITH REGIONAL MODEL LIFE TABLES

We compared our standard pattern to new model life tables for older ages that Coale and Guo[15] produced from the World Health Organization's compilation of quinquennial age data for the 1980s for many of the countries that we used in our analysis. These models are designed to replace the life tables for the North, West, South, and East regions contained in Coale and Demeny's regional model life table system.[16] We made the comparison by summing ${}_1m_x$ values over a five-year age range in our standard to produce a statistic comparable to $-\ln {}_5p_x$ in the model life tables. In both cases, division by five yields an average mortality rate for the age range.

To match our standard, which contains countries from all four of the regional families, to the appropriate life table in Coale and Guo's model system, we have chosen levels from each of the regional patterns that bracket the value of e_{45} implied by our standard. It is convenient to make the comparison by examining the difference between

[14] United States. National Center for Health Statistics. *Vital Statistics of the United States, 1980. Life Tables* Vol. ii, Section 6 (Washington, D.C. Government Printing Office, 1984).
[15] Coale and Guo, *loc. cit.* in fn. 6.
[16] Coale and Demeny (1983), *op. cit.* in fn. 6.

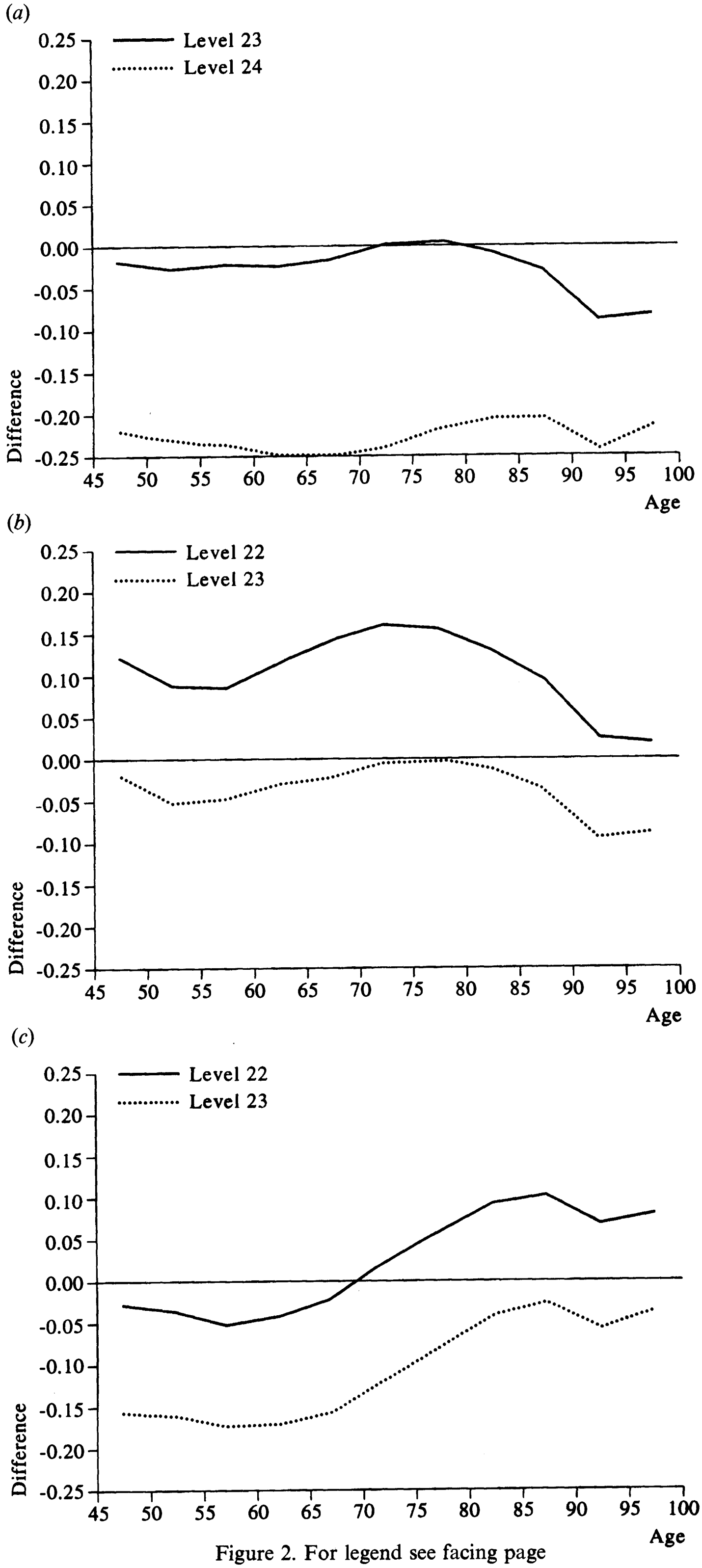


Figure 2. For legend see facing page

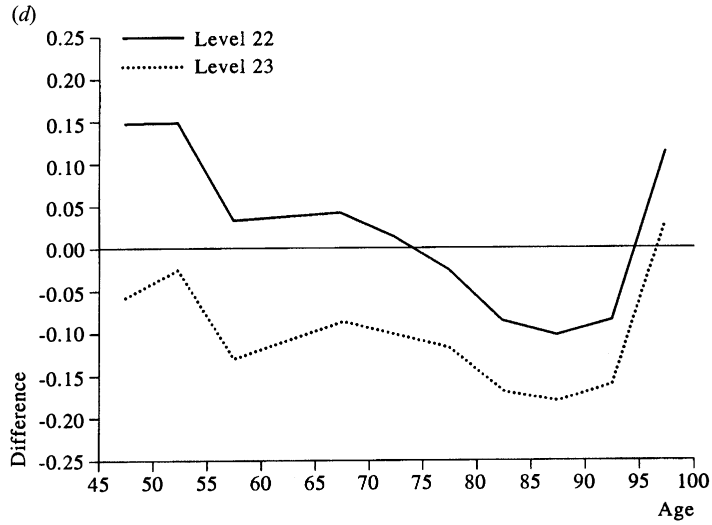


Figure 2. Differences between the logits of the death rates in model life tables and the standard, females. (*a*) West model life table; (*b*) east model life table; (*c*) south model life table; (*d*) north model life table.

