

# APPROACHES AND EXPERIENCES IN PROJECTING MORTALITY PATTERNS FOR THE OLDEST-OLD

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## ABSTRACT

In 1998 the United Nations Population Division extended the age format of its estimates and projections of population dynamics for all countries and areas of the world from 80 years and above to 100 years and above. The paper is based on experiences made during the implementation of relevant mortality projection methodologies and their application in two rounds of global population projections.

The paper first briefly addresses the need for the explicit inclusion of very old population segments into the regular UN estimates and projections. It is argued that since population aging is an important issue for both developed and developing countries, the need for more information regarding the elderly, and the oldest-old in particular, is significant.

The paper then documents the methods that have been evaluated and implemented, namely, the relational mortality standard proposed by Himes, Preston, and Condran, the Coale-Kisker extrapolation method for extending empirical age patterns of mortality to very high ages, and the Carter-Lee projection method for projecting model patterns of mortality to very high levels of life expectancy at birth. The methods are critically reviewed, and possible improvements to the methods are discussed.

The paper concludes with a discussion of different views regarding the future evolution of mortality at older ages, their regional variability, and the necessity to improve the coverage and quality of data collected in this area.

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## 1. INTRODUCTION

It is estimated that in 2001, 72 million of the 6.1 billion inhabitants of the world were 80 years or older (United Nations 2001). The population of the oldest-old (e.g., those 80 years and older) constitutes therefore 1.2% of the world's population, but, although it is a small fraction of the whole, it is the fastest growing segment of the population. Thus, whereas the world population is expected to increase by about 50% and to reach 9.3 billion by 2050, the number of people aged 80 years or older is expected to increase more than fivefold, to reach 379 million in 2050 (Figure 1). Most of the growth of the oldest-old population

will occur in the developing world, where their numbers are expected to increase almost eightfold, from 34 million in 2001 to 266 million in 2050. In the more developed countries, the number of oldest-old will likely triple, passing from 38 million to 113 million. By 2050, therefore, the majority of the oldest-old will be living in the less developed regions of the world.

Furthermore, because life expectancy continues to increase, not only are an increasing number of people surviving to very old ages, but also deaths of the oldest-old are accounting for an increasing proportion of all deaths. Thus, at the global level, 18 out of every 100 deaths expected in 2000–2005 will be to persons aged 80 years or older (i.e., 10 million out of the expected 55 million deaths). In the more developed regions, the proportion of deaths to persons aged 80 or over is expected to be much higher—42%—and those proportions are expected to keep on rising.

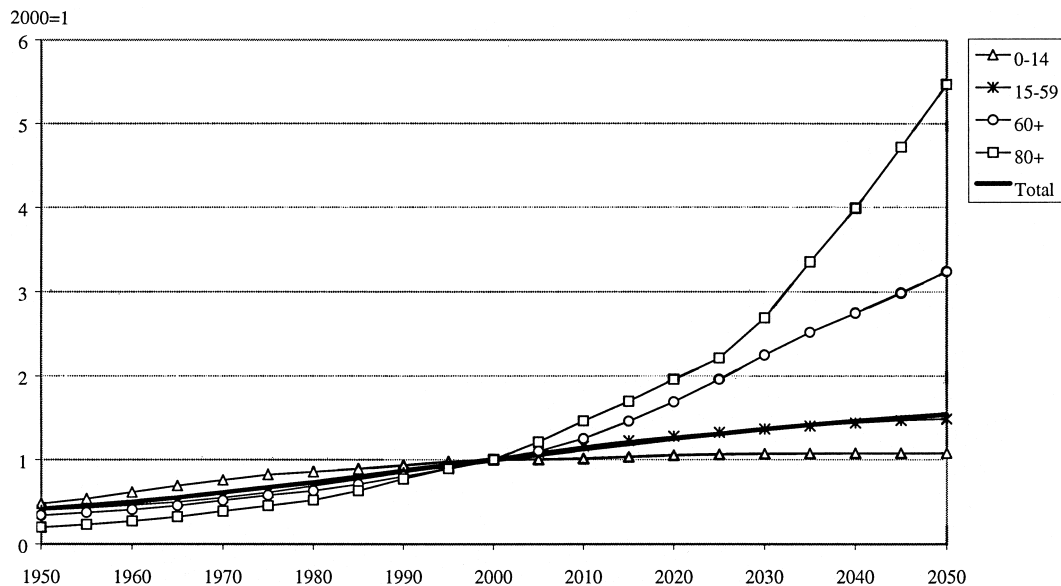
In view of such trends, it is important to have

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Figure 1  
**Growth of Broad Population Age Groups, World Total 1950–2050**



detailed information about the age structure of the oldest-old and about the population dynamics to which they are subject, namely, the risks of dying by age. However, until 1996, the estimates and projections of population produced by the United Nations Population Division did not provide an age breakdown for the group aged 80 years or older. In order to provide such information, it is necessary to obtain both data on the age distribution of the population classified by five-year age groups above age 80 and estimates of the mortality risks to which the population in those age groups is subject. Unfortunately, such data are not readily available for most countries. Developing countries, in particular, generally lack the necessary information either because reliable statistics on adult mortality in general and on old-age mortality in particular do not exist or because the available statistics on old-age mortality are unreliable, being biased by poor age reporting regarding both those alive and those who die (Condran, Himes, and Preston 1991; Kannisto et al. 1994). In a review of data availability, Hill (1999) concluded that the coverage of death registration had not improved between the early 1970s and the early 1990s. Although the proportion of developing countries lacking information on adult deaths by age group remained constant

at 44%, their share of the world population rose from 66% in the early 1970s to 69% in the 1990s. Furthermore, when it comes to both population age distributions and mortality rates among the very old, problems of data reliability are not confined to the developing world. Even in countries with advanced statistical systems, inconsistencies of age reporting between the ages of the living and those who die can bias the estimated rates of death for the oldest-old.

Therefore, to produce both estimates and projections of population with an open-ended interval of 100 years and over instead of the more traditional 80 years and over, the Population Division had to resort to models that could be adapted to the varied situations of the 187 countries whose populations are projected using the components method. This paper describes the methodology adopted by the Population Division for that purpose. It describes first the use of a relational mortality model with a standard proposed by Himes, Preston, and Condran to extend life tables beyond age 80. It focuses later on the projection of mortality using the method proposed by Lee and Carter. After a description of each method, an assessment of their performance and robustness is undertaken. A final section adds some observations regarding possible future

trends in survival among the oldest-old and necessary improvements of empirical data.

