An IPAT-type Model of Environmental Impact Based on Stochastic Differential Equations

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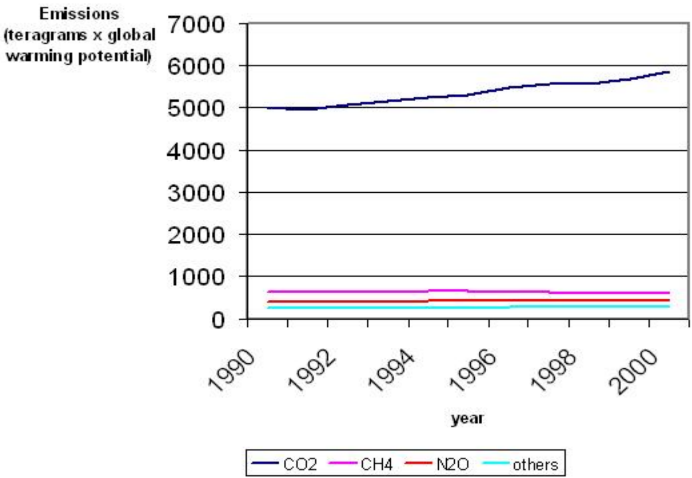
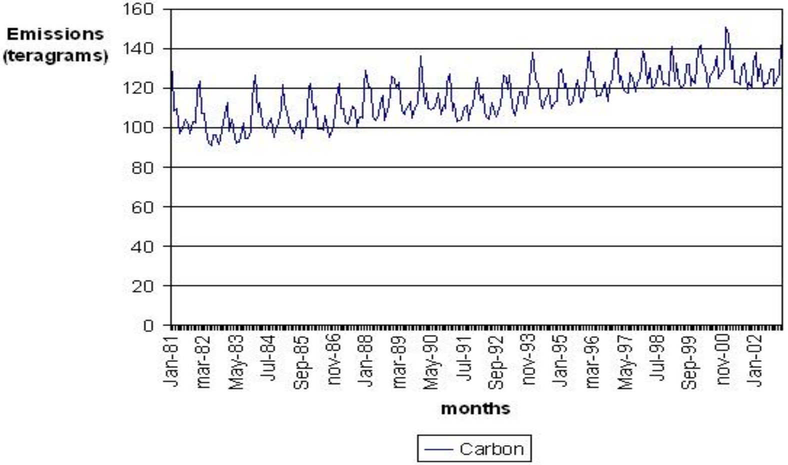
Outline

Recent Trends in the U.S.



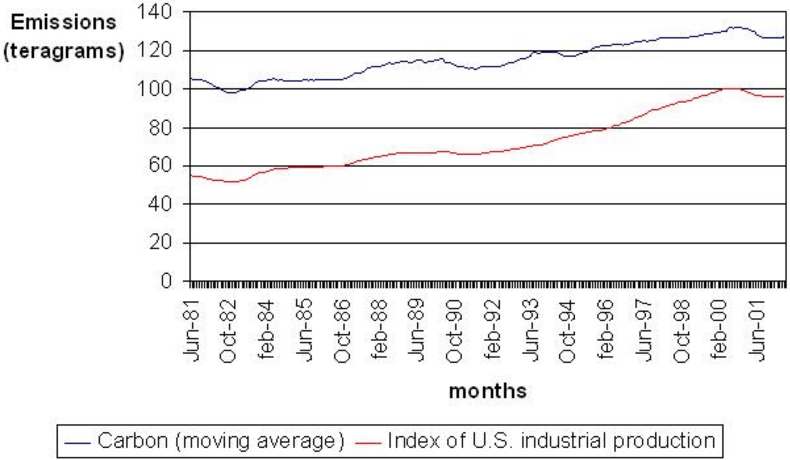
Data Source: UNFCCC, 2004

Seasonal pattern of U.S. carbon emissions



Data Source: Energy Information Administration, 2004

Recent trends in the U.S. industrial production



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Federal Reserve, 2004

Emissions of greenhouse gases in the U.S.

The IPAT model

An alternative representation of the IPAT model

An application in the context of decision theory

Possible developments and conclusions

The IPAT model (Ehrlich and Holdren, 1971)

- $$Env.\mathbf{I}mpact = \mathbf{P}opulation * \mathbf{A}ffluence * \mathbf{T}echnology$$

- $$I = P * \frac{Y}{P} * \frac{I}{Y}$$

  where
  I = Environmental Impact (CO2 emission);
  P = Population;
  Y = Income (GDP).