the logits of death rates in a model life table and in our standard pattern, $Y_{mlt}(x) - Y_s(x)$. Figures 2*a–d* display the differences between the four families of model life tables and our standard for females. The West and East regional models fit the standard developed here best, although in both mortality levels in the age range 75–90 are higher relative to those at younger ages than those observed in our standard. The South and North regional models display two extremes. Compared to our standard, mortality in the South model is quite high at older relative to young ages, while in the North, the opposite is true. None of the regional model life tables match our standard for males (Figures 3*a–d*) as well. In the West region, which replicates our model best, levels of male mortality at young ages relative to older ages are higher than the standard. The age pattern of mortality in the South model is particularly far from our standard, with relatively higher mortality at the youngest and oldest ages, and lower mortality between ages 60 and 75.

Except for the South and North models for males, where deviations from the standard are not monotonic, Coale and Guo's models may be approximated by choosing an appropriate slope coefficient, β, in Equation (1). However, the geographical patterns earlier described by Coale and Demeny and updated by Coale and Guo[17] are probably no longer necessary for describing old-age mortality in countries with low mortality. Coale and Guo noted that in recent data patterns for females differed little by region, and adopted a single age pattern similar to that in the North model to which all regions converge at the lowest levels of mortality. Regional differences in the age patterns of mortality at higher mortality levels chiefly reflect regional variation in the incidence of infectious diseases[18] and should be expected to diminish when chronic and degenerative diseases dominate the mortality profile.

The regional model life tables differ from one another in the 'shape' of their age-pattern of mortality at a particular 'level' of mortality. Since our relational system contains both a shape parameter β and a level parameter α, the importance of regional

[17] Coale and Demeny (1966 and 1983), *op. cit.* in fn. 6; Coale and Guo, *loc. cit.* in fn. 6.

[18] S. H. Preston, *Mortality Patterns in National Populations* (Academic Press, New York, 1976).

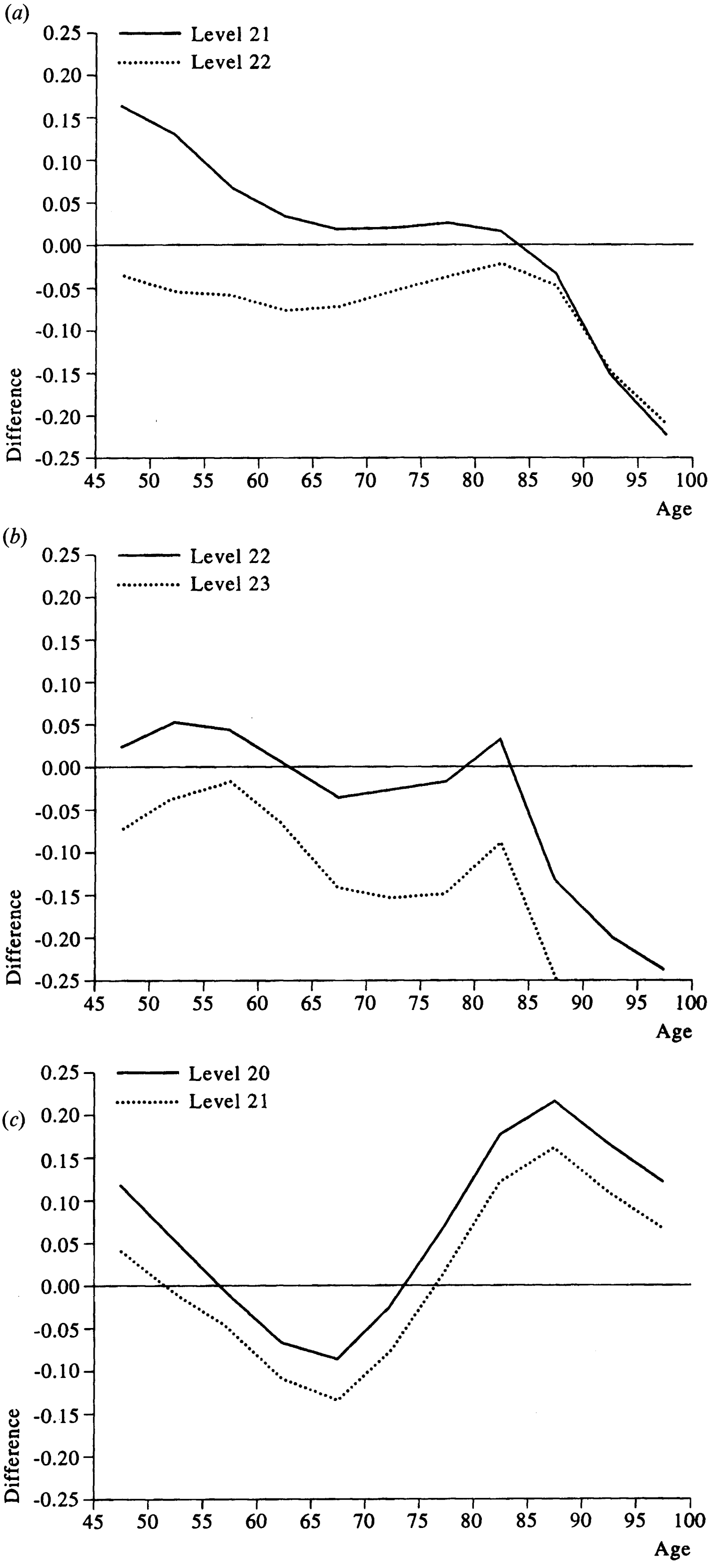


Figure 3. For legend see facing page

(d)