## 2. EXTENDING LIFE TABLES TO AGE 100 AND BEYOND

In 1997 the Population Division convened a meeting of the Working Group on Projecting Old-Age Mortality and Its Consequences to review the different options to extend age-specific mortality rates to older ages (United Nations 1997). Three approaches were examined in some detail, namely:

- The old-age mortality standard developed by Himes, Preston, and Condran (1994)
- The old-age term of the Heligman-Pollard mortality model (Heligman and Pollard 1980)
- The Coale-Kisker method of closure of life tables (Coale and Kisker 1990).

The working group recommended the use of a relational mortality model based on the old-age mortality standard developed by Himes, Preston, and Condran (HPC standard), mainly because that standard was derived from the observed old-age mortality patterns of a variety of populations with reliable data. However, because empirical data do not reflect as yet the very low mortality levels projected in the future, and mortality rates at very advanced ages are affected by random variation, it was later decided to replace the HPC standard at ages 95 and over with mortality rates derived using the old-age term proposed by Heligman and Pollard. Furthermore, in order to avoid random mortality crossovers between different model life tables at very advanced ages, the Coale-Kisker method was used to close the life tables.

### 2.1. The Himes-Preston-Condran Mortality Standard

Himes, Preston, and Condran proposed in 1994 a standard mortality schedule (HPC standard) representing the typical mortality pattern at advanced ages based on the patterns observed in a variety of countries and periods. The HPC standard was constructed by examining mortality rates by single years of age for the age range

45–99 from 16 low-mortality countries.<sup>1</sup> The mortality experience covered spanned the period 1948–85. Observed mortality data were subject to strict reliability and consistency tests to be included. In the end, the standard was derived from 82 different mortality schedules for each sex.

Figure 2 shows the HPC standard by sex as published in 1994. Two deficiencies are noticeable. First, the standard exhibits visible fluctuations above age 90 for both sexes and around ages 54 and 81 for males. Second, the standard does not cover age-specific mortality patterns above age 99. It was therefore necessary to remove fluctuations by smoothing the standard and to extend it beyond age 99.

A parameterization with the old-age term of the Heligman-Pollard mortality model and several moving averages were tested for smoothing the original HPC standard. The parameterization exhibited relative large deviations, especially at older age groups, and was therefore dismissed. The smoothest results—measured by second-order differences—were produced by a five-point moving average, of the form

$$\hat{m}_x = \frac{m_{x2} + 2m_{x1} + 3m_x + 2m_{x+1} + m_{x+2}}{9},$$

where  $m_x$  is the central mortality rate at age  $x$ , and  $\hat{m}_x$  is the resulting smoothed value for age  $x$ .

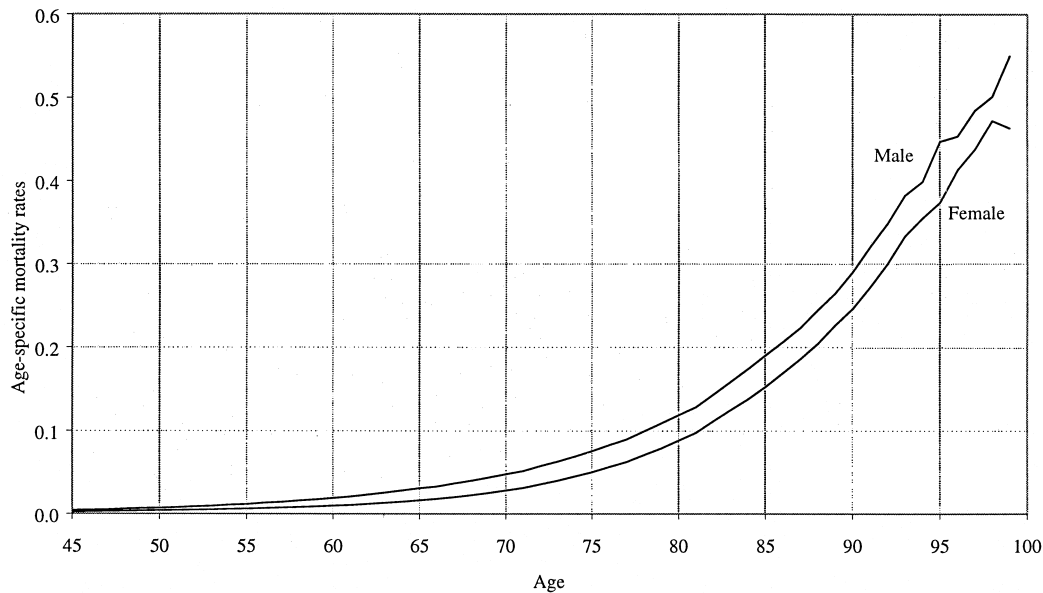
The smoothed HPC standard was extended up to an upper age of 115 by fitting a straight line to the logits of the age-specific mortality rates, following a suggestion made by Himes, Preston, and Condran. It can be shown that such a linear extension in the logit domain is equivalent to the old-age term of the Heligman-Pollard model. The linear function in the logit domain is

$$\text{Logit}(m_x) = \alpha + \beta x, \quad (1)$$

where the logit is defined as

<sup>1</sup> The 16 countries are Australia, Austria, Belgium, Canada, Denmark, England and Wales, Finland, Hungary, Italy, Japan, The Netherlands, New Zealand, Norway, Scotland, Spain, and Sweden. Czechoslovakia, Ireland, and Northern Ireland were excluded because of insufficient data quality. Data from France, East and West Germany, and the United States were not included because of data inconsistencies.

Figure 2  
Original HPC Mortality Standard, Expressed as Age-Specific Mortality Rates ( $m_x$ )



$$\text{Logit}(m_x) = \ln\left(\frac{m_x}{1 - m_x}\right). \quad (2)$$

By taking the antilogit and rearranging for  $m_x$  we obtain

$$m_x = \frac{e^{\alpha + \beta x}}{1 + e^{\alpha + \beta x}} = \frac{e^{\alpha} e^{\beta x}}{1 + e^{\alpha} e^{\beta x}} = \frac{GH^x}{1 + GH^x}, \quad (3)$$

where parameters  $G$  and  $H$  are

$$G = e^{\alpha},$$

$$H = e^{\beta}.$$

Using this procedure, the smoothed HPC standard was extended. The smoothed and extended HPC standard, covering the age range 45–115 years, is presented in Table A1 in the Appendix. Figure 3 shows the smoothed and extended HPC standard, expressed as age-specific mortality rates, together with the relative difference between the original and the smoothed standard.