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the equation can be simplified to:

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  $\epsilon \sim N(0,1)$
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Demographic and Economic assumptions

- Population grows exponentially on average:

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- Income grows exponentially on average:

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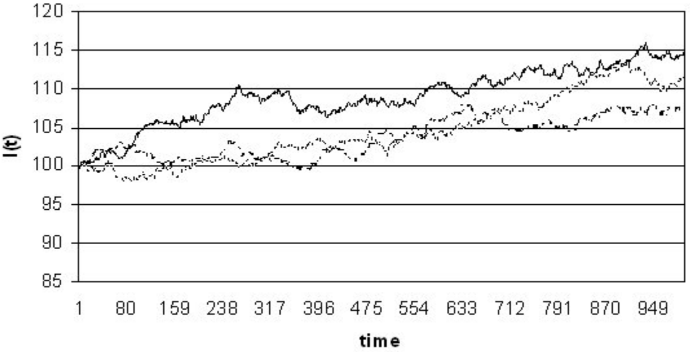
The model

$$\begin{cases} dI = (ac + be)Idt + \sigma_1 IdB_1(t) + a\sigma_2 IdB_2(t) + b\sigma_3 IdB_3(t) \\ dP = cPdt + \sigma_2 PdB_2(t) \\ dY = eYdt + \sigma_3 YdB_3(t) \end{cases}$$

- Three simulated trajectories:



An evaluation of the costs related to a reduction of the environmental impact

- International agreements on environment require countries to reduce their environmental impact, especially when the consequences of environmental impact are not confined within the country itself
    - Kyoto protocol (1997) requires developed countries to accept a legally binding commitment to reduce their emissions of greenhouse gases by at least 5%, compared to 1990 levels, by the period 2008-2012
- The representation of environmental impact that we derived before is suitable for the development of an analysis based on the conceptual scheme suggested by Black and Scholes (1973) in the context of the determination of the fair price of an option

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Assumptions

- The country is engaged in the reduction of a certain amount of its emissions, within a deadline set by an international agreement
- The commitment to reduce emissions has a cost that is directly related to the amount of national emissions (I). This cost represents the overall amount of money that has to be invested in order to either reduce national emissions or to buy permits to emit from other countries.
- The country receives financial aid from an international organisation that is proportional to its emissions, say $\alpha I(t)$, where α is the marginal cost of emissions for the country
- If the country did not meet its commitments, it would have to pay a 'fine' proportional to the difference between its actual emissions, at the deadline, and the target set by the international agreement (the fine may be seen in terms of emission trading costs)

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National spending on environment

- National Spending = Costs - Financial aid

$$S(t) = C(t, I(t)) - \alpha I(t)$$

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- The assumptions we made, and the differentiation of the national spending on environment $S(t)$ with respect to time, lead to the following equation:

$$C'_t + eC'_I I + \frac{1}{2}({\sigma_1}^2 I^2 + a^2 {\sigma_2}^2 I^2 + b^2 {\sigma_3}^2 I^2) C''_{II} - eC = 0$$

with the condition that the cost C at time T, the deadline for the reduction of emissions to a level $\bar{I}$, is expressed as:

$$C(T, I(T)) = max[\gamma(I(T) - \bar{I}), 0]$$

An example

- Consider a country whose population grows at a rate of 1% per year, whose income grows at a rate of 3% per year and whose starting level of emissions is 5000 teragrams
- Suppose also that:
  - $\mu_{I,P} = 0.7$
  - $\mu_{I,A} = 1.0$
  - $\mu_{I,T} = 0.8$
- Imagine that the country commits to reduce its emissions by 5% within 5 years
- How expensive would the commitment be if the country had to pay 20 $ per ton of emissions over the target set by the agreement? $\Rightarrow$ 12 billions $

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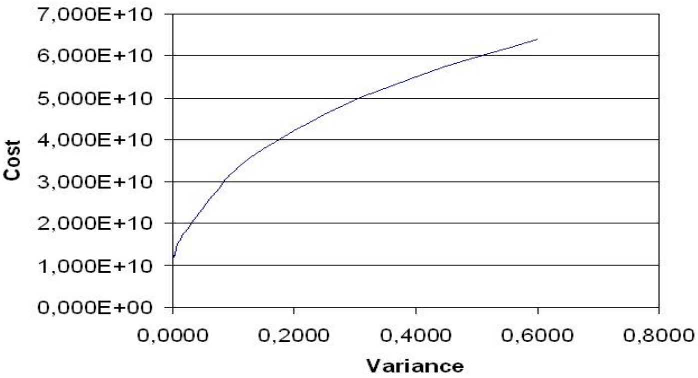
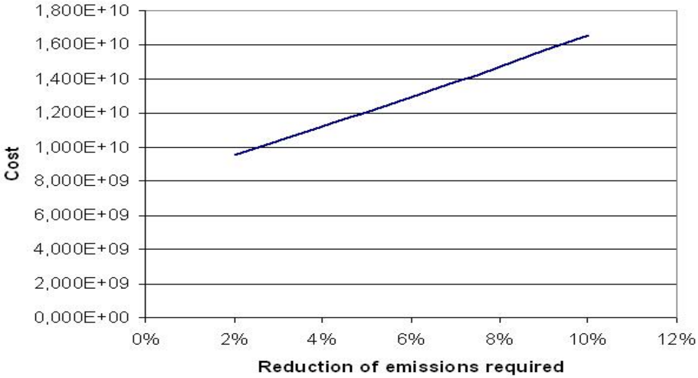
- Our uncertainty about the evolution of emissions over time, given our demographic and economic assumptions



- The amount of emissions that the country is supposed to reduce



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  - A structural model, instead of a single equation, may capture the interrelationships between explanatory variables
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Conclusions

- The model represents a straightforward stochastic representation of the IPAT equation alternative to the STIRPAT model
- The model allows for a meaningful representation of the uncertainty related to carbon dioxide emission: for instance, it is possible to compute the probability that the level of emissions will reach an upper bound within a certain date.
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