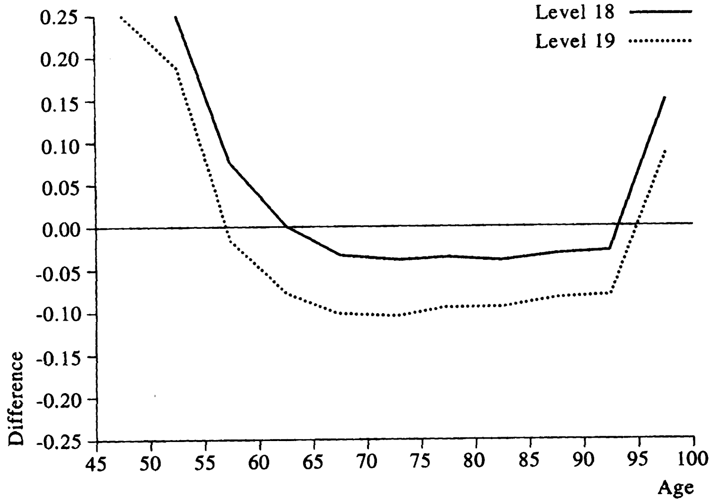


Figure 3. Differences between the logits of the death rates in model life tables and the standard, males. (*a*) West model life table; (*b*) east model life table; (*c*) south model life table; (*d*) north model life table.

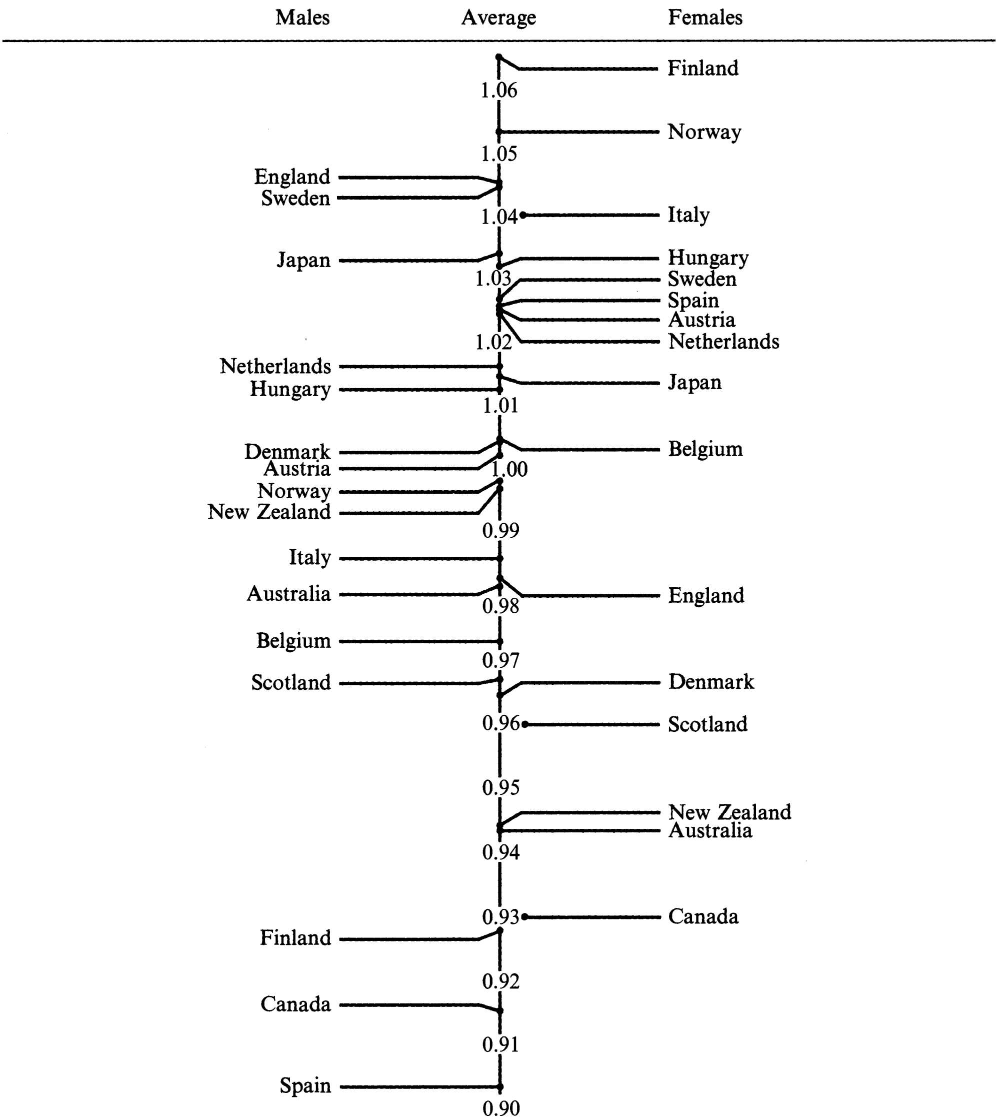
groupings can be investigated by examining the regional clustering of β-values. When we fit Equation (1) to each of the 82 populations in our data set, the deviations of individual mortality schedules from our standard pattern do not cluster appreciably into geographical groups. The value of β in this equation is the 'slope' of age-specific death rates and when greater than 1.00 indicates that, relative to the standard, mortality in a population is higher at older than at younger ages. Table 4 shows the value of β averaged across time periods in each of the countries in the data set.

High values of β for females occur in both northern and southern Europe. The countries with low values of β for females form a more coherent group, with the three former English colonies at the bottom of the list, and with values of β less than 1.00 in England and Scotland. Among males, the ordering of β's with respect to geography is essentially random. Countries that we might expect to show similar patterns of mortality – England and Scotland, New Zealand and Canada, Italy and Spain – occupy very different positions on the ladder. There is, however, considerable consistency over time in the value of β for a particular country. (See Table 5 below.) In sum, it would appear that regional factors play a relatively minor role in conditioning age patterns of mortality at older ages in low-mortality countries.