Once the HPC standard had been extended to advanced ages, the procedure used to extend any other set of  $m_x$  values made use of the empirical fact that mortality patterns, appropriately transformed, are often linearly related. In this case, the logit function was used as the linearizing transformation: that is, the logit transformation of the

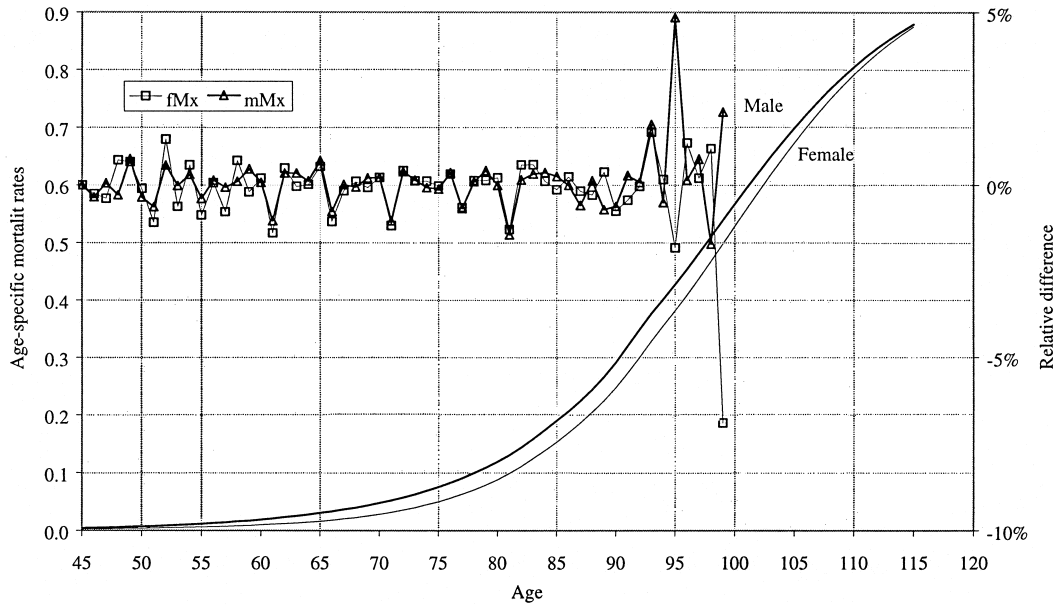
given set of  $m_x$  values would be linearly related to the logit transformation of the standard set of  $m_x$  value. Fitting a line to those pairs of values would provide the  $\alpha$  and  $\beta$  values (i.e., the regression coefficients) that would permit the estimation of the  $m_x$  values at advanced ages from those of the standard.

## 2.2. The Coale-Kisker Method

Coale and Guo (1989) used a novel method to close a life table that assumes that the exponential rate of mortality increase at very old ages is not constant, as in the classical Gompertz model, but declines linearly. This feature of mortality at very advanced ages has been empirically verified by a number of studies (Horiuchi and Wilmoth 1997). Coale and Guo applied this approach to close the extended version of the Coale-Demeny model life tables presented in five-year age groups. Later Coale and Kisker (1990) used the same approach to close empirical life tables by single years of age. Following common practice, the method first used by Coale and Guo and then by Coale and Kisker is henceforth referred to as the Coale-Kisker method.

The Coale-Kisker method has two parameters, namely, the Gompertz parameter  $k$  and a mortal-

Figure 3  
**Smoothed and Extended HPC Mortality Standard, Relative Difference between Original and Smoothed Mortality Patterns**



ity rate for the uppermost age, say, 110 years. Coale and Kisker set a value of 1.0 per 1,000 for  $m_{110}$  for males, and 0.8 per 1,000 for females.<sup>2</sup> The mortality differential by sex at age 110 was explicitly chosen to avoid a crossover between male and female mortality at very advanced ages.

Having set mortality at age 110, the Gompertz parameter is calculated from the given age-specific mortality rates  $m_x$  as follows:

$$k_x = \text{Ln}\left(\frac{m_x}{m_{x-1}}\right) = \text{Ln}(m_x) - \text{Ln}(m_{x-1}) \quad (4)$$

or, setting  $x = 85$ ,

$$k_{85} = \text{Ln}\left(\frac{m_{85}}{m_{84}}\right) = \text{Ln}(m_{85}) - \text{Ln}(m_{84}), \quad (5)$$

$$m_x = m_{84} * \exp\left[\sum_{y=85}^x k_y\right], \text{ for } x = 85, 86, \dots \quad (6)$$

If  $k_x$  were constant (e.g.,  $k_x = k$ ), then this equation becomes the classic Gompertz:

$$m_x = m_{84} * \exp[(x - 84) * k]. \quad (7)$$

Coale and Kisker assume that  $k_x$  is linear above a certain age, 85 years in this case, that is:

$$k_x = k_{85} + s * (x - 85). \quad (8)$$

Solving for  $s$  yields

$$s = -\frac{[\text{Ln}(m_{84}/m_{110}) + 26k_{85}]}{325}. \quad (9)$$

Age-specific mortality rates are then calculated using one of the two following formulae:

$$m_x = m_{84} * \exp\left[\sum_{y=85}^x (k_{85} + (y - 85) * s)\right], \quad (10)$$

for  $x = 85, 86, \dots$ ,

or, without the need to accumulate the  $k$  and  $s$  values,

$$m_x = m_{x-1} * \exp[k_{85} + (x - 85) * s], \quad (11)$$

for  $x = 85, 86, \dots$

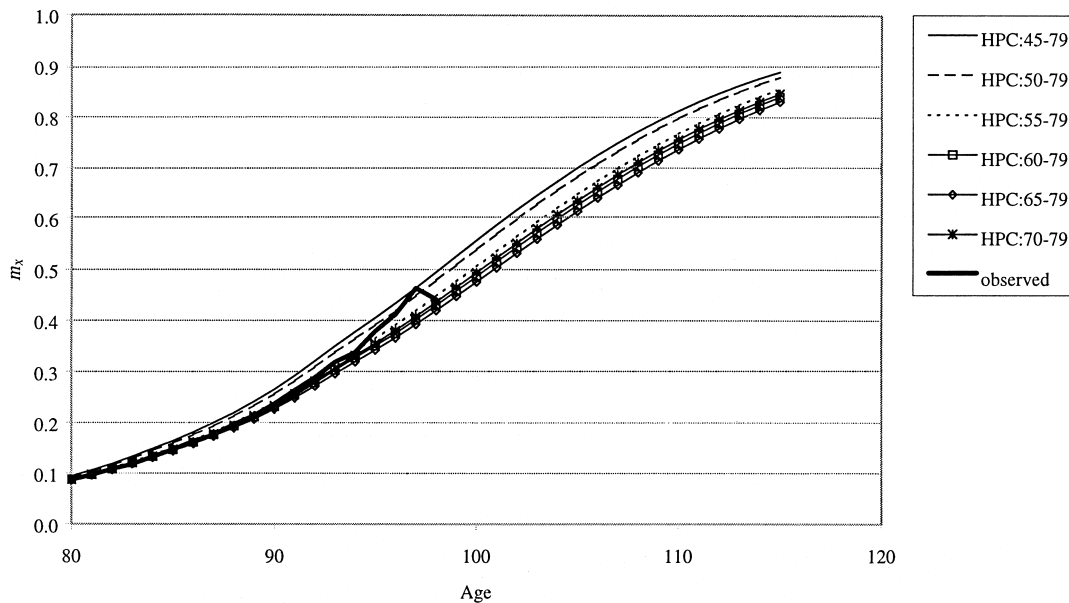
### 2.3. Discussion

Truncated mortality patterns are traditionally extended using a variety of approaches, be they an

<sup>2</sup> Wilmoth (1995) later extended the original Coale-Kisker method by transforming it into a regression model that can be used to estimate empirically the age specific mortality rate at  $m_{110}$ , for instance.



