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Review

Demographic forecasting: 1980 to 2005 in review

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Abstract

Approaches and developments in demographic and population forecasting since 1980 are reviewed. Three approaches to forecasting demographic processes are extrapolation, expectation (individual-level birth expectations or population-level opinions of experts), and theory-based structural modelling involving exogenous variables. Models include 0–3 factors (age, period and cohort). Decomposition and disaggregation are also used in multistate models, including macrosimulation and microsimulation. Forecasting demographic change is difficult; accuracy depends on the particular situation or trends, but it is not clear when a method will perform best. Estimates of uncertainty (model-based ex ante error, expert-opinion-based ex ante error, and ex post error) differ; uncertainty estimation is highly uncertain. Probabilistic population forecasts are based on stochastic population renewal or random scenarios. The approaches to population forecasting, demographic process forecasting and error estimation are closely linked. Complementary methods that combine approaches are increasingly employed. The paper summarises developments, assesses progress and considers the future.

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Keywords: Demographic modelling; Population forecasting; Mortality; Fertility; Migration; Extrapolation; Expectations; Causal models; Disaggregation

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1. Introduction

The last twenty-five years has been an exciting period in the history of demographic forecasting. It has witnessed the emergence from a period of relative inactivity, the realisation of previously unappreciated truths about traditional methods of population projection and the development of new probabilistic approaches to the problem. This heightened activity does not seem to be a result of more demanding users; rather it appears that demographers are adopting the methods of other disciplines and applying them to their trade. Perhaps, in a world of increasing statistical sophistication, the discipline could no longer afford to have population forecasting regarded as “an old-fashioned art form, somewhat embarrassing to the profession, like a disreputable relative” (Ryder, 1990:433). Population forecasting is, after all, the public face of the profession.

Demographic forecasting is an important topic: population, household and related forecasts form the basis of social and economic planning and are fundamental to many other forecasting exercises. The many uses of population forecasts give rise to choices on several dimensions. The time horizon may be as short as a year, more commonly a generation, increasingly a life span, or occasionally longer still.

The population in question may be local, national, regional or global. To provide the necessary detail, the population must be disaggregated by age and sex, and often by geographical region; yet further disaggregation may be desired. To achieve this, the three components of population change (mortality, fertility and migration) must be separately forecast and appropriately combined; each component may be decomposed and its parts independently forecast. Population forecasting is thus a highly complex and difficult undertaking (Keyfitz, 1985).

Population forecasting is also highly uncertain: as Keyfitz (1996:xii) remarked, “The best demographers do it, but none would stake their reputation on the agreement of their forecasts with the subsequent realization.” Uncertainty in demographic forecasting has been a major focus, contributing to the rapidly increasing volume of published research on the overall topic. It is impossible to include all this material in a single review. Several useful collections and studies have previously appeared (notably Ahlburg & Land, 1992a; Bongaarts & Bulatao, 2000; Lutz & Goldstein, 2004a; Lutz, Vaupel, & Ahlburg, 1999; *North American Actuarial Journal*, 1999; Rogers, 1995a).

During the twentieth century, fertility was the most important component in determining population size.

However, forecasting fertility proved to be difficult in the post-World War II era: neither the “baby booms” of the 1950s nor the “baby busts” of the 1970s were foreseen. Neither was the post-war rapid decline in mortality foreseen. Both mortality and migration forecasting were naïve: for decades, official population forecasts widely assumed that mortality would not improve, at least beyond the immediate future, and migration was treated as an uninteresting constant.

The historical booms and busts in childbearing have to a large degree provided the impetus for the more recent renewal of interest in demographic forecasting. The uneven and transitional age structure created by past fertility fluctuations is the dominant feature of present day demography in the developed world. Populations are ageing: not only is the large cohort of baby boomers moving into the post-retirement ages, but the baby-bust and younger cohorts are relatively small, creating fiscal problems for social security and health care provision for the elderly. In this context, forecasting mortality has gained prominence, fuelled by the fact that larger proportions will survive to older ages as life expectancy continues to rise. Further, replacement migration, seen as a solution to population ageing (United Nations, 2001), has increased the quantum of migration. In some countries, population growth due to migration now rivals natural increase, a clear indication that forecasting methods are needed for migration, notwithstanding the difficulties involved.

This paper reviews methodological developments in demographic forecasting during the last twenty-five years. It concentrates on forecasting the three components of population change and their combination in population forecasting, where applications are most common. The focus is on national forecasts, principally for industrialised countries where data availability permits greater statistical sophistication and where most research has been undertaken.

The paper is organised as follows. Section 2 briefly describes the state of the art of demographic forecasting that existed in 1980. Section 3 discusses approaches used in forecasting the demographic components. With this framework established, developments in modelling and forecasting mortality, fertility and migration are presented in Section 4. The combination of the components in probabilistic population forecasting is discussed in Section 5. The

direct forecasting of populations is addressed in Section 6. Finally, Section 7 assesses progress and considers possible future developments.

2. The state-of-the-art in 1980

Demographic forecasting has a long history (De Gans, 1999). By 1980 there was a well-established tradition of using the cohort-component method of population projection. This is a system of demographic accounting in which the population is advanced forward in time through the application of time-specific survivorship ratios by age and sex and the derivation of births from time-specific fertility rates of women by age; migration by age and sex can also be incorporated (Preston, Heuveline, & Guillot, 2001: 119–129). The problem of population projection is thus decomposed into the separate forecasting of these three components of population change.

Traditional population projections typically comprise three deterministic scenarios, based on combinations of assumptions about mortality, fertility and migration. If mortality, fertility and migration each have three alternative assumptions or variants, there are nine possible combinations from which three are chosen as the high, medium and low scenarios according to population size. Often the components are restricted to one or two variants; indeed, the difference between scenarios may be entirely due to one component (usually fertility). However derived, the high–low interval is generally portrayed as containing likely future population values.

A limitation of this scenario-based approach is that it is impossible to specify the probability for the high–low interval, even if probabilities could be specified for the range in component variants. Further, the assumptions embody an element of rigidity: component variants are typically high, medium or low for the duration of the forecast rather than varying within the range over time. Perfect correlations are also introduced across age by the use of fixed age patterns, and between components by the particular component combination for each scenario. These correlations result in the probabilistic inconsistency of population projections: the high–low interval for population size will be inconsistent with high–low intervals for other characteristics of the projected population, notably

ratios of age groups (Alho & Spencer, 1985; Lee & Tuljapurkar, 1994).

Both the patent inability of demographers to foresee demographic change and the rigidity of the scenario-based approach contributed to the assertion that traditional population projections are merely ‘what-if’ illustrations. Increasingly, however, demographers (e.g., Ahlburg & Lutz, 1999; Ahlburg & Vaupel, 1990; Stoto, 1983) regard any so-called projection intended to be a best guess of the future as a forecast, and this is the view taken here. Certainly users adopt this view: as Keyfitz (1972:353) remarked, “a demographer makes a projection, and his reader uses it as a forecast”.

In 1980, this traditional deterministic scenario-based approach was used by international bodies, such as the United Nations (UN) and World Bank, and national statistical offices almost without exception. Today, changes are occurring: statistical forecasting is gaining prominence. The developments that have led to these changes are reviewed in the following sections. The ongoing use of the traditional approach is not discussed.

3. Approaches to forecasting the demographic components

In forecasting demographic processes such as mortality, fertility and migration, three approaches or classes of methods are often identified: extrapolation, expectation and explanation. Extrapolative methods focus on the regularity of patterns and trends and extend these into the future without recourse to other knowledge in the form of exogenous variables. Methods based on expectation may use individual data (such as surveys of women’s future birth expectations) or the opinions of experts about future demographic developments. Methods seeking to explain demographic processes use structural models based on theories relating demographic quantities to other variables.

In practice, the distinction between methods is not always clear-cut. Subjective expert opinion may be incorporated into extrapolative methods, exogenous variables are sometimes used to enhance extrapolative models, and structural models may involve extrapolation. A method may also be used in conjunction

with decomposition and disaggregation. Disaggregation of the population serves to account for variation in the demographic process, while decomposition of the process simplifies this task and increases understanding. This ‘explanatory’ approach may be viewed as an alternative to causal structural modelling.

To some extent, the forecasting approach is linked to the measure used: events or rates (or probabilities), aggregate or age-specific, period or cohort. A major problem for age-specific forecasting is high dimensionality, especially when single years of age are used. Parametric models are widely used to address this problem.

The models used in forecasting can be classified according to the number of factors modelled: zero, one, two or three (following [Tableau, 2001](#)). A factor may be viewed as a classificatory variable intrinsic to the data. Zero-factor models are simply the time series of aggregate measures or age-specific rates (each treated independently). One-factor models express demographic rates as a function of age (or duration), making use of their characteristic and stable age patterns. Two-factor models most commonly take age (duration) and period (time) into account; they may model age and cohort but cohort models demand lengthy series of annual data. Three-factor or age-period-cohort (APC) models aim to distinguish between age, period and cohort effects, but suffer from the identification problem arising from the linear dependence of any one factor on the remaining two (see [Tableau, 2001](#)). If three-way classification (also known as doubly-classified data or non-overlapping cohorts) is used in data collection, this dependence is lost ([Willekens & Baydar, 1986](#)). The APC model is useful in describing the past but has not generally been successful in forecasting. The more complex models are used predominantly in extrapolation.

3.1. Extrapolative methods

Extrapolation is the most common approach in demographic forecasting. Extrapolative methods are essentially atheoretical; the only assumption is that the future will be (in some sense) a continuation of the past. This is their strength, but it is also their fundamental weakness: historical patterns may not be the best guide to the future, notably because

changes in the trend, or structural changes, may be missed. Extrapolative methods make no use of exogenous variables: they do not incorporate current knowledge about actual and prospective developments in relevant areas such as medicine and new diseases, lifestyles and the economy.

The most commonly-used method of extrapolation is univariate ARIMA modelling (Box & Jenkins, 1976). In the case of zero-factor models, such as a time series of total births, the series is directly modelled and forecast. When one-factor models are employed, the time series are the fitted parameters of the deterministic model (a parameterisation function or relational model). Erratic parameter time series and interdependencies among parameters present difficulties for forecasting. Forecast age patterns may be implausible, and if interdependencies are not taken into account (e.g., by a vector autoregression), forecast prediction intervals will be inaccurate.

Principal component models are multiplicative two-factor models used with time series extrapolation in forecasting. The estimated age parameters (or functions) are assumed to be fixed and time series methods are used to extrapolate the time-varying parameters. The most prominent example is the Lee–Carter model (Lee & Carter, 1992). Tabeau (2001) noted that the Lee–Carter model can be viewed as a statistical association model for a two-way cross-classification table, and as such belongs to the class of generalised linear modelling (GLM) and can be estimated by maximum likelihood estimation for Poisson-distributed errors. This link was first used by Wilmoth (1993) and has been developed by others.

