

Why is $\exp(\beta)$ the odds ratio?

Prof. Andrew Noymer

Sociology 221
University of California, Irvine

Consider a logistic regression model like:

$$\log\left(\frac{p}{1-p}\right) = \alpha + \beta x$$

This is logistic regression because *logistic regression* is *log odds regression*, as was covered in lecture on more than one occasion.

Exponentiate both sides:

$$\frac{p}{1-p} = \exp(\alpha + \beta x)$$

Consider the odds when $x = 1$ and when $x = 0$

When $x = 1$:

$$\frac{p}{1-p} = \exp(\alpha + \beta)$$

When $x = 0$, call the probability p^* to distinguish it from p (this is just a matter of notation — when $x = 1$ the probability is p and when $x = 0$ the probability is p^*).

So, when $x = 0$:

$$\frac{p^*}{1 - p^*} = \exp(\alpha)$$

The odds ratio is:

$$\frac{\frac{p}{1-p}}{\frac{p^*}{1-p^*}} = \frac{\exp(\alpha + \beta)}{\exp(\alpha)} = \frac{\exp(\alpha) \cdot \exp(\beta)}{\exp(\alpha)} = \exp(\beta)$$

If the last step above does not make any sense to you, it may help to refer to the “cheat sheet” on logs and exponentials that was handed out earlier.

So the odds ratio is $\exp(\beta)$. When x is not dichotomous, the mathematics is completely analogous — it’s the effect on the odds of a unit increase in x . When there are other x variables x_1, x_2, \dots, x_n , they all cancel out except the one of interest.