Figure 5

**Male Age-Specific Mortality Rates for France, 1992–1994, and Results of Several HPC Extensions**

meny, and Vaughn 1983), while the life tables for higher life expectancy added to this system by Coale and Guo (1989) used a non-Gompertzian closing method that is similar to the Coale-Kisker method discussed in this paper. Although the different closing methods had little impact on life expectancy at age 80, they can result in marked differences in the number of survivors to very high ages. Consequently, it was important to ensure that all model life tables used were closed by the same procedure. The Coale-Kisker method was used for this purpose because use of the relational model with the HPC standard resulted in inconsistencies at very advanced ages, that is, age-specific mortality rates above age 100 belonging to model life tables with contiguous life expectancies would cross over because of instability in the numerical fitting procedures. Use of the Coale-Kisker method of closure avoided that problem by allowing the analyst to set the mortality rates of the uppermost age in a way consistent with the overall order of life expectancy at birth. Once that uppermost mortality rate was set, the trajectories of age-specific mortality rates for advanced ages also ordered themselves properly.

### 3. PROJECTING MORTALITY PATTERNS AT VERY OLD AGES USING THE LEE-CARTER MODEL

To project mortality, the Population Division uses a two-step process. First, recent trends in overall life expectancy are established on the basis of national estimates, adjusted as necessary. When data on adult mortality do not exist or are severely deficient, life expectancy is established on the basis of estimates of mortality in childhood and an assumed model pattern of mortality. Assumptions about future trends in life expectancy are made on the basis of a set of models that ensure international consistency (United Nations 2000, p. 188).

Age-specific mortality for the projection period is calculated in a second step. In the absence of actual age-specific mortality data, mortality patterns are obtained from a model specified by the analyst. A choice can be made among nine different families of model life tables, the five in the United Nations models for developing countries, and the four in the Coale-Demeny set. If information on the actual age-specific mortality pattern of a population is available, that pattern is modified over the projection period until it eventually

converges to the pattern of a model selected by the analyst. In this process, the first step is to extend the actual age-specific mortality rates for the range 80 years and over.

In some cases, the analyst may choose to provide the age-specific pattern of mortality that is to be used for each five-year period of the projection span. In such a case, the eventual convergence to a model mortality pattern does not take place, but it is still necessary to extend the age-specific mortality data provided by the analyst to the range 80 and over.

Until 1996, the model life tables used by the Population Division had life expectancies whose upper limits were 82.5 years for males and 87.5 for females (United Nations 1989, pp. 13–19). However, as the projections were extended to 2050 and mortality continued to decline to very low levels in some developed countries, a higher upper limit became necessary. The revised model life tables, discussed below, have life expectancies going up to 92.5 years for both males and females. In order to generate internally consistent sets of model life tables for levels of life expectancy not yet observed, a projection method developed by Lee and Carter<sup>3</sup> (1992) was evaluated, amended, and then implemented in a simplified fashion.

The original Lee-Carter procedure projects past patterns of age-specific mortality change into the future using time series methods. The evolution of age-specific mortality rates is modeled as exponential rates of change of a normalized, or average, mortality pattern. The model has the form

$$f_{xt} = \ln(m_{xt}) = a_x + b_x k_t + \varepsilon_{xt}, \quad (12)$$

with the parameters

$a_x$  Standard age pattern of mortality, expressed as the average of the logarithm of the mortality rate  $m_{x,t}$  at age  $x$  over time  $t$

$b_x$  Age-specific pattern of mortality change

$k_t$  Time trend

$\varepsilon_{xt}$  Error term.

It has been established empirically that the

time-dependent term ( $k_t$ ) is often linear over most parts of the observation period. This is a useful feature since it allows for a relatively easy interpretation and projection. But even in cases where two or more distinct phases of the transition to lower mortality have been observed over longer periods of time,  $k_t$  has been found linear in those periods (Wilmoth 1993; Booth, Maindonald, and Smith 2001).

Inverting the logarithmic function, Wilmoth (1993) noted that the model can be written as (without the error term)  $m_{xt} = A_x B_x^{k_t}$ , where  $A_x = e^{a_x}$ ,  $B_x = e^{b_x}$ . According to this formulation, if  $k_t$  changes linearly over time, then each age-specific mortality rate changes at a constant exponential rate (Carter and Lee 1992, p. 396).

Projections are carried out by projecting the only time-dependent parameter  $k_t$  using appropriate statistical techniques. Lee and Carter used a random walk approach, which allowed for the calculation of confidence intervals for projected life expectancies.

However, in order to use the Lee-Carter approach to project mortality patterns on the basis of model life tables, some transformations are in order since the families of model life tables available are organized as collections of life tables at distinct levels of life expectancy of birth but do not contain a time reference. Therefore, the model's time index needs to be replaced with an index reflecting level of life expectancy. Hence the model becomes

$$f_{xl} = \ln(m_{xl}) = a_x + b_x k_l + \varepsilon_{xl}, \quad (13)$$

where the index  $l$  represents the level of life expectancy associated with the corresponding age-specific mortality rates ( $m_x$ ), and the parameter  $k_l$  represents the trend in the level of life expectancy at birth (in years).

The transformed Lee-Carter model was tested using the families of the Coale-Demeny model life table system. The model was fitted to series of model life tables spanning levels of life expectancy from 20 to 75 years, and then projected to a life expectancy of 92.5 years. The results were, at first sight, generally encouraging. The method produced a set of smooth and consistent age patterns of mortality that would pass a visual inspection. However, in contrast to the original model,

<sup>3</sup> This method has been successfully employed to project mortality for a number of countries, including the United States (Lee and Carter 1992), Japan (Wilmoth 1993), Chile (Lee and Rofman 1994), and Australia (Booth, Maindonald, and Smith 2001).



where the time trend ( $k_t$ ) conveniently is roughly linear, the trend parameter ( $k_l$ ), representing levels of life expectancy, takes here the form of a convex function, declining faster as life expectancy reaches higher levels. Such a nonlinear trend is not surprising, since it reflects the empirical observation that similar gains in life expectancy tend to take longer the higher the life expectancy. The original Lee-Carter model, formulated in the time domain, exhibits the same feature, but its trend parameter ( $k_t$ ) remains nearly linear and is therefore significantly easier to model.