An alternative approach to time series methods is to directly model demographic rates within the GLM framework with time as an explicit covariate. The zero-factor model is a constant and the one-factor model is a simple regression on time. Additive bivariate regression models constitute the two-factor case. An extension is the dynamic parameterisation model (Tabeau, Ekamper, Huisman, & Bosch, 2001) in which the parameters of an initial static base parameterisation function are linear or non-linear functions of time. Since time is a covariate, forecasting is achieved by using appropriate time values. However, two- or higher-degree polynomials in time are often required to attain a satisfactory fit, leading to implausible forecasts. A partial solution is to use

linear splines for subperiods, but this is restricted to short-term forecasting because turning points cannot be identified (De Beer, 1989).

3.2. *Methods based on expectation*

Methods based on expectation include the use of data on the expectations of individuals about their own behaviour, the use of data on the expectations of a group of experts about population-level behaviour based on historical data and research, and the less structured use of the expectations or informed judgment of experts in the forecasting process.

Demographic forecasting based on reported individual expectations is most common for fertility, where survey data on birth expectations are used (usually in a qualitative or ad hoc manner) to inform the forecast; however, the lead time is clearly limited. A problem with individual expectations data is that they tend to lag prevailing trends. Recent innovations in the collection of individual expectations data include direct questions about subjective probabilities of future events. For example, personal estimates of the chance of survival to a certain age produced similar survival probabilities to life table estimates (Hauser & Willis, 2005). The approach may provide useful information for probabilistic forecasting, particularly where historical data are lacking.

Individual expectations can be viewed as the outcome of unspecified complex psycho-social models that determine individual decision-making (Hendershot & Placek, 1981:307–310). In this sense, they are a substitute for the explicit theoretical models of structural modelling. Similarly, forecasts based on the expectations of a group of experts implicitly take theory into account (Lutz, 1996). The Delphi-based method developed by Lutz and colleagues (Lutz, Sanderson, Scherbov, & Goujon, 1996) combines the qualitative and quantitative opinions of a group of experts into quantitative forecasts for each demographic component. The forecast is defined by three points in the future, thus allowing for limited changes in trend; linearity is assumed between points (see also Section 5.5).

The proponents of this expert-based approach argue that it is preferable to the time series approach because expert opinion takes into account the possi-

bility of future structural change and unexpected events (Lutz, Goldstein, & Prinz, 1996:38). However, the conservativeness of expert opinion with respect to mortality decline is widespread; Alho and Spencer (1990a) found that US experts systematically forecast smaller declines than actually occurred. Expert opinion about future fertility has not been any more accurate (Lee, 1999). Similar to the lag in individual expectation, expert opinion is subject to assumption drag (Ahlburg & Vaupel, 1990). Further, the expert-based approach tends to be limited to aggregate measures, reflecting the difficulty in specifying such details as age-specific rates or parity-specific fertility, thereby precluding information that might be expected to be pertinent to identifying structural change. The approach is advantageous in developing countries because of its lesser demands for historical data.

Less formally, subjective judgment often influences the forecasts (Alders & De Beer, 2005). Informed judgment may be directly involved in constraining or adjusting an extrapolative forecast through targeting, where the expert specifies a value of the relevant measure, the year by which it will be reached and by which path (Olshansky, 1988). Targets may be based on the experience of a more advanced population or subpopulation (Pollard, 1987), a model such as the UN ‘ultimate life table’ (United Nations, 2000), or notions of eventual stability as in replacement fertility (Lee, 1990). Linear interpolation, or a more complex curve fitting procedure such as assuming logistic growth, is then used between current and target rates. Targets tend to be limited to aggregate measures, while model life or fertility tables (e.g., United Nations, 2000) provide the target age pattern; intermediate values are derived through interpolation with respect to level. In the case of developing countries, the method is informed by demographic and epidemiological transition theories, or in the case of targets drawn from subpopulations, by heterogeneity theory (Olshansky, 1988).

Informed judgment has formed the basis of many assumptions in traditional population projections, either in combination with extrapolation or as the sole input. However, such methods have tended to be unsystematic and in many cases inadequately documented, even in developed countries (Crujisen & Eding, 2001; Lutz, Vaupel, & Ahlburg, 1999; Olshansky, 1988).

3.3. *Structural modelling*

Structural models explain demographic rates in terms of the underlying socio-economic and proximate determinants. An advantage of structural feedback is that feedback mechanisms and limiting factors can be taken into account (Cohen, 1999; Lee, 1990). Unfortunately, the danger of model misspecification is high because of a less than secure theoretical basis: most if not all economic–demographic relationships are merely hypotheses (Ahlburg & Land, 1992b). Further, high autocorrelations in the dependent and independent variable series produce high correlations between series, even when there is no causal link. In this situation, a model may have high descriptive power, but little predictive power. Nevertheless, assuming the underlying hypotheses hold, an important use of structural models is the simulation of the effect of policy changes affecting the independent variables.

Models based exclusively on lagged variables constitute a means of forecasting, but lag time limits the forecast horizon. In order to extend the forecast horizon, structural modelling is combined with extrapolation (e.g., using transfer function models). The focus of the forecasting problem is shifted from the demographic variable to its determinants. However, as Keyfitz (1982a) noted, it may be no less difficult to forecast the determinants than to directly forecast the variable. Brass (1974) set stringent conditions for which structural models can be useful for forecasting: the association between variables must persist over time, the exogenous variables must be predictable (or lagged), and the association must be strong. Land (1986:898) pointed to “what is essentially an impossibility theorem” for forecasting with structural models, namely Ashley’s (1983) theorem that if the mean squared error of the forecast of the exogenous variable exceeds its variance, then including the forecasted exogenous variable will produce a worse forecast than omitting it. Unfortunately, the chances of the theorem applying increases with forecast length.

While forecasting with structural modelling is often regarded as the ideal, it has not on the whole produced more accurate forecasts. Keyfitz (1982a) concluded that knowledge has not improved the accuracy of forecasts. While Sanderson (1999) (and others, e.g., Heckman & Walker, 1989) found

evidence to the contrary, his conclusion was based on a comparison of several global and developing-country structural models with scenario-based UN population projections. For developed countries, comparisons with other statistical forecasting methods are required. Nevertheless, the ideal of improved accuracy through structural modelling remains: Ahlburg, Lutz and Vaupel (1999:191) call for the greater use of models that include other demographic events (such as marriage and morbidity), that involve greater disaggregation (by education, for example) and that take into account economic, social and environmental dynamics. For reviews of aggregate economic–demographic models, see Land (1986) and Ahlburg (1987a). Sanderson (1999) reviews structural models used in global and developing country forecasts.

The theoretical advantage of structural modelling, the ability to capture turning points or structural change, has not often been realised. For fertility in particular, identifying changes in direction ahead of time has been a major difficulty. The demographers' inability to foresee the peaks and troughs of the post-war baby boom and bust is legendary. Ahlburg (1982) showed that the US Census Bureau failed to foresee all 20 turning points in 11 forecasts of total births (of 5–15 year durations from base years of 1948 to 1975). Structural models, notably models based on Easterlin's (1968) hypothesis of baby booms and busts, should in theory be better able to forecast structural change, but consistent success is elusive (Tuljapurkar & Boe, 1999; Wachter, 1991).

3.4. *Decomposition and disaggregation*

Decomposition, such as the breakdown of mortality by cause of death or fertility by parity, is addressed by multiple-decrement life-table methods. The different parts may be best forecast by different methods. Disaggregation is the division of the base population beyond age and sex, for example, by education, ethnicity or region of residence. It is addressed by multistate modelling (Rogers, 1975, 1986, 1995b; Rogers & Woodward, 1991), which is the dynamic multidimensional extension of the cohort-component model; transitions between states are modelled and demographic rates are specific to each state. Lutz, Goujon, and Doblhammer-Reiter (1999) discuss the criteria for useful disaggregation; the ideal is to find

homogeneous subgroups (states) for which demographic rates are stable or changing in predictable ways so that extrapolation is appropriate. However, selection effects and unrecognised heterogeneity may limit the use of extrapolation (Vaupel & Yashin, 1985).

Multistate modelling with many dimensions of disaggregation is macrosimulation. Individual-level disaggregation is microsimulation; at this finely-detailed level of disaggregation, accounting for variation is maximised and the probability of an event occurring is often assumed to be time-invariant. Microsimulation produces forecasts of both the numbers of persons in different states and the length of time spent in those states. Two important advantages of multistate modelling (particularly microsimulation) and its focus on transitions between states are the incorporation of behavioural feedbacks and the simulation of hypotheses for assessing the effect of behavioural change. The use of microsimulation models in probabilistic forecasting is reviewed by Van Imhoff and Post (1998), including a comparison with macrosimulation models.

3.5. *Choice of approach*

The choice of approach depends on various factors including data availability and purpose; judgment also plays an important role (Pollard, 1987). Land (1986) confounded approach and purpose in defining three classes of methods for forecasting national populations: demographic accounting or cohort-component methods for long-term population forecasting, statistical time series methods for short-term forecasts of demographic rates, and structural modelling methods for the simulation and forecasting of policy changes. For long-term forecasts, Land questioned the use of statistical time series methods and thus advocated the use of informed judgment and scenarios. More recently, time series methods have increasingly been used for long-term demographic forecasting with some apparent success, admittedly in stable circumstances. Willekens (1990:17) contends that the distinction between short- and long-term forecasting is a false issue: the real issue is when a method can be expected to be accurate — when the demographic system is stable or when it is changing.

Whatever method is used for whatever period, it is important to draw the distinction between model fit and forecast accuracy. [Ahlburg and Land \(1992b\)](#) note that in-sample errors are not necessarily a good guide to forecast errors, and that for long-term forecasting the choice between models cannot reliably be based on historical goodness of fit. However, it is not clear which criteria should be used instead. Clearly there is a need for extensive analysis of out-of-sample or ex post forecast errors (see Section 5.6), but lengthy forecast horizons also limit the feasibility and usefulness of this approach. The measurement of forecast accuracy and the relevant loss function are related concerns ([Ahlburg, 1995](#); [Tayman & Swanson, 1999](#); [Tayman, Swanson, & Barr, 1999](#)).

[Willekens \(1990\)](#) noted the developmental path of forecasting in any discipline from the identification and extrapolation of stable patterns to an understanding of these patterns in terms of the causal mechanisms and processes involved: the search for regularity is transitional. Rather than concentrate on improved methods of extrapolation, [Willekens \(1990:11\)](#) advocated research aimed at understanding the causal factors and processes involved as the precondition for effective forecasting. However, given the long-term nature of such a quest, he agreed with [Keyfitz \(1982a:747\)](#) that “we cannot afford to be ashamed of extrapolating the observed regularities of the past”. This is as true today as it was then.

4. Component forecasting in practice

The framework established in Section 3 forms the organisational basis of the following discussion of developments during the last twenty-five years in modelling and forecasting mortality, fertility and migration. The direct forecasting of population is discussed in Section 6.

4.1. Mortality

Mortality forecasting has received a great deal of attention. Previous reviews include [Keyfitz, \(1982b\)](#), [Pollard \(1987\)](#), [Murphy \(1990\)](#), [Tuljapurkar and Boe \(1998\)](#) and [Wong-Fupuy and Haberman \(2004\)](#). See also [Bengtsson and Keilman \(2003\)](#) and [Keilman \(2005a\)](#).