COMPARISON OF THE RELATIONAL MODEL TO MATHEMATICAL REPRESENTATIONS

Equation (1) and the standard mortality schedules provide a two-parameter transformation of mortality rates that can be applied to data from any country. Both Gompertz's model, in which the logarithm of death rates is a linear function of age, and a model linear in logits of death rates, are also two-parameter models. It is, therefore, interesting to examine whether the relational standards that we have constructed fit the data better than these other two models. In order to make this comparison, we fitted each of the 82 age schedules of mortality to these other models by ordinary least squares regression and averaged values of R^2 across the entire data set, as we had previously done for our standard. Each of the models explains more than 99 per cent of the variance

Table 4. *The value of β averaged across time periods for countries included in the standard mortality pattern*



in age-specific mortality rates in the data for males and females. The average amount of variance left *unexplained* $(1-R^2)$ by the different models for each of the 82 populations is:

	Two-Parameter relational model	Death rates linear with age in logits	Death rates linear with age in logarithms
Females	0.00091	0.00564	0.00398
Males	0.00160	0.00195	0.00237

The relational model performs better than the other two for females, with only one quarter to one-sixth as much variance left unexplained. However, the relational model

Table 5. *The intercept (a) and slope (b) of the regression of each country/time period mortality schedule on the standard*

Country/ Time period[1]	Females		Males	
	α	β	α	β
Australia				
1961–1966	−0.11031	0.95109	0.08888	0.98442
1966–1971	−0.14383	0.94246	0.14549	0.99495
1971–1976	−0.30162	0.92731	0.08894	0.99479
1976–1981	−0.43681	0.93699	−0.16396	0.97083
Austria				
1961–1966	0.19487	1.01848	0.26370	1.04028
1966–1971	0.18434	1.01777	0.26968	1.02956
1971–1976	0.11778	1.02225	0.14148	1.00747
1976–1981	0.06862	1.03126	−0.09166	0.95581
Belgium				
1953–1957	0.16442	0.96023	−0.20774	0.89129
1957–1961	0.21300	1.01021	−0.03063	0.95908
1962–1966	0.14507	1.01116	0.05485	0.98056
1966–1970	0.11672	1.00851	0.09376	0.98913
1971–1975	0.06909	1.01242	0.16097	1.01723
1976–1980	−0.07182	1.00457	0.12547	1.02626
Canada				
1951–1956	−0.11553	0.93122	−0.26191	0.92132
1956–1961	−0.09328	0.95675	−0.34686	0.89590
1961–1966	−0.23481	0.94352	−0.24986	0.93202
1966–1971	−0.42152	0.91584	−0.27892	0.92616
1971–1976	−0.52751	0.90593	−0.32953	0.92000
1976–1981	−0.64057	0.90300	−0.38029	0.92778
Denmark				
1950–1955	0.18813	1.00068	−0.05668	1.02680
1955–1960	0.16248	1.01620	−0.02260	1.03711
1960–1965	0.11621	1.01963	0.03008	1.03949
1965–1970	−0.10556	0.97502	−0.04403	1.01729
1970–1975	−0.39637	0.91865	−0.17221	0.97980
1976–1980	−0.46797	0.90931	−0.13740	0.98960
1981–1985	−0.56424	0.88714	−0.23586	0.96470
England and Wales				
1951–1956	0.18075	0.99647	0.31953	1.03909
1956–1961	0.13644	1.00317	0.37324	1.06104
1960–1966	0.04188	0.98994	0.37493	1.06381
1965–1971	−0.08383	0.96939	0.35138	1.06320
1971–1976	−0.18337	0.95323	0.16774	1.02681
1976–1981	−0.21621	0.95940	0.17834	1.04710
Finland				
1951–1955	0.60283	1.05375	0.09780	0.92082
1956–1960	0.57833	1.07118	0.08275	0.93124
1961–1965	0.63809	1.09474	0.05544	0.91795
1966–1970	0.58627	1.10096	0.06174	0.91781
1971–1975	0.00358	1.05752	−0.07610	0.89856
1976–1980	−0.06482	1.02978	0.01735	0.95664
1981–1985	−0.23262	1.01930	−0.01125	0.98854
Hungary				
1960–1964	0.39924	1.04399	0.21419	1.04672
1965–1969	0.41719	1.05862	0.27787	1.05774
1970–1974	0.23470	1.01474	0.13827	0.99981
1975–1979	0.16993	0.99123	0.07927	0.95662
Italy				
1951–1956	0.37886	1.03913	0.02435	0.99487
1956–1961	0.26135	1.03630	−0.02445	0.98828

Table 5. (*cont.*)

Country/ Time period[1]	Females		Males	
	α	β	α	β
1961–1966	0.15251	1.02524	−0.03408	0.98517
1966–1971	0.09005	1.04356	−0.07274	0.98728
Japan				
1955–1960	0.11681	0.94280	0.26384	1.01023
1960–1965	0.18937	0.99284	0.29238	1.04320
1965–1970	0.14083	1.01648	0.19799	1.04932
1970–1975	0.05395	1.03480	0.10830	1.06149
1975–1980	−0.10887	1.04557	−0.11113	1.04209
1980–1985	−0.32135	1.03672	−0.29393	1.02128
Netherlands				
1956–1960	0.19784	1.05226	0.10611	1.09104
1961–1965	0.03925	1.03501	−0.08851	1.01820
1966–1970	−0.06123	1.01670	−0.15708	0.99035
1971–1975	−0.13771	1.01247	−0.11777	1.00026
1976–1980	−0.34975	0.98833	−0.16532	1.00324
New Zealand				
1951–1956	−0.03313	0.95520	0.00070	1.00675
1956–1961	0.02310	0.99170	0.09550	1.03360
1961–1966	−0.08687	0.95541	0.09832	1.00972
1966–1971	−0.26036	0.91622	0.07632	0.99574
1971–1976	−0.28823	0.92292	0.02728	0.99126
1976–1981	−0.44202	0.90173	−0.08235	0.98373
Norway				
1960–1965	0.15388	1.07000	−0.12932	1.01912
1965–1970	0.05502	1.05778	−0.16160	0.99902
1970–1975	−0.09302	1.04145	−0.16198	1.00179
1975–1980	−0.22652	1.02542	−0.23769	0.98826
Scotland				
1951–1956	0.30428	0.97963	0.18113	0.97156
1961–1966	0.17958	0.97823	0.24348	0.98935
1971–1976	−0.19175	0.90989	0.06055	0.95583
Spain				
1961–1965	0.12767	1.02153	−0.54944	0.92165
1966–1970	0.03779	1.02075	−0.67477	0.90024
Sweden				
1951–1955	0.18137	1.01177	0.07021	1.06743
1956–1960	0.16413	1.03491	0.07695	1.07904
1961–1965	0.13719	1.04985	0.05957	1.07760
1966–1970	0.00136	1.03649	−0.04233	1.04999
1971–1975	−0.19181	1.01077	−0.14792	1.01904
1976–1980	−0.26812	1.00908	−0.18125	1.00988
1981–1985	−0.34876	1.01140	−0.18920	1.03176