Although the results looked acceptable when analyzed graphically, the projected age patterns of mortality differed noticeably from recent evidence, as embodied, for example, in the revised model life tables prepared by Coale and Guo (1989) and in the two ultimate life tables prepared by the Population Division (United Nations 1989, p. 19) and the U.S. Bureau of the Census (Arriaga 1994, pp. 362–63). Apparently, therefore, the patterns of change embodied by the existing families of model life tables are not well suited to infer future trends in the evolution of age-specific mortality patterns.

Indeed, Coale and Guo (1989) decided to revise the original Coale-Demeny life tables for that very reason. The original model life tables contained tables for levels of life expectancy for which no empirical evidence had been available at the time of preparation (with life expectancies 75 years or higher) and that had been obtained by extrapolation. These extrapolated life tables consistently underestimated mortality rates at young ages and overestimated mortality for older persons as comparison with actual mortality patterns for low-mortality countries revealed. One possible reason that extrapolating the existing model life tables did not match recent age patterns of mortality is that the original Coale-Demeny model life table system is based on national life tables that cover approximately 100 years of mortality experience, more than half of which are from periods before 1945, and none of them is based on periods after the 1960s. Therefore they had no basis for reflecting the impact of changes in cause-of-death composition, public health interventions, and changes in lifestyles that occurred later. The findings of Wilmoth (1993) and Booth (2001) regarding the long-term evolution of mortality in Japan

and Australia also indicated that  $k_t$  does not necessarily follow a linear trend and suggests that past experience is not necessarily the best predictor of the future.

Regardless of the adequacy of the data used and independent of whether the Carter-Lee model was formulated in the time domain or not, it was also found that the Lee-Carter model exhibited a general tendency to produce extremely low mortality rates for younger age groups when used to project life tables for high levels of life expectancy. Although the model effectively prevents age-specific mortality from becoming negative since it is modeled in the logarithmic scale, the rates can nevertheless become virtually zero. In other words, the model gradually “forgets” the reference age-pattern of mortality as it approaches lower mortality. The very low projected mortality rates for children are not of direct relevance for mortality at advanced ages. However, the possibility of introducing lower bounds by age group might be considered to enhance its performance.

A variation of the Lee-Carter method that uses such lower bounds was therefore developed. Assume that there are some intrinsic lower limits of mortality by age. The Lee-Carter model can incorporate such lower bounds by restricting the modeling to that part of mortality by age that is subject to change. To do this, we can subtract the lower bounds of mortality from the empirical mortality rates and fit the model on the remainder. After the model is fitted, and mortality patterns of the remaining mortality are projected, the lower bounds are added back. This procedure is equivalent to an age-specific Makeham correction.

The extended Lee-Carter model with lower bounds incorporates the following models as specific instances:

1. The original Lee-Carter model if the lower mortality bounds are set to zero
2. A Makeham-corrected Lee-Carter model if the lower bounds are set to a fixed value for all age groups
3. A Makeham-corrected Lee-Carter model that approaches a limiting age specific mortality pattern; for example, the lower mortality bounds are set equal to a limiting life table. This approach assumes that there is an age-

specific intrinsic mortality that cannot be reduced.

In view of the specific requirements related to projecting model life tables to very high levels of life expectancy, a simplified version of the Lee-Carter model was employed. Instead of basing the projection of age-specific mortality patterns on past experience as embodied in the series of model life tables, the Lee-Carter model was used to simply interpolate geometrically between a reference model life table with a life expectancy of 75 years and an ultimate life table with very low mortality that reflected likely mortality patterns based on current experience (United Nations 1988, see Tables 2 and 3). Since previous rounds of UN population projections also used an ultimate life table, its use in generating a new set of model life tables had the benefit of ensuring much needed consistency between subsequent revisions of the UN world population projections. Model life tables with life expectancies ranging from that of the reference table (with a life expectancy of 75 years) and that of the ultimate life table were obtained by iteratively modifying the level parameter of the Lee-Carter model ( $k_t$ ) until the desired life expectancy was reached. For levels of life expectancy higher than that of the ultimate life table, the pattern of change between the reference table and the ultimate life table estimated by the Lee-Carter procedure was extrapolated in a similar fashion.

Concerned with the numerical stability of mortality projections at very old ages, the inclusion of limit life tables (Duchene and Wunsch 1988a, 1988b), were tested, as suggested in option three of the amended Lee-Carter model. However, it was found that such a provision is not required in order to ensure reasonable projections results. Moreover, since such limit life tables are highly speculative, it was decided to implement only a simple Makeham correction, with a lower bound of  $m_x$  set to 0.00002 for all age groups except the first one, where it was set to 0.00023 for males and 0.00038 females (Duchene and Wunsch 1988a, 1988b). As an illustration, the resulting life tables for a life expectancy of 92.5 years for the North Model of the Coale-Demeny system of model life tables and for the General Pattern of the UN system of model life tables are presented in the Appendix.

## 4. CONCLUSIONS

In the past, mortality has exhibited distinct regional patterns, in both their shapes and their patterns of change over time. For this reason, several families of model life tables have been constructed and successfully used. The question arises whether patterns of mortality and mortality change in the future will ultimately converge to one or very few characteristic patterns, or whether a substantial variety will persist. Current projection practices favor convergence (United Nations 1988). Coale and Guo (1989) suggested that mortality patterns worldwide might converge to a pattern similar to the North model of the Coale-Demeny system. Convergence to very similar patterns seems more likely for age-specific mortality rates that are increasingly moving toward the lowest levels possible, such as those relative to childhood or adolescence. Theories of natural limits to survival would lend some support to this assumption. However, the same argument cannot be made about mortality at older ages, which is far from reaching a lower limit, and whose patterns may end up being fairly diverse even at comparable levels of life expectancy.