The simplest method of forecasting mortality is to extrapolate life expectancy (a zero factor model), or some other life-table measure, and to use empirically-based model life tables to obtain the age pattern; this has been facilitated by the expansion of life tables to include older ages ([Coale, Demeny, & Vaughan, 1983](#)) and lower mortality ([Coale & Guo, 1989](#)). The independent extrapolation of age-specific rates commonly involves mortality reduction factors or some fraction of the reduction factor ([Goss, Wade, Bell, & Dussault, 1998](#); [Pollard, 1987](#)).

Parameterisation (one-factor) functions of mortality have a long history ([Forfar, McCutcheon, & Wilkie, 1988](#); [Gavrilov & Gavrilova, 1991](#); [Keyfitz, 1982b](#); [Tabeau, Willekens, & van Poppel, 2002](#)). Among the best-fitting are the three-term functions capturing the age pattern of mortality in childhood, young adulthood (the accident hump) and senescence. [Mode and Busby \(1982\)](#) proposed an eight-parameter version, in which the three terms are estimated independently. The Heligman–Pollard model ([Heligman & Pollard, 1980](#)) also has eight parameters; each term takes positive values only at relevant ages, the whole function being estimated in one step. The parameters have meaningful interpretations; however, high correlations between parameters, particularly for male mortality (see [Hartmann, 1987](#)), compromise this interpretability. Though the model fits well ([Hartmann, 1987](#); [Kostaki, 1988](#); [Mode & Busby, 1982](#); [Rogers & Gard, 1991](#)), only [Forfar and Smith \(1987\)](#) found it useful for forecasting. Others encountered difficulties, particularly in determining the best base period for projecting the parameters ([Keyfitz, 1991](#); [Pollard, 1987](#)). [McNown and Rogers \(1989\)](#) modelled the eight parameters as univariate ARIMA processes.

The Heligman–Pollard model represents senescent mortality using the Gompertz function; three variants were also proposed. [Congdon \(1993\)](#) demonstrated over-parameterisation in one nine-parameter variant; a reduced model also exhibited instability. Modifications at other ages have been made. [Brooks, Sams, and Williams \(1980\)](#) and [Chauhan \(1997\)](#) simplified the childhood term. [Kostaki \(1992\)](#) introduced a ninth parameter to improve the fit at young adult ages. [Carrière \(1992\)](#) proposed a three-term model with eight interpretable parameters which gave a better fit to US data than the Heligman–Pollard model; a four-

term version with eleven parameters gave significant improvements. Carrière's use of a mixture of survival functions is equivalent to a multiple-decrement life table; the generalised framework can accommodate extra terms and other functions. The flexibility of this model has potential advantages in forecasting.

Rogers and Planck (1983) modelled mortality with the four-term nine-parameter version of the multi-exponential function, comprising a constant, exponentially declining child mortality, a double exponential accident hump (the Coale and McNeil (1972) function) and Gompertzian senescent mortality. Rogers and Little (1994) found slightly poorer fits than the Heligman–Pollard model. In forecasting, McNown, Rogers and Little (1995) found interdependencies among parameters; five were fixed, and four were modelled by three univariate ARIMA processes and a bivariate autoregression. McNown and Rogers (1992) forecast total mortality and five cause-specific mortalities by fixing six parameters and modelling only the level parameters by univariate ARIMA models.

Siler (1983) proposed a five-parameter model that was found to fit better than either the Heligman–Pollard or Mode–Busby models (Gage & Mode, 1993); only the ten-parameter Mode–Jacobson (1984) model provided a better fit. Hannerz (1999, 2001a) also proposed a five-parameter model that fitted recent Swedish female mortality better than models which include a term for the accident hump. For males, however, the persistence of the accident hump necessitated an additional three-parameter term (Hannerz, 2001b). It appears that these have not been used in forecasting.

The relational model of mortality (Brass, 1971) linearly relates the logit transformations of observed and standard mortality. Forecasts based on this model include those by Golulapati, De Ravin, and Trickett (1984) for Australian male cohorts (see also Pollard, 1987), and Keyfitz (1991) for Canadian data. Zaba (1979) and Ewbank, Gómez de León, and Stoto (1983) developed four-parameter models to increase flexibility. Congdon (1993) adopted the Zaba model and forecast the relatively stable parameters by univariate ARIMA models. Hannerz (2001c) combined the features of relational models with parameterisation functions and model life tables in a regression model.

The most well-known method of mortality forecasting is the Lee–Carter method (Lee & Carter, 1992) for long-term forecasting; see also Gómez de León (1990). The underlying two-factor model describes a one-parameter system with fixed age effects and has a homoscedastic additive Gaussian error structure. A unique least squares solution is found by singular value decomposition (SVD). Lee and Carter incorporated an adjustment of the level parameter so that fitted deaths match observed total deaths in any year; this avoids discrepancies arising from modelling on the logarithmic scale (Lee, 1992). The adjusted level parameter is modelled by time series methods; in almost all applications, a random walk with drift has been found to be applicable. Girosi and King (2006:40–49) review the method in detail.

Wilmoth (1993) developed a weighted SVD solution to the Lee–Carter model, providing a good fit without the need to adjust the level parameter, and a maximum likelihood solution; these methods gave almost identical results to the unweighted SVD in the case of Japan (Wilmoth, 1996). Carter and Lee (1992) addressed divergence by sex by estimating a joint level parameter while retaining sex-specific age effects. Lee and Nault (1993) jointly forecast provincial mortality. Lee (2000a) discusses these and other extensions of the method; data reconstruction for years where only total deaths are available is also possible (Lee & Rofman, 1994). Tuljapurkar, Li, and Boe (2000) applied the method (without adjustment of the level parameter) to the G7 countries, finding a common pattern of linear decline in the level parameter. Lundstrom and Qvist (2004) used the method to examine changing trends in the Swedish mortality decline during the twentieth century.

The stability of the Lee–Carter method to structural change and initial conditions was examined by Carter (1996, 2000) and Carter and Prskawetz (2001). Tuljapurkar (2005) further demonstrates the robustness of the method. Li and Chan (2005) propose an outlier-adjusted method. Lee and Miller (2001) noted the influence of the adjustment procedure on forecast bias. Three modifications were introduced: the fitting period was restricted to post-1950 to reduce structural shifts, adjustment of the level parameter was by matching life expectancy, and observed rates were used as jump-off rates. Booth, Maindonald, and Smith (2002) also modified the method after finding

historical departures from linearity in the Australian mortality decline: they proposed a method for determining the optimum fitting period for use in shorter-term forecasting when the relatively recent trend is linear, and adjusted the level parameter by fitting to the age distribution of deaths (a conditional maximum likelihood procedure). In a twenty-population comparison of the original method and these two variants, [Booth, Tickle, and Smith \(2005\)](#) found the Lee–Miller and Booth–Maindonald–Smith variants to perform roughly equally well overall over a 15-year forecast period, but their ranking varied by sex; in other words, performance depends on situation. The Lee–Miller variant has been widely adopted as the standard Lee–Carter method.

Recent developments extend the applicability of the Lee–Carter method. [Li, Lee, and Tuljapurkar \(2004\)](#) demonstrate how, by assuming a linear trend in the level parameter, the method can be applied to populations with limited data at unequal time intervals. [Li and Lee \(2005\)](#) develop an augmented common factor method for overcoming the divergence problem, using a common factor to model group mortality and an additive population-specific factor. Such approaches make use of demographic convergence of mean levels; [Edwards and Tuljapurkar \(2005\)](#) note that substantial differences in variances should also be taken into account.

The Lee–Carter method has close similarities to the principal components approach used by [Bell and Monsell \(1991\)](#), and [Bell \(1997\)](#) discusses the similarities and differences in detail, demonstrating the importance of bias adjustment and the superiority in short-term forecasts of Lee–Carter over both Heligman–Pollard and principal components using all components. Whereas the Lee–Carter method uses only the first component, the principal components approach typically uses several, thereby allowing for greater flexibility in forecasting change. Higher order terms in the Lee–Carter method were modelled by [Booth, Maindonald, and Smith \(2001, 2002\)](#) and modelled and forecast using univariate ARIMA processes by [Renshaw and Haberman \(2003a\)](#). [Hyndman and Ullah \(2004\)](#) extend the principal components approach by adopting a functional data paradigm combined with nonparametric smoothing (penalised regression splines) and robust statistics, and fit univariate time series models to each compo-

nent coefficient (or level parameter); the Lee–Carter method is shown to be a special case of this generalized approach.

[Wolf \(2004\)](#) proposed a Lee–Carter variant based on a first difference specification that integrates estimation of the Lee–Carter and time series models. [De Jong and Tickle \(2006\)](#) generalise and extend the Lee–Carter method by introducing a state space framework that also integrates model estimation and forecasting; they use B-splines to build in the expected smooth behaviour of mortality over age, and outline further possible extensions.

Integrated estimation and forecasting is a feature of modelling within the GLM framework. [Renshaw, Haberman, and Hatzoupoulos \(1996\)](#) proposed a two-factor model with two multiplicative terms: a Gompertz–Makeham graduation term and an age-specific trend adjustment term (see also [Renshaw, 1991](#)). This model was used to forecast UK mortality at ages 65+ with qualified success: the optimum fitted model parameters did not necessarily generate plausible forecasts, for which lower-order polynomials are often required ([Sithole, Haberman, & Verrall, 2000](#)). This study included a comparison with the standard actuarial practice of fitting the Gompertz–Makeham class of functions (see [Forfar et al., 1988](#)). [Currie, Durban, and Eilers \(2004\)](#) employed bivariate penalized B-splines to smooth over both age and time within a penalized GLM framework with extrapolation of the fitted surface over time; comparison with Lee–Carter revealed a much slower mortality decline.

In modelling mortality reduction factors using GLM, [Renshaw and Haberman \(2000\)](#) identified the conditions under which the underlying structures of the GLM and Lee–Carter models are identical; they later demonstrated the use of the Lee–Carter methodology for forecasting the reduction factors ([Renshaw & Haberman, 2003b](#)). [Renshaw and Haberman \(2003c\)](#) developed a GLM-based approach that parallels the Lee–Carter method, including matching observed and expected total deaths. The important difference between the two approaches is in the treatment of time: in the Lee–Carter method time is a factor estimated by SVD, while under the GLM approach time is a known covariate. The GLM approach is based on a heteroscedastic Poisson (non-additive) error structure. [Brouhns, Denuit, and Vermunt \(2002\)](#) proposed a similar bilinear approach in which the Lee–Carter

model forms the systematic component (predictor) in the Poisson error setting (cf Wilmoth, 1993). Renshaw and Haberman (2003a) compare the Lee–Carter, linear, and bilinear approaches with and without age-specific enhancement: in the Lee–Carter case such enhancement is achieved by including the second term, in the GLM case it involves a break point or hinge to allow for greater emphasis on recent trends, and in the bilinear case the two-term Lee–Carter model is implemented as a double bilinear predictor.