[1] Calendar years of deaths used in the numerator of the mortality rates. Five full years of deaths were used for each rate; six years are listed when population censuses did not occur on January 1.

that we constructed has relatively little advantage over the other models in representing mortality of men. For females, the relational model incorporates the systematic curvature in the relation between logarithms (or logits) of death rates and age in the data, a feature that accounts for its better fit relative to the other two representations. Logarithms or logits of death rates were more linear with age among males (and in the standard for men), and therefore there is less to gain by using the relational model for males. These results demonstrate that representations of (transformed) death rates that

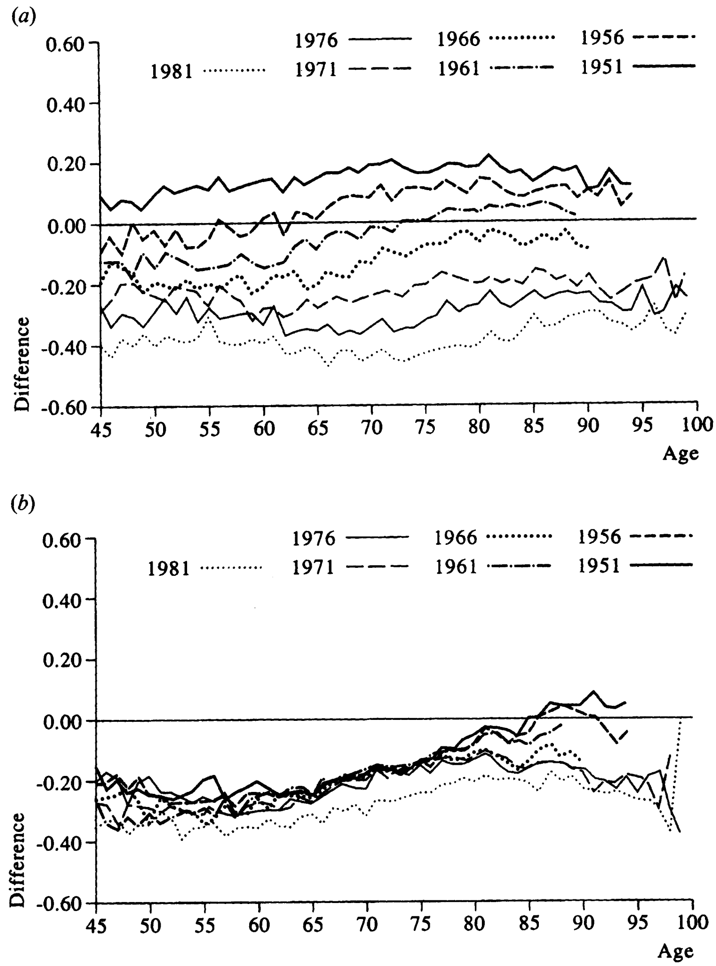


Figure 4. Differences between the logits of the death rates in Sweden and the standard. (*a*) Females; (*b*) males.

are linear with age are more successful (in terms of R^2) for males than for females. Ironically, the non-linearities for females are systematic enough for the relational model, which embodies them, to perform better for females in predicting age-specific death rates in a particular country/period combination than it does for males.

TRENDS IN MORTALITY PARAMETERS

Rather than focusing on one mortality indicator, such as life expectancy at birth, we can summarize recent trends in age-specific death rates at older ages by reference to two parameters: the level of mortality, α, and its slope, β. A change in α shifts the logits of all age-specific death rates by an equal amount, while a change in β shifts the logits of all death rates below age 100 for females and below age 98 for males in the same direction, but more at older than at younger ages. Results of this procedure are shown in Table 5.

For females, α declines over time in nearly every country. There is no country in which α has risen since 1960–65 nor one in which the value in the latest period studied is above