Today there are two alternative views about the future evolution of mortality at older ages: compression versus expansion (sometimes also called rectangularization versus steady progress). Mortality compression would occur if age-specific mortality were to continue declining over a widening range of adult ages, but would meet natural limits for very advanced ages (Bourgeois-Pichat 1978; Fries 1980; Gavrilov and Gavrilova 1991). As a result, the survivor curve would approach a rectangle, and mortality across countries may indeed converge to similar patterns. The modified Lee-Carter projection model for age-specific mortality patterns would operate under such hypothesis when setting the mortality bounds to these limits. In the case of steady progress, there would be no "natural" limits to further reductions in mortality at higher ages, or the age at which natural limits set in could move upward (Olshansky, Carnes, and Grahn 1998). Consequently, all age groups, especially at higher age groups, would continue to experience declining mortality. The age pattern of mortality would not change substantially, but the age range would expand (Manton, Stallard, and Tolley 1991; Manton 1992; Vau-

pel and Lundstrom 1994; Olshansky, Carnes, and Grahn 1998). In this case, the Lee-Carter model could be used without specification of age-specific lower bounds. More research is needed to verify these models of change.

Last, although detailed data on old age mortality are collected in most countries of the developed world, they are not so commonly available for developing countries. Furthermore, even in developed countries, the quality of age reporting deteriorates among the very old. National statistical offices do not evaluate regularly the quality of these data, and it is not evident how to correct any biases that might be detected. The consistent evaluation of the quality of data on the elderly and a wide dissemination of the findings of such evaluations are needed. Indicators of data quality for these data need to expand on those suggested by UN recommendations.<sup>4</sup> In addition, it is crucial to add detailed documentation on the techniques used to construct life tables to the publications in which those data are disseminated. It is also important to ensure that data on the oldest-old are published with sufficient detail. Use of 100+ as the open-ended age group should be standard in the preparation of tabulations of population and deaths by age and sex. With the number of persons in advanced ages growing so rapidly in modern populations, detailed demographic characteristics of this group should become part of standard tabulations.

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<sup>4</sup> Quality of registered data is measured in two categories: virtually complete (at least 90% of the events each year are represented) or incomplete (less than 90% representation).

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*Discussions on this paper can be submitted until January 1, 2003. The author reserves the right to reply to any discussion. Please see the Submission Guidelines for Authors on the inside back cover for instructions on the submission of discussions.*

APPENDIX

Table A1

Smoothed and Extended Himes-Preston-Condran Mortality Standard for Older Ages

Age	Logit ( $m_x$ )		$m_x$	
	Female	Male	Female	Male
45	-5.945580	-5.429400	0.00261055	0.00436658
46	-5.857527	-5.331900	0.00285016	0.00481161
47	-5.763043	-5.231552	0.00313170	0.00531680
48	-5.670024	-5.131108	0.00343593	0.00587529
49	-5.581900	-5.030498	0.00375128	0.00649312
50	-5.496322	-4.931224	0.00408507	0.00716594
51	-5.413219	-4.830740	0.00443749	0.00791743
52	-5.326582	-4.728463	0.00483714	0.00876258
53	-5.243806	-4.627376	0.00525239	0.00968566
54	-5.159373	-4.527179	0.00571248	0.01069550
55	-5.077452	-4.429220	0.00619713	0.01178329
56	-4.988427	-4.329780	0.00677023	0.01299924
57	-4.897846	-4.230487	0.00740737	0.01433678
58	-4.802196	-4.131314	0.00814482	0.01580785
59	-4.709610	-4.035228	0.00892787	0.01737444
60	-4.614074	-3.940151	0.00981408	0.01907437
61	-4.517492	-3.844479	0.01079848	0.02094929
62	-4.413752	-3.745297	0.01196477	0.02308319
63	-4.308781	-3.645761	0.01327143	0.02543758
64	-4.203867	-3.549564	0.01471785	0.02793440
65	-4.100339	-3.457029	0.01629707	0.03055993
66	-3.995889	-3.367137	0.01805897	0.03333846
67	-3.886589	-3.274687	0.02010279	0.03644987
68	-3.773873	-3.181136	0.02244749	0.03988183
69	-3.659886	-3.087139	0.02508976	0.04364089
70	-3.544304	-2.993253	0.02807758	0.04773160
71	-3.427037	-2.898531	0.03146110	0.05222622
72	-3.304468	-2.799941	0.03541824	0.05732736
73	-3.181927	-2.701239	0.03985155	0.06290029
74	-3.059083	-2.601832	0.04482694	0.06902059
75	-2.939676	-2.504512	0.05022675	0.07554246
76	-2.819954	-2.405958	0.05625535	0.08271951
77	-2.700589	-2.306993	0.06293862	0.09054543
78	-2.578348	-2.205550	0.07054499	0.09925320
79	-2.457149	-2.105523	0.07891733	0.10856114
80	-2.333558	-2.003857	0.08838159	0.11879859
81	-2.209010	-1.899341	0.09894430	0.13018307
82	-2.079846	-1.786644	0.11107121	0.14348462
83	-1.953067	-1.671617	0.12421935	0.15820875
84	-1.829471	-1.558163	0.13830129	0.17391035
85	-1.710777	-1.450972	0.15306301	0.18985199
86	-1.593031	-1.345761	0.16895787	0.20656424
87	-1.474910	-1.240494	0.18619747	0.22434993
88	-1.356319	-1.130236	0.20483923	0.24411763
89	-1.235362	-1.013617	0.22524429	0.26627266
90	-1.111783	-0.888717	0.24753857	0.29137473
91	-0.980186	-0.756307	0.27285497	0.31944867
92	-0.846790	-0.627570	0.30010666	0.34806174
93	-0.716903	-0.506954	0.32807525	0.37590775
94	-0.598880	-0.400472	0.35459997	0.40119889
95	-0.485864	-0.294863	0.38086828	0.42681366
96	-0.371227	-0.187069	0.40824465	0.45336869
97	-0.251530	-0.073442	0.43744695	0.48164769
98	-0.129453	0.041082	0.46768179	0.51026911
99	-0.007377	0.155608	0.49815584	0.53882364
100	0.114701	0.270132	0.52864388	0.56712537
101	0.236779	0.384657	0.55891971	0.59499574
102	0.358857	0.499180	0.58876364	0.62226661
103	0.480933	0.613703	0.61796824	0.64878512
104	0.603010	0.728228	0.64634464	0.67441625
105	0.725087	0.842752	0.67372615	0.69904455
106	0.847163	0.957277	0.69997175	0.72257622
107	0.969241	1.071800	0.72496821	0.74493908
108	1.091319	1.186323	0.74863000	0.76608285
109	1.213397	1.300848	0.77089940	0.78597763
110	1.335473	1.415372	0.79174455	0.80461190
111	1.457551	1.529897	0.81115784	0.82199119
112	1.579629	1.644420	0.82915195	0.83813547
113	1.701707	1.758943	0.84575750	0.85307727
114	1.823783	1.873467	0.86101948	0.86685889
115	1.945860	1.987990	0.87499451	0.87953033