Forecasts based on cohort models are relatively few because of heavy data demands; where data are available the model may depend on the (inappropriate) experience of cohorts born in the nineteenth century if the entire age range is considered (e.g., Tabeau et al., 2001). This problem is reduced when only adult mortality is of interest. The cohort approach is free of tempo distortions (caused by changes in timing). Bongaarts and Feeney (2002, 2003, 2005) propose an adjustment for tempo distortions in period life expectancy, with implications for forecasting. Other aggregate measures of mortality (Bongaarts & Feeney, 2003; Guillot, 2003; Sanderson & Scherbov, 2005) may be considered.

In developing countries, restricted time series of observations limit the application of most forecasting methods. Lutz, Sanderson, Scherbov, and Goujon (1996) overcome this problem by deriving target life expectancy as the average expectation of experts. Girosi and King (2006) address the problem in the context of cause-of-death forecasting by incorporating prior information as covariates in a linear age–period regression model. The covariates, which may differ by age, may be lagged observed exogenous variables or forecasts of reliable longer time series; the model may thus be regarded as structural. Smoothness over both age and time can be incorporated as priors, based on empirical evidence or informed judgment. The method improves on existing Bayesian approaches by borrowing strength (or partial pooling) based on expected values rather than on coefficients, and maximises the use of prior information, hence maximising automation.

Regression models are easily extended to three factors, but (as noted above) age–period–cohort (APC) models must accommodate the identification problem (see also Van Hoorn & De Beer, 2001). To address this, Wilmoth (1990) developed a modified

model involving additive age and period effects and several multiplicative interaction terms; see also Wilmoth (2001). Tabeau (2001) concluded that mortality forecasting based on APC models is not feasible because of the difficulty in assuming future period effects (although age and cohort effects can be assumed to be fixed); only in forecasts of specific diseases would sufficient epidemiological knowledge be available. Caselli (1996, 2002) used the APC model to forecast mortality from leading causes.

Forecasting by cause of death has been advocated from a theoretical perspective as a means of gaining accuracy (e.g., Crimmins, 1981), but experience has largely proved otherwise. Little is gained from decomposition because of similar age patterns in the main causes; cause-of-death reporting is unreliable at older ages where most deaths occur; and cause-reduction may have minimal effect on total mortality (Murphy, 1995). Further, model misspecification and the presence of leading indicators (where changes in one cause systematically precede changes in another) can result in reduced accuracy from decomposition (Alho, 1991). The short time series of cause-of-death data also limits extrapolation. Using the multi-exponential model, McNowen and Rogers (1992) found no consistent discernible gain in accuracy from cause-of-death decomposition. Wilmoth (1995a) demonstrated that, for proportional rates of change models, mortality forecasts based on the sum of cause-specific forecasts will always be higher than those based on aggregate data because causes of death that are slow to decline come to dominate as other causes are more rapidly diminished. Using APC models for ages 60+, Caselli (1996) found this to be true for females but reversed for males. Tabeau et al. (2001) also found this difference between the sexes for France, Italy and the Netherlands, but not for Norway. Stoto and Durch (1993), Tuljapurkar (1998) and Wilmoth (2005) further discuss these issues.

Mortality forecasting based on (partial) cause-elimination and cause-delay models make use of targeting and informed judgment (Manton, Patrick, & Stallard, 1980; Olshansky, 1987, 1988); Kunst, Mackenbach, Lautenbach, Oei, and Bijlsma (2002) incorporated competing causes of death. These methods have often led to conservative forecasts of mortality reduction. Le Bras (2005) elaborates a cause-delay model of mortality change.

Guterman and Vanderhoof (1998) argue the case for structural models of cause-specific mortality change that take medical and other factors into account, despite the difficulties involved. Structural models of mortality at older ages relate lifestyle and other risk factors to functional status and mortality using vector autoregression, achieving some improvement over traditional time series and informed judgment methods (Manton, Stallard, & Tolley, 1991; Manton, Stallard, & Singer, 1992). However, their forecasting potential is limited by the short time series of risk factors, the large number of parameters and the non-linear interactions generating the mortality forecast. Epidemiological, structural and multistate approaches to cause-of-death forecasting are reviewed by Van Den Berg Jeths, Hoogenveen, De Hollander, and Tabeau (2001); (see also Van Genugten, Hoogenveen, & De Hollander, 2001).

The forecasting of AIDS mortality is particularly difficult due to data limitations and inaccuracies, but methods have evolved rapidly. An early method involves the extrapolation of AIDS cases, while another uses a model of the progression from HIV infection to the onset of AIDS to back-calculate HIV infections, which are then forecast. The limitations of these methods are discussed by Bloom and Glied (1992) and Bos and Bulatao (1992). A third method incorporates a model of HIV transmission, and a fourth adds a behavioural model (Auvert, 1991; Bongaarts, 1989; Bos & Bulatao, 1992; Bulatao, 1991). Oliveira and Mexia (2004) propose a method which uses data on infection and AIDS mortality without recourse to transmission and behavioural models.

For some purposes, forecasts of senescent mortality suffice. Using a two-factor regression-based approach, Heathcote and Higgins (2001a,b) modelled the log-odds of lexis-parallelogram mortality at ages 40+ by an expression involving time, a quadratic in age and an age–time interaction with up to 24 additional predictors; in forecasting, naïve extrapolation gave implausible divergence by sex and subjectively-adjusted coefficients were used. The approach by Bongaarts (2005) adopts a shifting logistic model of the pattern of change over time in age-specific senescent mortality at ages 25+. This one-factor parameterisation function makes use of the (nearly) constant slope parameter, or the equivalence of

mortality decline and a shift of the senescent force of mortality schedule to older ages. In forecasting, the age pattern of change is fixed (constant proportionate decline) and the remaining two parameters are independently extrapolated: these are the levels of senescent and age-invariant background mortality. A shifting logistic is combined with the Lee–Carter method in long-term UN forecasts (Buettner & Zlotnik, 2005; United Nations, 2004). A relatively constant age pattern of change at 25+ was recently recommended for US mortality (Lee, 2000b).

A difficulty in senescent mortality forecasting is the adequacy of both the data and the models at the oldest ages, at which most deaths occur (Kannisto, 1994; Kannisto, Lauritsen, Thatcher, & Vaupel, 1994; Tuljapurkar & Boe, 1998). The Gompertz function has been widely employed (Olshansky & Carnes, 1997; Pollard & Valkovics, 1992). However, the Gompertz and other models were developed on truncated data and are not recommended for use much beyond the age of truncation: this is 85+ in the Heligman–Pollard case (Heligman & Pollard, 1980). At older ages, the Gompertz is inadequate (Boleslawski & Tabeau, 2001). Willekens (2001) discusses the Gompertz function in the contexts of survival analysis and event-history analysis, with implications for modelling and forecasting. Models incorporating theoretical concepts of ageing are reviewed by Yashin (2001) and Yashin, Iachine, and Begun (2000).

Where data are truncated, extension to the oldest ages is necessary. Wilmoth (1995b) found the Coale and Kisker (1990) targeting method to fit well at the oldest ages, while Thatcher (1999) demonstrated good fits for the logistic model. Modelling by Renshaw and Haberman (2003c) supports the Coale and Guo (1989) method which also uses a target. A relational model was developed by Himes, Preston, and Condran (1994); no assumption is made about the upper limit. Buettner (2002) examined various methods including an adaptation of the Lee–Carter model. Boleslawski and Tabeau (2001) reviewed the efficacy of eleven functions in representing mortality at ages 80–110, concluding that the Coale and Kisker (1990) method is superior when reliable data are available to at least age 90 (see also Alho & Nyblom, 1997), but if data are reliable only for age 85, the use of polynomials to extrapolating the age pattern is recommended. Buettner and Zlotnik (2005) found the Coale–Kisker

method to be too restrictive in very long-term forecasts; they preferred the logistic model because it does not require specification of an upper limit.

4.2. Fertility

It is generally agreed that fertility and births are non-stationary series (Alho, 1992a; Ermisch, 1992). The difficulty in fertility forecasting stems from structural change, seen in the trajectory of total fertility (quantum), changing age patterns (tempo) and the complex association between the two. Forecasting success has been limited.

Zero-factor models are relatively common in fertility forecasting. Early forecasts focussed on events. McDonald (1979, 1981) used time series methods to forecast total births and first nuptial confinements, easily outperforming economic–demographic structural models in the very short term. Improvements were achieved by incorporating transfer functions linking total births to females of childbearing age and first nuptial confinements to marriages (thus, in effect, forecasting rates).

Forecasting fertility rates rather than births has several advantages including, in deriving births, making use of the fact that the number of women is known with near certainty for the first 15 years. Miller (1986) forecast total fertility and the mean age at childbearing by a transfer function model linking past quantum to current tempo. Age-specific fertility rates have been forecast by Congdon (1980, 1989) using regressions and ARIMA models incorporating periodic time and relative cohort size (in line with the Easterlin hypothesis), and by McDonald (1983) using simple time series models, with greater success than structural models.

Ortega and Poncela (2005) address the difficulty arising from structural change by jointly modelling total fertility trends for a (subjectively-defined) homogeneous group of countries. They use dynamic factor models to estimate one or two common factors capturing the non-stationary average total fertility trajectory and a stationary deviation from the average; they forecast these factors using time series methods. Compared with univariate models and a vector autoregression, the approach yielded substantial gains in forecast accuracy, particularly over longer horizons.

The approach of Lutz, Sanderson, Scherbov, and Goujon (1996) implicitly deals with the problem of past and potential future structural change by deriving target total fertility as the average expectation of a group of experts. This approach has also been used with disaggregation by education by Lutz and Scherbov (2004).

Various theories underpin the structural modelling of fertility; brief reviews are found in Keyfitz (1982a) and Ermisch (1992). Ahlburg (1982) achieved greater short-term accuracy for total births than US official forecasts; his model included marriage, divorce and female labour force participation. Ahlburg also forecast US births using a simple Easterlin relative cohort size model, identifying cycles of alternate generations (Ahlburg, 1983), and forecast Canadian births using a similar model based on births both one and two generations ago (Ahlburg, 1986).

Structural modelling of age-specific fertility rates generally involves separate modelling by age but estimation as ‘seemingly unrelated regressions.’ Ermisch (1983) modelled three age groups in this way as functions of women’s and men’s earnings, relative cohort size and female lifetime employment rates, and forecast them. Poorer results for women aged 30–34 than 20–24 were attributed to heterogeneity in fertility responses to economic change by parity with, the changing parity distribution of women producing unstable responses (Ermisch, 1992). Frameworks that build in the sequential nature of childbearing are thus preferred (De Cooman, Ermisch, & Joshi, 1987; Ermisch, 1988). Ermisch (1992) adopted such a model of fertility rates by birth order; short-term forecasts based on this model were not sensitive to assumptions about the explanatory variables, but neither were they a substantial improvement on time series models, and in the longer term, the main advantage was not in forecasting but in exploring different scenarios for policy purposes (Ermisch, 1992:220).

