Odds Ratios Considered Harmful

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Romeo walks down the street 100 times and falls in love twice. Juliette walks down the same street the same number of times and falls in love once.

	Romeo	*Juliette*
Falls in love	2	1
Does not fall	98	99

What is the odds ratio? It's 2.0, right, since Romeo has twice the chance of falling in love? No. Because odds are not probabilities, it's not 2.0. Romeo's probability is 0.02 so his odds are $0.02/(1-0.02) = 0.0204$. Similarly, Juliette's odds are $0.01/0.99 = 0.0101$. The odds ratio is $0.0204/0.0101 = 2.02 \neq 2.00$. Odds are not probabilities. But the differences are modest when chances are small. Remember, we could have also shortcut to this result by the cross-product formula: $(2 \times 99) \div (1 \times 98)$.

If both parties become more romantic, how do matters change?

	Romeo	*Juliette*
Falls in love	8	4
Does not fall	92	96

Romeo's probability of falling in love is now double that of Juliette, 8%. But Juliette's probability has also doubled, and is now 4%. Since both have

doubled, the odds ratio should remain at 2.02, right? Again, no. The odds ratio is now $(8 \times 96) \div (4 \times 92) = 2.09$.

Suppose things become considerably more amorous for both, and we have the following table:

	Romeo	*Juliette*
Falls in love	80	40
Does not fall	20	60

Romeo's probability of falling in love is still double that of Juliette. Odds ratio is $(80 \times 60) \div (40 \times 20) = 6.00$. The notion that the odds ratio is approximately the probability ratio does not apply for common outcomes.

What if Juliette's probability remains the same and Romeo's is changing all the time?

	Before		After	
	Romeo	*Juliette*	*Romeo*	*Juliette*
Falls in love	8	4	16	4
Does not fall	92	96	84	96

Here Romeo doubles his probability between the two periods while Juliette's probability does not change. The odds ratios are 2.09 and 4.57; the latter is more than double the former.

Odds ratios are not probability ratios. Odds ratios are odds ratios.