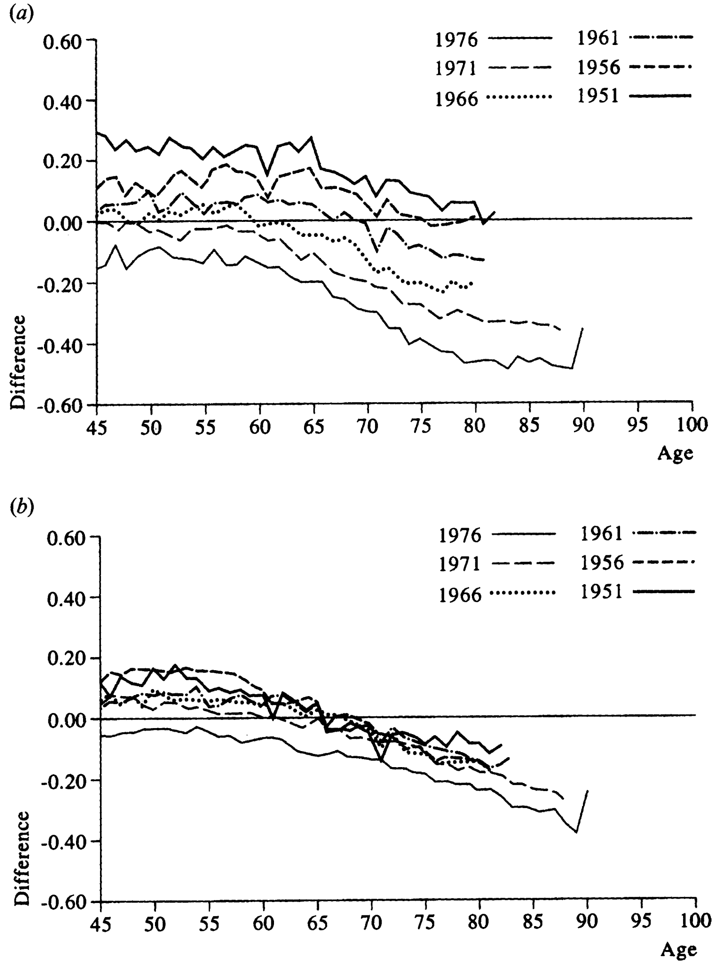


Figure 5. Differences between the logits of the death rates in Canada and the standard. (*a*) Females; (*b*) males.

that in the earliest. Declines were generally fastest between 1965–70 and 1970–75, when they averaged 0.161 for the 14 countries with available data for both those periods. Declines of 0.70 or more over the entire period examined (1950 to 1980) were observed in Denmark and Finland.

Declines in the values of α were smaller and less steady for males than for females. In Belgium, mortality was higher during the last period, 1975–80, than the first, 1950–55. The level of mortality declined by less than 0.15 in Canada, England and Wales, Finland, Hungary, Italy, New Zealand, Norway, and Scotland. In only one country, Austria, was the decline in α as small as this for females.

In a series of articles, Fries[19] has argued that the slope of the age pattern of mortality above age 50 or so should become steeper as sources of 'premature' mortality are eliminated, while genetically programmed senescence makes it very difficult to reduce mortality at the very old ages. Crimmins[20] questioned this hypothesis in a study of U.S.

[19] e.g. J. F. Fries, *loc. cit.* in fn. 3.; 'The compression of morbidity; Near or far?', *The Milbank Quarterly* **67** (1990), pp. 208–232.

[20] E. Crimmins, 'Life expectancy and the older population: demographic implications of recent and prospective trends in old age mortality'. *Research in Aging*, **6** (1984), pp. 490–514.

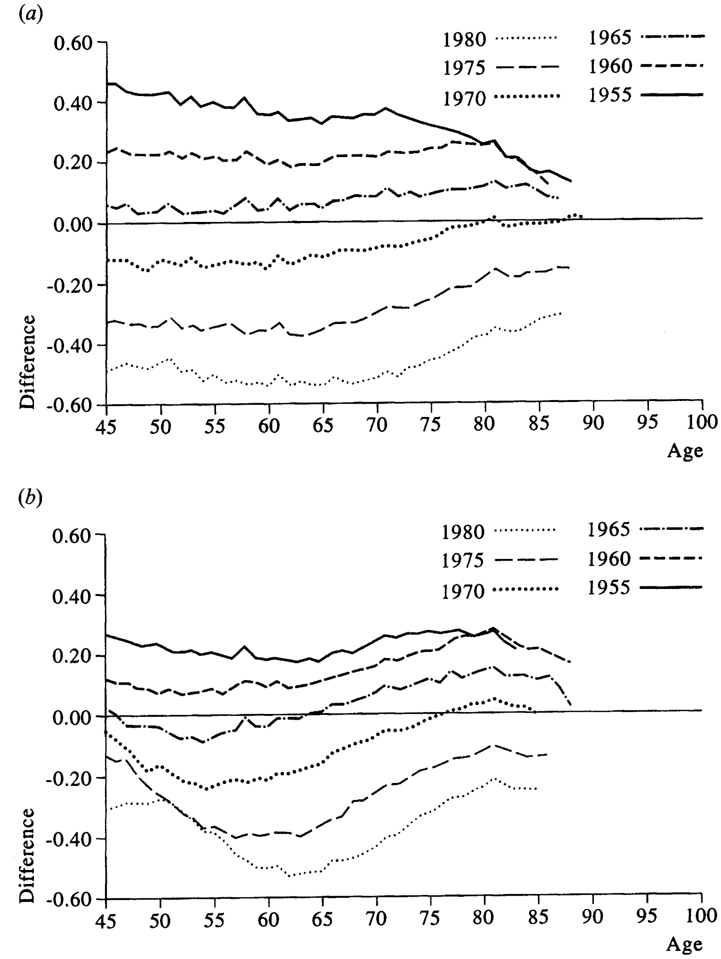


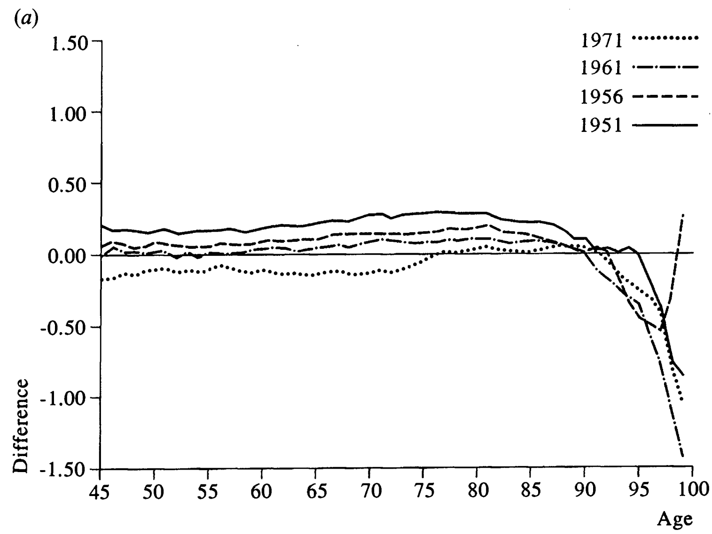
Figure 6. Differences between the logits of the death rates in Japan and the standard. (*a*) Females; (*b*) males.

mortality. Whether mortality has declined less at older ages than at younger, as Fries would suggest, is a question that we can consider by using high-quality international data.