Several parameterisations have been applied in the modelling of age-specific fertility rates, including the beta, gamma and Hadwiger functions. Hoem et al. (1981) compared several functions, finding the gamma density and the Coale–Trussell (Coale & Trussell, 1974) function to be equally superior except for highly parameterised splines; they noted the Coale–Trussell advantage of parameter interpretability for forecasting. Rogers (1986) considered the Coale–

Trussell model to be overly complex and suggested the direct use of the double exponential function; in other words, the third term of the multi-exponential function (Rogers & Little, 1994). The double exponential was found to fit better to fertility patterns that were relatively symmetrical, and less well to the flatter curves of the 1980s (Knudsen, McNown, & Rogers, 1993), whereas the Coale–Trussell model was superior in cases of higher fertility in the 1960s (Rogers, 1986:51). The four parameters of the double exponential are not readily interpretable; in the absence of time series data, Rogers (1986) regressed each on the gross reproduction rate (GRR) and assumed future GRR in forecasting. Knudsen et al. (1993) partially reparameterised the model, the new parameters being the mode and modal value, and related total fertility to the remaining two parameters (via the gamma density), considerably improving forecastability; univariate ARIMA models performed well in out-of-sample forecasts of US fertility, despite strong interactions among parameters.

The more tractable relationship between the three pattern parameters of the gamma density and the mean and variance of childbearing was made use of by Thompson, Bell, Long, and Miller (1989) for short-term forecasting of age-specific fertility; they directly forecast the level, mean and standard deviation of childbearing using a vector autoregression. The gamma density was also used by Keilman and Pham (2000) in long-term interval forecasts of Norwegian fertility involving a vector autoregression of three of the four parameters (governing level and age pattern, the parameter representing the minimum age of childbearing being constant).

Congdon (1990) used the Hadwiger function, forecasting the four parameters using univariate ARIMA models; with hindsight, this compared favourably with a structural time series model incorporating cyclical and trend factors, relative cohort size and female job opportunities. Congdon (1993) made a similar comparison using the reduced-form Hadwiger function for both period and cohort fertility. He noted that all four-parameter functions are over-parameterised; of the reduced-forms, the beta gave a better fit but the Hadwiger has the advantage of parameter interpretability. Chandola, Coleman, and Hiorns (1999, 2000) used the reduced Hadwiger function to model European fertility: while it provided

a good fit for several countries, for others it was unable to capture the slight hump at young ages that has recently developed. For these countries, a mixture model was used to combine Hadwiger functions for non-marital and marital fertility, making use of disaggregation. The general approach has potential for forecasting.

Brass (1974, 1981) developed the relational Gompertz model which linearly relates observed fertility to a suitable standard. The model is used with incomplete cohort data, or to produce series of the level and two pattern parameters for forecasting. Parameter interpretation was improved by Zeng, Zhenglian, Zhongdong, and Chunjun (2000) in relating the pattern parameters to the median age and interquartile range. Murphy (1982) investigated the use of relational Gompertz models in forecasting. In general, structural change limits the use of parameterisation functions and relational models for forecasting, especially where (as is desirable) vector autoregressions are employed.

The Coale and McNeil (1972) double exponential has been widely shown to fit first births well (Bloom & Trussell, 1984). Bloom (1982) used this model to forecast first births for incomplete cohorts, with limited success for younger cohorts. Trussell and Bloom (1983) allowed the parameters to depend on covariates; Bloom and Trussell (1984) then forecast childlessness for incomplete cohorts. The Coale–McNeil function was elaborated by Kaneko (2003) as the generalized log gamma distribution and used to forecast first marriage and parity-specific fertility for incomplete cohorts.

Evans (1986) used linear regression to predict first birth fertility after age 25 from the proportion attaining parenthood by age 25 (quantum) and the ratio of fertility at 15–19 and 20–24 (tempo); overall fertility was similarly forecast. Martinelle (1993) forecast first birth rates and childlessness using a regression model of incomplete cohort fertility that took education into account. Chen and Morgan (1991) and Morgan and Chen (1992) showed that the Bloom approach was sensitive to censoring below age 30, while the Evans approach was in fact based on period effects, and concluded that it was preferable to base forecasts on the period life table model which assumes that current rates will persist into the future.

Ryder (1990) also advocated a period approach, incorporating continued patterns of change. The dominant effect on fertility change in the short- to medium-term is period rather than cohort; the relationship between the two and their relative advantages in forecasting have been extensively discussed (e.g., [Ní Bhrolcháin, 1992](#); [Ryder, 1983](#)). [Bongaarts and Feeney \(1998, 2005\)](#) proposed an adjustment of period total fertility to take tempo effects into account when data by parity are available. [Kohler and Philipov \(2001\)](#) extended this adjustment to include variance effects. While unadjusted fertility remains the basis of population forecasting, adjusted measures aid understanding and inform future trends. Any potential gains in forecast accuracy are important because long-term population forecasts depend heavily on long-term total fertility forecasts.

[Kohler and Ortega \(2002\)](#) proposed a tempo-adjusted period parity progression measure that can be used to forecast the fertility of incomplete cohorts conditional on a level of fertility and a postponement pattern derived from past period trends. [Sobotka \(2005\)](#) used this method to derive tempo-adjusted first birth probabilities, which he used as the low assumption contrasted with the high assumption based on unadjusted probabilities, separating tempo and quantum effects.

[Lee \(2005\)](#) charts his 30-year journey in fertility forecasting, describing how the very idea of modelling (post-transition) fertility as a random process was widely rejected by demographers for the first 20 years! Forecasters were more receptive: in an elaboration of earlier ideas, [Lee \(1992, 1993\)](#) modelled age-specific fertility rates over time using a single time-varying fertility index (the method parallels the Lee–Carter method for forecasting mortality). To address the problems arising from structural change, lower and upper bounds and an ultimate (average) level were imposed ([Lee, 1993](#); see also [Lee, 1999](#)). [Lee and Tuljapurkar \(1994\)](#) used this model with a different ultimate level and no bounds. [Tuljapurkar and Boe \(1999\)](#) further evaluated the performance of the approach. [Carter and Lee \(1986\)](#) used the approach in a joint model of nuptiality (age and period) and marital fertility (duration and period).

The Lee method is a principal components method. Principal components methods were used by [Bozika and Bell \(1987\)](#) to forecast age-specific fertility, using

the first four components and the level in a vector autoregression. A principal components approach was also used by [Sivamurthy \(1987\)](#). [Hyndman and Ullah \(2004\)](#) also adopted this general approach for forecasting Australian fertility using functional data methods; the logarithms of fertility rates were, after smoothing, modelled by three basis functions, and the coefficients of these orthogonal functions were forecast using univariate time series models. [Bell \(1992\)](#) discusses the use of principal components and various other models in time series forecasting of age-specific rates; see also [Bell \(1997\)](#).

Cohort forecasting of fertility makes lesser demands on data than mortality, but may be compromised by structural change. [Li and Wu \(2003\)](#) modelled fertility for completed cohorts by age and cohort using the [Lee \(1993\)](#) model, and combined the estimated fixed age effect and incomplete cohort observations to forecast the cohort effect, thereby completing that cohort's fertility. The method is restricted to completing cohort fertility when certain assumptions are met.

[De Beer \(1985\)](#) developed the CARIMA (cohort-ARIMA) model for short-term forecasting which was successful in identifying turning points 6 or 7 years ahead in first and second order births. The model forecasts age- or duration-specific fertility rates for cohorts using time series methods, taking into account error covariances and additive period effects, both of which are modelled as ARIMA processes. The unobserved fertility of incomplete cohorts is forecast on the basis of observations at younger ages and for older cohorts. As with ARIMA models in general, a disadvantage of the CARIMA model is the difficulty in interpreting its parameters.

Using post-1950 data for four European countries, [De Beer \(1989\)](#) compared four models: the CARIMA model, a multiplicative APC-ARIMA model ([Willekens & Baydar, 1986](#)), cubic spline models of age-specific fertility rates and an ARIMA forecast of total fertility. An advantage of the APC-ARIMA model was its greater parameter interpretability, but its usefulness is strictly limited to short-term forecasts. The short-term CARIMA and APC-ARIMA forecasts performed slightly better than the ARIMA forecast of total fertility, while the CARIMA forecasts were more accurate than those from the APC-ARIMA model when parity-specific rates were used

(except anomalously for second births). The relative accuracy of the spline and CARIMA models was highly dependent on the position of turning points in the fitting period: either model could produce large forecast errors.

Birth expectations might be regarded as a potentially useful exogenous variable in cohort fertility forecasting, particularly in forewarning of changing trends (Willekens, 1990), but this is often not the case: not only are the expectations data relatively unstable over time, but they also tend to lag rather than lead actual fertility (Ahlburg, 1982; Bongaarts & Bulatao, 2000:93; Lee, 1980; Ryder, 1990; Westoff, 1981). Poor correspondence between stated expectations and later births has been found using record linkage (Noack & Østby, 1985) and longitudinal surveys (Calhoun & De Beer, 1991; Quesnel-Vallée & Morgan, 2003). Though correspondence is greater at the aggregate level because of compensating discrepancies, expectations data are generally regarded as unreliable (Morgan, 2001; Van Hoorn & Keilman, 1997). Non-response contributes to the problem (Morgan, 1981, 1982; Van de Giessen, 1992; Van Hoorn & Keilman, 1997). Further, Lee (1980) demonstrated that minor fluctuations in desired completed family size will generate major fluctuations in period fertility, which determines annual births and consequent age structures (with effects on births a generation later).

Birth expectations data are often used qualitatively; this was the case for many years in US official forecasts (Long, 1981, 1984). Short-term forecasts of total US births were found to be less accurate than those derived by structural modelling (Ahlburg, 1982). Based on extensive US evidence, Hendershot and Placek (1981:312) concluded that whether birth expectations data were useful or not depended on the period considered and on informed judgment.

Given the discrepancies between expectations and realisations, the direct use of expectations data is ill-advised. Models (e.g., De Beer, 1991; Lee, 1980, 1981) are used to ‘correct’ the data for forecasting, but none is considered ideal and their use is limited (Van Hoorn & Keilman, 1997). Calhoun and De Beer (1991) developed a bivariate econometric model of children ever born and additional expected births allowing for the structural relationship between the two. Morgan and Chen (1992) dismissed birth expectations data as a basis for first birth forecasting.

4.3. Migration

International migration has been largely neglected in demographic modelling and forecasting. Wilson and Rees (2005) note the difficulties involved. A review of the practice in 30 industrialised countries (George & Perreault, 1992) found that most used simple assumptions (at best considered a naïve forecast) of net migration of zero or continuing at the current level with a fixed age pattern. Howe and Jackson (2005) found that little had changed, though there were some exceptions: the UK and Japan now use extrapolation with informed judgment, while the Netherlands combines time series and informed judgment in deriving assumptions for 12 migrant groups. Theories of international migration have not often been quantified in forecasting (Howe and Jackson, 2005); informed judgment has played a central role.