Table A2  
UN Ultimate Life Table, Males

Age	$m_x$	$q_x$	$l_x$	$d_x$	$L_x$	$T_x$	$e_x$	$a_x$
0	0.005021	0.004997	100,000	500	99,529	8,207,454	82.075	0.057
1	0.000147	0.000588	99,500	59	397,859	8,107,925	81.486	1.577
5	0.000065	0.000325	99,442	32	497,123	7,710,066	77.533	2.353
10	0.000059	0.000296	99,409	29	496,976	7,212,943	72.558	2.583
15	0.000104	0.000518	99,380	51	496,787	6,715,967	67.579	2.804
20	0.000194	0.000972	99,329	97	496,419	6,219,180	62.612	2.683
25	0.000265	0.001322	99,232	131	495,843	5,722,761	57.671	2.582
30	0.000328	0.001641	99,101	163	495,115	5,226,918	52.743	2.603
35	0.000463	0.002314	98,938	229	494,157	4,731,803	47.826	2.667
40	0.000747	0.003728	98,709	368	492,702	4,237,646	42.931	2.704
45	0.001278	0.006371	98,341	627	490,276	3,744,944	38.081	2.717
50	0.002228	0.011086	97,715	1,083	486,102	3,254,668	33.308	2.718
55	0.003904	0.019348	96,632	1,870	478,887	2,768,566	28.651	2.716
60	0.006842	0.033682	94,762	3,192	466,499	2,289,679	24.162	2.710
65	0.011966	0.058226	91,570	5,332	445,580	1,823,179	19.910	2.699
70	0.020833	0.099363	86,238	8,569	411,303	1,377,599	15.974	2.679
75	0.035979	0.165845	77,669	12,881	358,014	966,296	12.441	2.645
80	0.061284	0.266961	64,788	17,296	282,225	608,282	9.389	2.588
85	0.102341	0.407386	47,492	19,348	189,051	326,056	6.865	2.498
90	0.167970	0.581670	28,145	16,371	97,463	137,005	4.868	2.358
95	0.270484	0.762454	11,774	8,977	33,188	39,542	3.358	2.139
100	0.440203	1.000000	2,797	2,797	6,353	6,353	2.272	2.272

Table A3  
UN Ultimate Life Table, Females

Age	$m_x$	$q_x$	$l_x$	$d_x$	$L_x$	$T_x$	$e_x$	$a_x$
0	0.003956	0.003942	100,000	394	99,630	8,750,205	87.502	0.062
1	0.000083	0.000331	99,606	33	398,344	8,650,575	86.848	1.590
5	0.000038	0.000190	99,573	19	497,815	8,252,231	82.876	2.363
10	0.000036	0.000178	99,554	18	497,728	7,754,416	77.892	2.595
15	0.000060	0.000299	99,536	30	497,615	7,256,688	72.905	2.768
20	0.000105	0.000527	99,507	52	497,411	6,759,073	67.926	2.675
25	0.000146	0.000729	99,454	72	497,097	6,261,662	62.960	2.598
30	0.000187	0.000933	99,382	93	496,687	5,764,565	58.004	2.609
35	0.000262	0.001310	99,289	130	496,140	5,267,879	53.056	2.660
40	0.000414	0.002068	99,159	205	495,322	4,771,738	48.122	2.698
45	0.000699	0.003489	98,954	345	493,980	4,276,416	43.216	2.714
50	0.001212	0.006041	98,609	596	491,684	3,782,436	38.358	2.719
55	0.002120	0.010547	98,013	1,034	487,706	3,290,752	33.575	2.719
60	0.003717	0.018430	96,979	1,787	480,814	2,803,046	28.904	2.716
65	0.006518	0.032109	95,192	3,057	468,961	2,322,232	24.395	2.711
70	0.011404	0.055563	92,135	5,119	448,901	1,853,271	20.115	2.700
75	0.019867	0.094960	87,016	8,263	415,919	1,404,369	16.139	2.681
80	0.034342	0.158883	78,753	12,512	364,346	988,450	12.551	2.649
85	0.058833	0.257754	66,240	17,074	290,207	624,104	9.422	2.599
90	0.100966	0.403611	49,167	19,844	196,543	333,898	6.791	2.516
95	0.173428	0.595387	29,322	17,458	100,665	137,354	4.684	2.368
100	0.323372	1.000000	11,864	11,864	36,689	36,689	3.092	3.092

Table A4  
**UN Model Life Table for Life Expectancy at 92.5 Years, General Pattern, Males**

Age	$m_x$	$q_x$	$l_x$	$d_x$	$L_x$	$T_x$	$e_x$	$a_x$
0	0.000362	0.000362	100,000	36	99,965	9,249,994	92.500	0.044
1	0.000021	0.000084	99,964	8	399,836	9,150,028	91.533	1.652
5	0.000021	0.000103	99,955	10	499,751	8,750,193	87.541	2.500
10	0.000021	0.000104	99,945	10	499,700	8,250,441	82.550	2.519
15	0.000023	0.000113	99,935	11	499,647	7,750,741	77.558	2.597
20	0.000033	0.000166	99,923	17	499,579	7,251,094	72.567	2.652
25	0.000047	0.000234	99,907	23	499,479	6,751,516	67.578	2.620
30	0.000059	0.000294	99,884	29	499,347	6,252,037	62.593	2.589
35	0.000072	0.000359	99,854	36	499,186	5,752,690	57.611	2.632
40	0.000111	0.000555	99,818	55	498,964	5,253,504	52.631	2.699
45	0.000187	0.000934	99,763	93	498,601	4,754,541	47.658	2.713
50	0.000310	0.001547	99,670	154	497,997	4,255,940	42.700	2.723
55	0.000547	0.002733	99,515	272	496,965	3,757,942	37.762	2.747
60	0.001018	0.005077	99,244	504	495,100	3,260,978	32.858	2.782
65	0.002141	0.010655	98,740	1,052	491,399	2,765,878	28.012	2.814
70	0.004704	0.023283	97,688	2,274	483,476	2,274,479	23.283	2.818
75	0.010344	0.050564	95,413	4,824	466,398	1,791,003	18.771	2.789
80	0.020880	0.099685	90,589	9,030	432,492	1,324,605	14.622	2.735
85	0.039449	0.180762	81,558	14,743	373,714	892.113	10.938	2.689
90	0.076568	0.323716	66,816	21,629	282,484	518,399	7.759	2.615
95	0.147071	0.535808	45,186	24,211	164,623	235,915	5.221	2.468
100	0.294213	1.000000	20,975	20,975	71,292	71,292	3.399	3.399