Changes in the β coefficients, presented in Table 5, indicate whether mortality levels (in logits) have declined more at younger than at older ages. While there are other ways to operationalize Fries's hypothesis, the change over time in the slope, β, is a convenient and plausible one. It basically indicates whether the odds of dying $({}_1m_x/(1-{}_1m_x))$ have fallen proportionately more at younger than at older ages. The changes in β over the period of observation for the populations in our data set are summarized below.[21]

Absolute change in slope between earliest and latest observation	Number of countries	
	Females	Males
Increase in β greater than $+0.010$	2	5
No change $(-0.10 < \Delta\beta < 0.10)$	3	1
Decline in β larger than -0.010	10	9

[21] Spain was omitted because observations for this country are only five years apart. In all other countries, the earliest and latest observations are separated by at least 15 years.



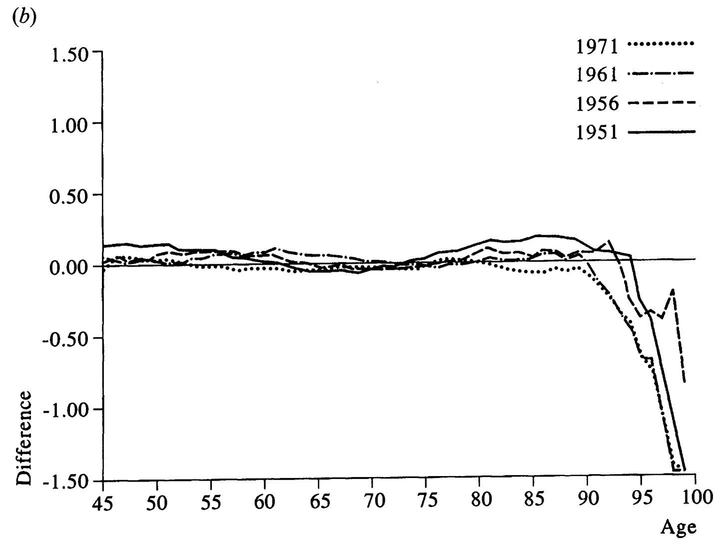


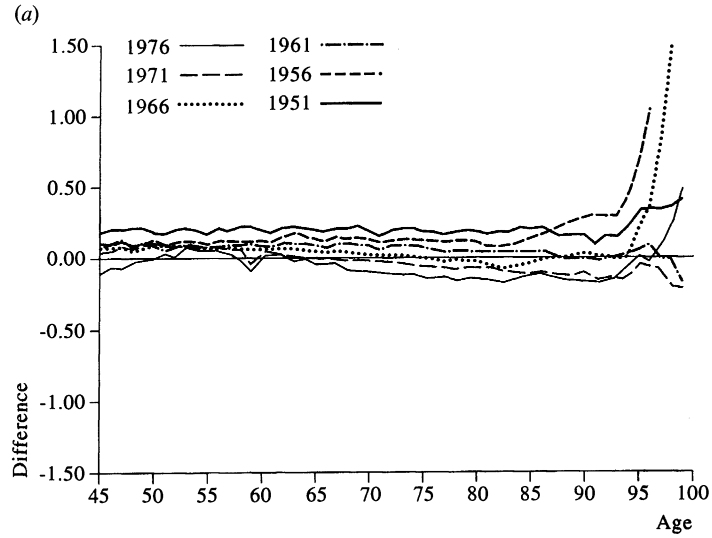
Figure 7. Differences between the logits of the death rates in Italy and the standard. (*a*) Females; (*b*) males.

The declining slopes for both males and females in a sizeable majority of countries suggest that, relative to the standard age pattern, mortality has been improving faster at the very old than at the younger old ages. The decline in slopes has been relatively continuous in most countries. These results are probably related to Coale and Guo's observation that during the past several decades age-specific mortality schedules of females used in their analysis have approached the 'North' pattern, in which among the regional groups mortality is lowest at very old ages relative to the middle ages.[22] However, unlike Coale and Guo, we find a similar pattern for males. One reason why slopes of logits of age-specific death rates have decreased may be that greater progress has been made against cardiovascular diseases than against cancer, whose age-slope of death rates is substantially less steep.[23]

We find little support for Fries's hypothesis in data from low-mortality countries

[22] Coale and Guo, *loc. cit.* in fn. 6.

[23] Preston, *op. cit.* in fn. 18.



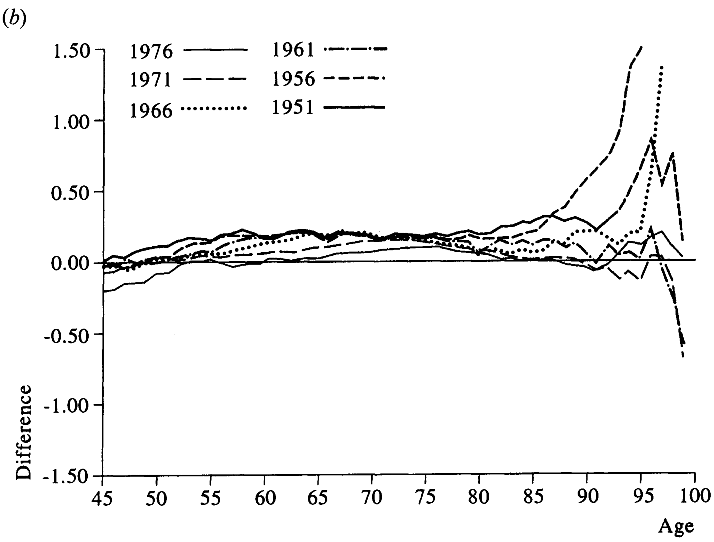


Figure 8. Differences between the logits of the death rates in England and Wales and the standard. (*a*) Females; (*b*) males.

during the post-war period, even though these are clearly the populations in which one would expect to find such support. The persistently rising slope in Japan between 1955 and 1979, an exception to our general finding of declining slopes, is noteworthy, because Fries has frequently used Japanese data to support his arguments. However, mortality at the oldest ages in most other countries has declined sharply (in logits) during the last three decades relative to mortality among the younger old. To have treated mortality at the very old ages as irreducible, at least in the immediate past, would have been a serious mistake.