Increasing sophistication in the use of informed judgment reflects the growing importance of migration in determining population change. De Beer and Van Wissen (1999) grouped European countries according to five demographic patterns, and adopted two assumed net migration rate targets in 2050 characterised by ‘uniformity’ under a strong economy and ‘diversity’ under a flagging economy. Bijak, Kupiszewski, and Kicing (2004) also grouped European countries and used target net migration rates for intra- and extra-European migration with exponential interpolation of migrant numbers. A detailed study of European migration and its correlates recommended that immigration and emigration are best forecast separately using one generic framework, taking into account reasons for immigration and qualitative and quantitative information in country-specific approaches (Hilderink et al., 2002). The use of qualitative and quantitative information can be formalised by the expert-based approach of Lutz and colleagues (Lutz, 1996).

The reason for migration is taken into account by decomposition; levels and age–sex patterns are forecast for the different types of migrants, such as labour, dependents and asylum seekers (note that these are not population characteristics). Disaggregation of immigrants by citizenship is also important (Hilderink et al., 2002). Decomposition and disaggregation are increasingly employed as data become more readily available. This approach permits in-

formed judgment to focus on the circumstances of specific groups. In general, however, the unpredictable nature of economic and political developments, including migration policies, present considerable uncertainties for forecasting.

De Beer (1997) used time series methods to model aggregate immigration and emigration, both as first-order autoregressive processes. Additionally, net migration was modelled by a first-order moving average. The three models gave consistent results, but informed judgment considered the net forecast (which was only two-thirds of the average observed over the last decade) to be too low. Informed judgment was incorporated as the target in a five-parameter “extrapolation-target” model; three parameters govern the extent to which the forecast depends on past observations, the fourth is the target and the fifth is the speed at which the forecast approaches the target. The parameters are all interpretable and in principle can be derived from fitting the model to observations. Keilman and Pham (2004) also forecast immigration and emigration by time series methods; the ARIMA models were (1,0,1) for immigration and (0,1,0) for emigration. The forecast series were randomly broken down by sex; the age patterns were based on six-parameter multi-exponential models (see below).

Miller (2002) used time series methods in forecasting total net migration to California: for domestic migration, the net rate was modelled by an AR(1), and for international migration, the proportion of legal migrants intending to reside in California was also an AR(1); adjustments were made for illegal migration and the age distributions were assumed to be fixed. Miller and Lee (2004) used a similar approach. Wilson and Bell (2004) also applied time series methods in forecasting numbers of immigrants and emigrants for Australia.

The multi-exponential function originated in modelling the age pattern of migration (Rogers & Castro, 1981; Rogers, Raquillet, & Castro, 1978; Rogers & Watkins, 1987). The four-term model comprises a constant, exponential child migration, double-exponential labour migration, and retirement migration (zero, exponential or double exponential), giving 7–11 parameters in total. With the addition of a post-retirement term (Rogers, 1988), the full model has 13 parameters (Rogers & Little, 1994). Congdon (1993)

suggested a three-parameter bell-shaped curve for retirement migration. The model does not easily lend itself to forecasting (Congdon, 1993; Rogers, Castro, & Lea, 2005), but see also George (1994): fitting requires non-linear regression methods with good guesses for the initial values of parameters which are highly correlated and potentially highly variable over time. Rogers et al. (2005) present three alternative linear estimation procedures for “simplified” and “complete” models which produce greater temporal stability and some improvements in interpretability; these may present possibilities for forecasting.

Zaba (1985, 1987) extended relational modelling to migration by developing three-parameter models of immigration and emigration based on a simple linearizing transformation. Congdon (1993) applied this method to out-migration data; the fit was further improved by adding quadratic and cubic terms in the standard. Though some fluctuation occurred in the parameters, trends were apparent, and the potential for forecasting was noted.

5. Probabilistic population forecasting

Many of the forecasting methods discussed above have been used to obtain only deterministic point forecasts, particularly in the earlier period. More recently, the variability of forecasts has become of central concern (Lee, 1999; Lee & Tuljapurkar, 2000). The importance of taking component uncertainty into account through probabilistic population forecasting has increasingly been recognised. Exactly how to do this has been the subject of much recent debate, not least because estimating the uncertainty is itself uncertain (Keilman, 2001a; Lutz, Sanderson, & Scherbov, 1999). Several approaches have been proposed, the methodological details of which are still evolving. Previous reviews of this work include De Beer (2000), Lee (1999, 2000b) and Keilman (2005b); see also Lutz & Goldstein (2004a).

5.1. Stochastic population renewal

From a statistical viewpoint, the ideal approach to population forecasting is the fully probabilistic approach; its advantages overcome the limitations of traditional deterministic projections (Section 2).

Importantly, probabilistically consistent prediction intervals are produced: uncertainty is expressed as a probabilistic prediction interval for any desired function of the forecast age distribution, and any probability can be chosen. The use of probability in the definition of intervals provides a means of ensuring consistency among component intervals, and separates specification of the population prediction interval from component intervals. Further, the use of probability increases the utility of population forecasts in risk assessment (Tuljapurkar, 1992); the forecast may be accessed as a database (Alho & Spencer, 1991). A prediction interval is, however, conditional on the forecaster's particular choice of method and the assumptions or judgment used.

The fully probabilistic population forecast is generated through stochastic population renewal in one of two ways; both employ the cohort-component method. The analytical approach involves the specification of a stochastic Leslie matrix (with relevant covariances); this approach was pioneered by Sykes (1969) and further developed by others (Alho, 1990; Alho & Spencer, 1985, 1991, 1997; Cohen, 1986; Lee & Tuljapurkar, 1994). This approach permits the assessment of the relative contribution of errors in mortality, fertility and migration to population error (Alho, 1992a). Using stochastic renewal theory, Tuljapurkar (1992) showed that population renewal is not a closed recursion in the presence of autocorrelation and that the forecast population size (or part thereof) is (asymptotically) lognormal. A problem with the analytical approach is its complexity and the need for approximations and simplifying assumptions. The second approach avoids these problems by using Monte Carlo simulation in which each step of the forecast is generated by randomly drawn parameters for the distributions of mortality, fertility and migration. The population distribution is derived from the percentiles of the set of forecasts. This method is commonly used.

The cohort-component model does not itself give rise to forecast uncertainty as it is an accounting system. However, errors are propagated by the process of population renewal, producing complex combinations of errors in forecast population size and structure (Alho & Spencer, 1985). These errors may cancel each other out depending on the degree to which they are correlated. Thus in (fully) probabilistic population

forecasts, correlations among errors in component forecasts affect the degree of uncertainty of the population forecast.

5.2. *Correlations among forecast errors*

Four types of correlation among forecast errors may occur: temporal or serial (auto)correlation, correlation across age, correlation between demographic components, and correlation among populations by sex or within a coherent region.

First-order autocorrelation of errors is relatively high for annual demographic rates. However, it is the long-run mean (of fertility in particular) rather than annual values that determine population growth, and over the long term (a century or so), temporal correlation is weak (Lee, 1993). Such correlations must be taken into account.

Correlations in forecast errors across age are generally positive because forecasting methods often take advantage of the regularity of age patterns or assume a fixed age pattern. Further, the conditions resulting in high or low annual demographic rates often apply over the whole age range. Correlations across age in mortality contribute directly to larger errors in forecast populations. For fertility the effect is less important because it is the number of births that matters rather than the age of the mother. Alho (1998) found correlations of at least 0.89 between neighbouring ages for mortality and fertility rates. Lee and Tuljapurkar (2000) and others have concluded that perfect correlation can be assumed.

Correlations between demographic components (and the jump-off population) are only likely to be important for developing countries where development factors may underlie changes in all three components (e.g., Lutz, Sanderson, & Scherbov, 1996). For industrialised countries such correlations are negligible (Alho, 1992a; Keilman, 1997; Lee & Tuljapurkar, 1994). Correlations among populations within regions cannot be ignored: for example, correlations related to common patterns of change in mortality or fertility must be taken into account in sub-Saharan and European forecasts (Lee & Tuljapurkar, 2000; Lutz, Sanderson, et al., 1996). Positive correlations between the sexes in mortality and migration also arise; the common assumption of perfect correlation results in some overestimation of

population forecast uncertainty (Keilman, Pham, & Hetland, 2002).

5.3. Sources of error

In forecasting demographic rates, Alho (1990) classified error into four different, but conceptually dependent, sources: model misspecification, errors in parameter estimation, random variation and errors in informed judgment (see also Keilman, 1990). Model misspecification may occur with respect to the assumed underlying model (e.g., a parameterisation function) or the time series model or both. Informed judgment in statistical modelling refers to ‘prior’ beliefs about model parameters and the weight given to them in forecasting; these weights can be estimated by mixed estimation (Alho, 1992b). In addition, the basic data are subject to random and non-random measurement error (Stoto, 1988); see also Lee (1999). Alho (2005) provides a useful discussion of the roles of probabilities and judgment. For population forecasts, an additional source is measurement error in the jump-off population, though this is usually negligible for industrialised countries.

5.4. Model-based *ex ante* error estimation

Stochastic population renewal makes considerable demands in terms of variance–covariance error structures. One way to estimate this uncertainty is to use *ex ante* estimates produced by extrapolative methods. However, estimating uncertainty in this way is highly uncertain because the estimate depends on the particular forecasting method used (Cohen, 1986; Lee, 1974; Sanderson, 1995). Standard methods may lead to implausibly wide intervals, and judgment is required in limiting the series or imposing constraints (Keilman, 2002; Keilman & Pham, 2000; Lee, 1993; Tuljapurkar & Boe, 1999). Time series methods do not take into account this and several other sources of uncertainty (Lee, 1999), nor were they developed for long-term forecasting, for which both point forecasts and prediction intervals may be unrealistic (Sanderson, 1995). They can, however, provide a standard against which other estimates are compared (De Beer, 1988).

An alternative is to model forecast errors per se (Alho & Spencer, 1985, 1990a,b, 2005; Cohen, 1986).

Alho and Spencer (1985) constructed prediction intervals for short-term horizons from a parametric model of the main sources of error: model misspecification, parameter estimation and informed judgment. Estimates of uncertainty based on naïve forecasts provide a conservative baseline for the assessment of forecast errors. Alho (1998, 2003) related errors produced by historical naïve forecasts for Finland to forecast length, and used this to inform prediction intervals for new Finnish forecasts. Alho (1997) borrowed information on the variance and autocorrelation of errors from one country for forecasting fertility for another, and also modelled inter-country correlations. Alho and Spencer (1997) describe how to implement this approach in population forecasting using the analytical approach.

5.5. Expert-based *ex ante* error estimation

In many countries, data availability limits the estimation of variance–covariance structures for fully probabilistic population forecasting. The expert-based approach circumvents this problem. Component target forecasts (see Section 3.2) are supplemented by expert specifications of high and low values, which on average are assumed to correspond to 90% prediction intervals of a normal distribution (Lutz, Sanderson, et al., 1996; Lutz, Sanderson, & Scherbov, 1997; Lutz & Scherbov, 1998). The method is robust to changes in the probability and the distributional form (Lutz, Sanderson, & Scherbov, 1999; Lutz & Scherbov, 1998), but is not suited to short-term forecasting where variation may be greater than can be captured by the combined assumptions of linearity and normality (Lutz, Sanderson, et al., 1996); Alders and De Beer (cited in De Beer & Alders, 1999:4) show that this problem is not negligible in the long term.