Table A5  
**UN Model Life Table for Life Expectancy at 92.5 Years, General Pattern, Females**

Age	$m_x$	$q_x$	$l_x$	$d_x$	$L_x$	$T_x$	$e_x$	$a_x$
0	0.001416	0.001414	100,000	141	99,866	9,249,982	92.500	0.054
1	0.000028	0.000113	99,859	11	399,406	9,150,115	91.631	1.522
5	0.000023	0.000114	99,847	11	499,208	8,750,709	87.641	2.500
10	0.000023	0.000115	99,836	11	499,151	8,251,501	82.651	2.564
15	0.000031	0.000155	99,824	15	499,086	7,752,350	77.660	2.666
20	0.000051	0.000255	99,809	25	498,985	7,253,264	72.672	2.665
25	0.000068	0.000342	99,783	34	498,835	6,754,279	67.689	2.603
30	0.000084	0.000418	99,749	42	498,647	6,255,443	62.712	2.604
35	0.000112	0.000562	99,708	56	498,407	5,756,796	57.737	2.655
40	0.000176	0.000881	99,652	88	498,057	5,258,390	52.768	2.708
45	0.000306	0.001531	99,564	152	497,474	4,760,333	47.812	2.739
50	0.000556	0.002776	99,411	276	496,436	4,262,858	42.881	2.749
55	0.001020	0.005087	99,135	504	494,542	3,766,422	37.993	2.749
60	0.001859	0.009255	98,631	913	491,095	3,271,880	33.173	2.742
65	0.003323	0.016493	97,718	1,612	484,945	2,780,786	28.457	2.738
70	0.006013	0.029660	96,107	2,851	474,086	2,295,840	23.889	2.738
75	0.011082	0.054045	93,256	5,040	454,792	1,821,754	19.535	2.721
80	0.019363	0.092685	88,216	8,176	422,266	1,366,962	15.496	2.699
85	0.034943	0.161603	80,040	12,935	370,164	944,696	11.803	2.678
90	0.064535	0.279800	67,105	18,776	290,942	574,532	8.562	2.626
95	0.121696	0.466819	48,329	22,561	185,388	283,590	5.868	2.506
100	0.262400	1.000000	25,768	25,768	98,202	98,202	3.811	3.811

Table A6  
**Coale-Demeny Model Life Table for Life Expectancy at 92.5 Years, North Model, Males**

Age	$m_x$	$q_x$	$l_x$	$d_x$	$L_x$	$T_x$	$e_x$	$a_x$
0	0.000527	0.000527	100,000	53	99,950	9,249,975	92.500	0.044
1	0.000023	0.000091	99,947	9	399,770	9,150,026	91.549	1.857
5	0.000020	0.000102	99,938	10	499,666	8,750,256	87.557	2.500
10	0.000020	0.000101	99,928	10	499,615	8,250,590	82.565	2.501
15	0.000020	0.000102	99,918	10	499,564	7,750,975	77.573	2.515
20	0.000022	0.000109	99,908	11	499,512	7,251,411	72.581	2.548
25	0.000026	0.000128	99,897	13	499,453	6,751,899	67.589	2.561
30	0.000029	0.000146	99,884	15	499,385	6,252,446	62.597	2.606
35	0.000043	0.000213	99,869	21	499,299	5,753,061	57.606	2.718
40	0.000083	0.000417	99,848	42	499,150	5,253,762	52.618	2.815
45	0.000194	0.000967	99,807	97	498,819	4,754,612	47.638	2.784
50	0.000326	0.001631	99,710	163	498,197	4,255,794	42.682	2.833
55	0.000959	0.004786	99,547	476	496,715	3,757,596	37.747	2.854
60	0.001806	0.008996	99,071	891	493,365	3,260,882	32.915	2.768
65	0.003532	0.017521	98,180	1,720	487,078	2,767,516	28.188	2.779
70	0.007139	0.035128	96,460	3,388	474,666	2,280,439	23.641	2.748
75	0.012465	0.060591	93,071	5,639	452,409	1,805,772	19.402	2.704
80	0.021559	0.102645	87,432	8,974	416,271	1,353,363	15.479	2.673
85	0.035404	0.163424	78,457	12,822	362,153	937,092	11.944	2.650
90	0.063061	0.274147	65,635	17,994	285,340	574,939	8.760	2.619
95	0.117947	0.455648	47,642	21,708	184,047	289,598	6.079	2.505
100	0.245699	1.000000	25,934	25,934	105,551	105,551	4.070	4.070

Table A7  
**Coale-Demeny Model Life Table for Life Expectancy at 92.5 Years, North Model, Females**

Age	$m_x$	$q_x$	$l_x$	$d_x$	$L_x$	$T_x$	$e_x$	$a_x$
0	0.001868	0.001865	100,000	186	99,824	9,250,012	92.500	0.056
1	0.000034	0.000134	99,814	13	399,224	9,150,188	91.673	1.730
5	0.000023	0.000117	99,800	12	498,971	8,750,964	87.685	2.500
10	0.000023	0.000114	99,788	11	498,914	8,251,993	82.695	2.546
15	0.000029	0.000146	99,777	15	498,851	7,753,078	77.704	2.638
20	0.000044	0.000221	99,762	22	498,760	7,254,228	72.715	2.649
25	0.000060	0.000299	99,740	30	498,631	6,755,467	67.731	2.618
30	0.000078	0.000390	99,711	39	498,461	6,256,836	62.750	2.632
35	0.000112	0.000562	99,672	56	498,228	5,758,375	57.773	2.669
40	0.000176	0.000878	99,616	87	497,879	5,260,148	52.804	2.720
45	0.000324	0.001620	99,528	161	497,278	4,762,269	47.849	2.749
50	0.000583	0.002911	99,367	289	496,187	4,264,991	42.922	2.763
55	0.001151	0.005741	99,078	569	494,116	3,768,804	38.039	2.763
60	0.002081	0.010357	98,509	1,020	490,236	3,274,688	33.243	2.738
65	0.003680	0.018250	97,489	1,779	483,400	2,784,452	28.562	2.728
70	0.006452	0.031791	95,709	3,043	471,607	2,301,052	24.042	2.719
75	0.011244	0.054807	92,667	5,079	451,680	1,829,445	19.742	2.706
80	0.019371	0.092706	87,588	8,120	419,175	1,377,764	15.730	2.689
85	0.033838	0.156825	79,468	12,463	368,304	958,589	12.063	2.670
90	0.061509	0.268381	67,005	17,983	292,365	590,285	8.810	2.628
95	0.115524	0.448789	49,022	22,001	190,443	297,919	6.077	2.515
100	0.251421	1.000000	27,022	27,022	107,476	107,476	3.977	3.977