Table 6. *Fitted values of standard mortality schedule and above at ages 95*

	Females		Males	
Age	Logit of the death rate	Death rate based on logit	Logit of the death rate	Death rate based on logit
95	−0.49568	0.37856	−0.30249	0.42495
96	−0.37361	0.40767	−0.18797	0.45315
97	−0.25153	0.43745	−0.07344	0.48165
98	−0.12945	0.46768	0.04108	0.51027
99	−0.00738	0.49816	0.15561	0.53882
100	0.11470	0.52864	0.27013	0.56713
101	0.23678	0.55892	0.38466	0.59500
102	0.35886	0.58876	0.49918	0.62227
103	0.48093	0.61797	0.61370	0.64879
104	0.60301	0.64634	0.72823	0.67442
105	0.72509	0.67373	0.84275	0.69904
106	0.84716	0.69997	0.95728	0.72258
107	0.96924	0.72497	1.07180	0.74494
108	1.09132	0.74863	1.18632	0.76608
109	1.21340	0.77090	1.30085	0.78598
110	1.33547	0.79174	1.41537	0.80461
111	1.45755	0.81116	1.52990	0.82199
112	1.57963	0.82915	1.64442	0.83814
113	1.70171	0.84576	1.75894	0.85308
114	1.82378	0.86102	1.87347	0.86686
115	1.94586	0.87499	1.98799	0.87953

DEVIATIONS ABOUT THE STANDARD IN INDIVIDUAL COUNTRIES

By examining the difference between death rates in a particular country and the standard, we can conveniently compare the mortality experience of individual countries both with the average and with other countries. Graphs of the differences between death rates in a particular country and the standard are a useful visual tool for identifying idiosyncracies in individual countries. For example, in Figures 4*a–b* the data for Sweden, (among the highest quality in our data set), show that the age pattern of death rates for Swedish women was close to that in the standard, i.e. the slopes of the lines connecting the deviations of age-specific death rates from the standard are close to zero, which implies that the slope coefficient, β, from Equation (1) is close to unity for Sweden. The lines shift downward, but are approximately parallel over time, indicating that mortality decline was very uniform across ages. Relative death rates at very old ages for Swedish males are somewhat higher than at younger ages and decline much less over the period.

The Canadian data (Figures 5*a–b*) also show larger declines for females than for males, and a rather parallel pattern of decline for both sexes. The slope, however, is uniformly negative, indicating better mortality relative to the standard as age advances and a slope coefficient, β, less than one. Note that, as higher ages begin to pass the data quality check in later periods, the general patterns established at earlier ages are maintained.

Japanese mortality (Figures 6*a–b*) deviates far more radically and less systematically from the standard than that of any other country; its patterns of deviations are non-monotonic and change over time. For females, the slope of the age pattern of deviations changes sign, from negative to positive over time; mortality improves much less at very old than at younger ages. For males, the pattern of deviation is bowed, and shows evidence of a cohort trough for those aged 60–65 in 1980–85.[24]

Another use of the standard is to identify bad data for individual countries. We already have one indication of data quality from our tests of consistency between population counts in two censuses and intercensal deaths. It is informative to examine whether, for those ages that failed this test in a particular data set, the excluded data deviated systematically from the standard pattern. In each case that we examined, the answer was positive. Typically, the difference between logits of the excluded death rates and logits of the standard at that age verged sharply downward as age advanced. In other words, death rates calculated from the excluded data appear to be too low at older ages, probably reflecting age overstatement. Figures 7*a–b*, which contain data from Italy, display the typical pattern. The lines are nearly parallel with a slope near zero until around age 90, whereupon they plummet. Note that the vertical scale is enlarged relative to earlier graphs so that the extent of departure from conventional patterns can be fully revealed. Italian data began to fail the consistency test somewhere between ages 82 and 87 (Table 2).

As shown in Figures 8*a–b*, an opposite pattern was observed in the case of England and Wales; in earlier years, the difference between logits, after remaining quite level over a long stretch of ages, rises sharply above age 85 or 90. This pattern may reflect worse age overstatement in deaths than in population counts, or some other deficiency in the latter at older ages. During later years, the series become much more level, although they are still somewhat erratic above age 95. The data-quality checks reported in Table 2 showed a sharp improvement in consistency at older ages between 1951–70 and 1971–90, which is probably related to the greater regularity in death rates during later years. It appears that, without performing the painstaking data-quality checks that we have applied, it may often be possible to identify data of poor quality through a pattern of sharply divergent deviations (in logits of death rates) from the standard pattern.

EXTENSION TO HIGHER AGES

The standard pattern is based on relatively few observations in the high 90s, and there are none above age 99. There are purposes for which a standard pattern will be useful at higher ages, so we have fitted a line to our observations in order to extend the pattern.

In particular, we used weighted least squares regression on the logits of the standard pattern presented in Table 2, beginning at age 80. Weights are the number of observations (country/period combinations) available for each age. We added an age-squared term to the equation, but its contribution to variance explained was insignificant and it was dropped. Coefficients of the lines are the following:

$$\text{Females} \quad \text{logit } {}_1m_x = -12.0930+0.12208 \text{ age}$$
$$\text{Males} \quad \text{logit } {}_1m_x = -11.1823+0.111452 \text{ age}$$

Although the transformed death rates for females increase faster with age than those for males, the two sets of death rates do not intersect below age 115. Extrapolation of the lines shows them intersecting at age 120.5, beyond the highest age to which anyone has been certified as surviving. Logit and untransformed values of the standard for the higher ages are presented in Table 6.

[24] See also Horiuchi and Coale, *loc. cit.* in fn. 12.