In population forecasts, the path of each component is a random line (or piecewise linear path) determined by a single random draw from the above-defined normal distribution. Thus, for example, total fertility is positioned at the same percentile over the entire period, which may imply confined changes in trend but does not allow, for example, the meanderings of a random walk. These random lines are randomly combined into scenarios: each random scenario is the result of the three random numbers determining the trajectories of mortality, fertility and

migration. The distributions of population size, structure and other output variables are obtained from the set of repeated scenarios.

The differences between this random scenario approach and the fully stochastic approach are discussed in detail by Tuljapurkar, Lee, and Li (2004); see also Tuljapurkar (1997). The criticism that the perfect autocorrelations in the random lines do not permit a truly random sample path (e.g., Lee, 1999) contributed to the relaxation of the assumption of perfect autocorrelation by the addition of a random term (Lutz, Sanderson, & Scherbov, 1999, 2001). Further, in response to criticism that expert expectation is overconfident and produces overly narrow prediction intervals, which occurs particularly when the recent trend is unchanging (Keilman, 1990), the methodology was adapted to place greater emphasis on expert argument rather than opinion (Lutz, Goujon et al., 1999; Lutz, Sanderson, & Scherbov, 2004). Doubt is also cast on the ability of experts to specify values representing probability bounds, and on whether they consider the bounds of annual point forecasts or temporal averages (Lee, 1999). Alho (2005) also raises the problem of ‘expert flocking’ resulting from the common information base and a desire to conform. As already noted, Lutz, Goldstein et al. (1996) argue that the expert-based approach is the only meaningful way to capture future uncertainty because structural change and unexpected events are likely to occur. Others argue that such occurrences are reflected in the error variances of time series models (Lee, 1999).

Alders and De Beer (2005) discuss the use of expert knowledge in the specification of uncertainty in forecasting Dutch mortality. As different estimation methods produced substantially different prediction intervals, judgment was used based on four considerations: smoking trends by sex, possibilities for mortality decline at young ages, ongoing rectangularisation of the survival curve, and the role of medical advances. These and other considerations are built into the deterministic forecasting model (Van Hoorn & De Beer, 2001, cited in Alders & De Beer, 2005:50). The approach is described as ‘argument-based forecasting’ (Alders & De Beer, 2005:57). Expert opinions have also been used in Finland to derive prediction intervals for migration (Alho, 1998, 2005).

5.6. *Ex post error estimation*

Errors in past forecasts can be used to estimate the variability in future forecasts, on the assumption that the future will not differ from the past and that errors can be modelled. Uncertainty over the forecast period is derived from the overall estimate of the variance according to the estimated (or assumed) degree of autocorrelation. A form of targeting may be used, where the variance is specified for the last year of the forecast and interpolation is used for earlier years, for example by specifying the 95% prediction interval and assuming that variance increases with time as in a random walk. Different patterns of evolution may be required for different components, and for cohort and period fertility (De Beer & Alders, 1999). Extrapolative forecasts of ex post errors may be made (e.g., De Beer, 1997).

Prediction intervals based on ex post errors are attached to a separately-derived (deterministic) forecast, with which they should be consistent (Van Hoorn & De Beer, 2001). The method provides uncertainty estimates for fully probabilistic population forecasting (assuming perfect correlation across age and zero correlation between components). It is advantageous in situations where a judgemental point forecast is preferred, for example when recent structural change limits extrapolative methods. However, ex post errors for periods of structural change may not be relevant for future forecasts (De Beer & Alders, 1999).

Ex post errors were first applied to official population forecasts (rather than component forecasts), where they are essentially a measure of the accuracy of the traditional cohort-component medium scenario. Keyfitz (1981) and Stoto (1983) used variability in ex post errors in the population growth rates of UN projections to derive probability distributions for population size. Issues about the correlation of errors do not arise, and uncertainty in the age–sex structure is not addressed, though the method could be applied independently to any measure. These and other studies of errors in UN population projections (Bongaarts & Bulatao, 2000; Keilman, 1999, 2001b; Pflaumer, 1988) found evidence that errors in forecast growth rates were greater for high growth regions and earlier base (jump-off) years, and some evidence that accuracy improved over time; improved data accuracy and more stable demographic conditions contributed

to these findings. Errors in the estimated base population were not insignificant. Further, forecast accuracy was generally found to decline with increasing forecast horizon and to be better for large populations and worse at young and old ages. Based on such findings and cross-country correlations, the study reported by [Bongaarts and Bulatao \(2000\)](#) developed a statistical model for the uncertainty in national and global population forecasts.

Detailed studies of national population forecasts for developed countries ([De Beer, 1997](#); [Keilman, 1990, 1997](#)) examined errors in mortality, fertility and migration rates and events, and used multivariate methods (an APC model) to separate out the effects of the length of forecast horizon, the difficulty of the particular period and the base year. Other relevant considerations include the interaction between difficulty and forecast method; the role of luck and the reasoning behind the choice of method; the causes of forecast error (for example, model misspecification or an actual change in trend); and bias towards older forecasts because recent forecasts can only be analysed in the short term ([De Beer, 2000](#)). [Keilman \(1990, 1997\)](#) found no clear evidence of improvement over time in forecasting birth and death rates.

5.7. *Complementarity of approaches*

The three approaches to the estimation of forecast uncertainty were at first regarded as more or less mutually exclusive, but many applications have made use of and demonstrated their complementarity (e.g., [Alders & De Beer, 2004, 2005](#); [Alho, 1997](#); [De Beer & Alders, 1999](#); [Keilman et al., 2002](#); [Lee, 1993](#); [Lutz, Sanderson, & Scherbov, 2001](#); [Van Hoorn & De Beer, 2001](#)). [Keilman and Pham \(2000\)](#) derived prediction intervals for age-specific fertility in the short to medium term which take into account uncertainty arising from sampling variability, parameter estimation in both the gamma functional representation of the age pattern and the time series forecasting model, and residual variance in the time series model. [Keilman and Pham \(2004\)](#) used ARCH time series models (assuming the variability of the past 100 years) to derive prediction intervals for total fertility in Nordic countries and compared them with intervals based on ex post errors and the naïve (constant) forecast: the three estimates were similar,

indicating that time series estimates are not excessively wide by this measure.

In comparing estimates from different approaches, it is important to bear in mind the distinction between time series intervals for annual point forecasts and intervals for temporal means. Assumptions about future demographic rates refer more to temporal means than to annual meanderings, so that ex post errors and expert-based errors are comparable with ex ante errors for temporal means ([Lee, 1993](#); [Lee & Tuljapurkar, 2000](#)).

6. Direct population forecasting

Rather than forecast population size by the cohort-component method, simple or direct methods may be used. [Ahlburg \(1987b\)](#) used an autoregressive model. [Leach \(1981\)](#) re-examined the use of the logistic function, concluding that it provides an acceptable model of long-term change that can be more accurate than the cohort-component method; he attributed this to an ability to model the ‘mechanism for self-correction’ (p.102). Further, Leach argued that component forecasts (based on long-term trends) should be constrained to match population change forecasts by the logistic model. Similar views were expressed by [Pflaumer \(1992\)](#) who also advocated the use of time series methods to provide a baseline against which more complex methods could be gauged. [McNown et al. \(1995\)](#) applied this test to point and interval cohort-component forecasts based on components forecast using the multi-exponential model.

Many studies have examined the relative performance of simple and cohort-component models of population forecasting. Most have found constant growth models or standard time series models of population size or growth to be at least as accurate (e.g., [Cohen, 1986](#); [Long, 1995](#); [Murphy, 1984](#); [Pflaumer, 1992](#); [Smith, 1997](#); [Smith & Sincich, 1992](#); [Stoto, 1983](#)). [Keyfitz \(1981\)](#) found complexity to be an advantage. [Bongaarts and Bulatao \(1999\)](#) found that most of the variation in World Bank traditional cohort-component population projections can be accounted for by simple models. [Lee, Carter, and Tuljapurkar \(1995\)](#) discuss the relative merits of cohort-component and direct forecasting, including the possibility of disaggregating by age and sex after

forecasting the total. Rogers (1995c) reviewed the evidence over a broader range of models and provided a typology of model complexity: a main conclusion, already noted, is that much depends on the particular trends in the historical period that forms the basis of the forecast.

Smith (1987) compared three direct forecasting methods for population size, finding the base period to be influential and larger forecast errors for longer forecast horizons, high base period growth rates and smaller populations. The relationship between forecast accuracy and forecast horizon was approximately linear, but no consistent relationship was found for bias (Smith & Sincich, 1991). The length of the base period has also been found to have an (inverse) effect on forecast errors (Smith & Sincich, 1990). A non-linear inverse relationship between population size and forecast accuracy was also found among small populations (Tayman, Schafer, & Carter, 1998).

For very long-term deterministic forecasting, ergodicity allows the initial age–sex structure to be ignored, except for its effect on population momentum (see Tuljapurkar and Lee (1997) for the stochastic case). Goldstein and Stecklov (2002) proposed an analytical model that adopts a formula for population momentum under changing fertility (Li & Tuljapurkar, 1999, 2000) with extensions to allow for non-replacement ultimate total fertility and changing mortality. The method requires only aggregate measures: birth and growth rates and life expectancy.

7. Progress and progression

It has been seen that a considerable volume of research has been undertaken in demographic forecasting over the last twenty-five years. New models and methods have been developed and probabilistic forecasting has been introduced. Much of this research is recent: methods are still in development and many have yet to be fully evaluated. Further developments can be expected in the coming years.

7.1. Overview of methodological developments

Several broad conclusions about methodological developments in demographic forecasting can be drawn from this review. First, zero-factor models of

aggregate measures are inadequate, not least because the assumption of a fixed age distribution is unrealistic. The independent forecasting of age-specific rates may produce irregular and implausible age patterns. Second, one-factor models have not generally been successful in forecasting because of parameter uninterpretability and over-parameterisation; forecasts based on these methods may produce implausible trends and age patterns, especially where parameter time series models are univariate. The shifting logistic model of senescent mortality (Bongaarts, 2005) has the advantage of independent parameters; this method has yet to be evaluated.

Third, two-factor GLM methods, where time is a covariate, often produce implausible forecast trends. Fourth, the Lee–Carter method involving a single time-varying parameter appears to produce plausible trends and permits a changing age pattern, though the fixed pattern of change may produce implausible age patterns in the longer term (Giroi & King, 2006:45). Fifth, individual expectations are not a good basis for demographic forecasting; the validity of expert expectations is also open to question and has yet to be fully evaluated. Last, structural models have not proved useful for forecasting except perhaps for epidemiological models used in short-term forecasting of mortality from certain causes.

It would appear then that little progress has been made along the developmental path of forecasting referred to by Willekens (1990). Many of the models and methods developed in recent times focus on the extrapolation of stable patterns and the associated estimation of uncertainty in the transitional first stages. Others focus on the use of expert opinion, qualitatively but not quantitatively taking demographic knowledge into account. Forecasting with explanatory structural models has not been significantly advanced. Neither have decomposition and disaggregation been used to advantage. While multistate forecasting has been employed for spatial disaggregation, it has not been widely adopted for other dimensions. Microsimulation is limited (computationally) to manageable sample populations so as to accommodate more detailed disaggregation. Further, many microsimulation models are based on constant transition probabilities (or rates), so that the forecast is determined only by the initial distribution of the sample population.

However, other progress has been made: forecasting knowledge has been substantially increased through the high level of research activity and the ensuing debates. In addition, improved access to high-quality data through the development of databases (e.g., the Human Mortality Database, <<http://www.mortality.org/>>), increased computer power, the wider (though still not sufficient) availability of specialised software (e.g., <<http://www.robhyndman.info/Rlibrary/demography/>>), and the (partial), adoption of R as the common programming language are major contributions to the development of the research environment. If little progress has so far been made along the developmental path, demographers have at least acquired many of the skills and equipment for their future progression.

7.2. Progress in forecasting performance

The performance of demographic forecasts is often assessed against a range of criteria taking both quantifiable and qualitative factors into account (Ahlburg, 1995; Long, 1993, 1995; McNow, Rogers, & Little, 1995; Rogers, 1995c; Swanson & Tayman, 1995).

For any forecast worthy of the name, accuracy is a main goal. However, as Keyfitz (1981:580) observed, accuracy was of little ex post concern to population forecasters. In fact, little is known about what makes one demographic forecasting method more accurate than another, or indeed which can reliably be so distinguished. The fact that there is no evidence that overall accuracy in mortality and fertility forecasting has improved over time (e.g., Keilman, 1997) suggests that methods per se have not become more accurate. Indeed, an oft-found conclusion is that forecast accuracy is situation-specific. Little progress has been made in advancing knowledge about which methods can be relied upon when conditions are unstable.

A difficulty for the assessment of accuracy in demographic forecasting is the length of the time horizon: long-term methods, such as the Lee–Carter method and the expert-based approach cannot be readily evaluated. This is particularly the case for the expert-based approach, because out-of-sample evaluation based on historical data cannot be used (except perhaps in well-controlled blind experiments). Nev-

ertheless, evaluations are required based on both historical data and the (short) time periods that have elapsed. More comparative research focussed on methods would assist in this endeavour (e.g., Booth et al., 2005); the multivariate analysis of ex post errors (Keilman, 1997) is a useful approach.

A second criterion is accuracy in the estimation of uncertainty. Advances have been made in identifying and estimating the different sources of error in forecasts of demographic processes. However, the estimation of uncertainty remains uncertain: different methods produce different estimates, only partly because they take into account different subsets of error sources. In population forecasting, the replacement of the high–low scenarios of traditional population projections by a probabilistic prediction interval is a major advance; understanding and quantification of the propagation of error in population forecasts is an important contribution. The official agencies of some of the most statistically advanced countries are now introducing probabilistic forecasting; the Netherlands is a pioneer in this respect, and the US has recently chosen to employ formal time series modelling of the demographic components with stochastic population renewal (Long & Hollmann, 2004). Similar developments can be expected elsewhere.

A third criterion is consistency. Multistate methods produce consistency between national and subnational population forecasts. A major advantage of fully probabilistic population forecasts is the consistency of prediction intervals for different indices. On this criterion, significant progress has been made. Other criteria for the evaluation of forecasting performance broaden the concept considerably; parsimony, face-validity and legitimacy are largely specific to the particular method and data used. Utility has been increased by virtue of the probabilistic prediction interval and the consistency that this affords. Transparency, which is essential if new methods are to be adopted and valid comparisons are to be made, has been increased in that methods and their assumptions are now documented and extensively discussed.

A final criterion is user acceptability. Probabilistic population forecasts are often viewed as too complex to be understood by users who are accustomed to using the medium scenario of traditional population projections and appear to have no use for the high and low scenarios (Lutz & Goldstein, 2004b). This view

neglects the possibility that users realize the limitations of traditional population projections and simply make the best use of a poor product. Well-explained forecasts with probabilistic prediction intervals might indeed be welcomed by users: at last they would have a user-defined probability range of population futures that are consistent, rational and valid. User acceptability requires independent evaluation, distinguishing it from the “reluctance of forecasters to make the leap from using scenarios to fully stochastic forecasts” (Goldstein, 2004:94).

By most criteria, therefore, forecasting performance has been improved over the period under review. Only accuracy remains essentially unchanged, or in the longer-term unproven. Thus, the improvement of forecast accuracy and the concomitant reduction of uncertainty remains an important focus for research. Greater certainty in the estimation of uncertainty is also an ongoing aim.

7.3. *Competition, combination and complementarity*

Improving the accuracy of demographic forecasting is a major challenge. Two ideas borrowed from forecasting in other disciplines are competition and combining. Competition in demographic forecasting has been both advocated and criticised. Land (1986), Ahlburg and Vaupel (1990), Ahlburg (1995) and Rogers (1995c) advocated livelier competition or formal tournaments to compare forecasting accuracy. The idea was short-lived, however, perhaps because of the focus on uncertainty and the lack of both a passionate advocate and an appropriate institutional backing. Ahlburg and Lutz (1999:5) found the emergence of informal competition characterised by “a struggle for the survival of the fittest model, where fitness is judged primarily by forecast accuracy. . . or by the strength of the institution behind the forecast to be counterproductive.” Ahlburg et al. (1999:192) regard any exercise aiming “to find the best method” as “futile.”

Combining has not fared any better. The combining of forecasts from different models involves averaging or some other procedure as a simple and pragmatic way to possibly produce better forecasts (Granger, 1989, 2001), and has been repeatedly advocated as a means of increasing the accuracy of demographic forecasts (Ahlburg, 1995; Ahlburg & Land, 1992b; Ahlburg & Lutz, 1999; Willekens, 1992). However, the idea has

not been embraced. In rare exceptions, De Beer (1988) found combining to be advantageous only in the very short term, while Sanderson (1999) found it beneficial for forecasting the rate of natural increase. It is noted that probabilistic consistency would be forfeited if cohort-component-based population forecasts were combined.

While combining per se has not found favour, complementarity is increasingly recognised (e.g., Tuljapurkar et al., 2004:198). In forecasting demographic processes, the complementary combination of useful aspects of approaches would appear to be more fruitful than trying to find a single best method or combining forecasts from competing approaches. As has already been noted (Section 5.7), recent research has emphasised the complementarity of approaches at least as far as the estimation of uncertainty is concerned (e.g., Alders & De Beer, 2004). This pragmatic complementary approach avoids the complications involved in combining the prediction intervals of competing forecasts (Granger, 2001); it also facilitates the more complete estimation of uncertainty than a single approach might permit.

Complementarity has also been adopted in population forecasting; several methods combine probabilistic forecasts with the scenario approach. In the modified scenario approach (Goldstein, 2004), traditional population projections are calibrated to stochastic forecasts by adopting the prediction limits of life expectancy and average fertility forecasts as high and low assumptions, achieving similarity of population uncertainty estimates (adjustment is needed in the case of ratios). Conditional probabilistic population forecasting takes various forms. One such method involves classifying simulated population trajectories according to their randomly-generated fertility and mortality levels, permitting what-if questions to be examined (Sanderson, Scherbov, O’Neill, & Lutz, 2004); applications include Lutz and Scherbov (2002) and O’Neill (2004). Another method involves classifying forecasts according to the level of the forecast after an initial period, thus addressing policy questions about whether to delay action (Sanderson et al., 2004).

7.4. *Forecasting demographic change*

In order to improve forecast accuracy, methods are needed that have the capacity to capture structural

change. Both the period-specificity of forecast accuracy and the lack of improvement stem from the inadequacy of existing methods to forecast change. Extrapolative time series methods, such as the Lee–Carter method, do not purport to forecast structural change but produce wide prediction intervals that encompass variability from past change.

Neither does the expert-based approach claim to forecast structural change; Lutz, Goldstein, et al. (1996) merely state a belief that structural change is likely to happen (p. 38) and that in determining the predictive probability distribution, experts can ‘consider possible alternative stories that might happen in the future’ (p. 37). In practice, in this approach change is expressed only as an aggregate target in a specific year; the timing of change is not predicted, nor is the age pattern permitted to change. If this approach is to better predict change, it must be further developed to incorporate the timing of changes in trends and to formalise the decision-making processes involved in making use of demographic knowledge, perhaps through some form of model incorporating expert-based quantified effects. The wide availability of standardised survey data (e.g., Demographic and Health Surveys) facilitates such modelling.

There is no doubt that our capacity to forecast demographic change will gain from a greater understanding of the causal factors and processes that determine the level, sequence and timing of demographic events, as Willekens (1990) affirmed. Significant progress has in fact been made over the last twenty-five years in increasing such understanding. However, this has not resulted in theory-informed forecasting; current demographic theory is too general to be of such use. Rather, it would appear that demographic behaviour, like most human behaviour, is too complex to be easily modelled and forecast; indeed, it may be inherently unpredictable. Only mortality, which is largely biological, lends itself at all to an explanatory structural approach.

The complexity of demographic behaviour would suggest that disaggregation and decomposition is an appropriate and potentially useful approach, at least in the short- to medium-term. The increased understanding of demographic life-course processes and their determinants, combined with increased computer power, facilitates added dimensions of disaggregation and decomposition and the development of both

increasingly detailed multistate (macrosimulation) models and individual-level microsimulation.

The new European MicMac project combines the macrosimulation and microsimulation approaches to produce forecasts of both the population in multidimensional states (defined by age, sex and various other statuses such as participation in the labour force) and the length of time spent in those states (Van der Gaag, De Beer, & Willekens, 2005; Willekens, 2005; Willekens, De Beer, & Van der Gaag, 2005). The macrosimulation involves the construction of cohort biographies at the population level, while the microsimulation produces synthetic individual biographies which are aggregated to determine population characteristics. The bridge between the cohort and individual biographies is the common use of transition rates estimated conditionally on covariates (occurrence–exposure rates); thus the dynamics at the macro- and micro-levels are internally consistent. Informed by an increased understanding of demographic processes, transition rates are modelled and forecast either by regressions (structural models) or by recourse to (further) disaggregation. Thus, structural modelling is being incorporated into demographic forecasting. The MicMac approach is complementary and pragmatic: it makes use of causality where possible, and otherwise of statistical association. Further complementarity is likely to be found in the estimation of uncertainty and in the methodological details.

It remains to be seen to what extent MicMac will improve forecast accuracy by capturing structural change. This depends in part on the validity of the causal models. How can the theory embedded in these models be tested and further theory developed? One approach is to make use of agent-based or behavioural modelling; this differs from empirically-based microsimulation in that the computational model is based on pre-defined behavioural rules. Agent-based computational models simulate feedbacks and behavioural hypotheses, as can microsimulation, but the purpose is to verify whether the rules of individual behaviour can produce macro-level regularities and variability (e.g. Murphy, 2003); thus theory is tested and developed. Though agent-based computational methods have only recently been explored in demography (Billari & Prskawetz, 2003), they have the potential to assist in the development of demographic behavioural theory

and thus to contribute to progress along the path towards forecasting demographic change.

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