Abstract

This paper investigates whether health, a nontradeable risky asset, can explain reductions in financial risk-taking after retirement. A theoretical model is proposed in which individuals who care about consumption and health are initially endowed with health but risk becoming unhealthy. If adverse health shocks increase the marginal utility of consumption, health risk prompts individuals to lower their risky portfolio shares because they become more risk averse. Patterns in aggregate and individual-level data suggest that health risk may explain 60 percent or more of the decline in financial risk-taking after retirement. (JEL: G11, I12, J14; Keywords: portfolio choice, health, insurance, background risk)
1 Introduction

Classic portfolio choice theory as stated by Merton (1969, 1971) and Samuelson (1969) recommends that as long as stock returns display no mean-reversion, investors should place a constant share of their wealth in risky assets regardless of their ages or time horizons. This contrasts both with traditional personal investment advice, which often proposes that risky portfolio shares should be 100 minus the investor’s age, and with empirical evidence on the actual portfolio behavior of individuals, which generally exhibits declining risk-taking through age (Ameriks and Zeldes, 2001; Guiso, Haliassos and Jappelli, 2002). Why traditional advice and empirical patterns deviate so much from basic theory is the subject of this paper.

In the years since Merton and Samuelson articulated the basic theory, numerous researchers have revisited the theoretical underpinnings of portfolio choice. A large body of literature explores the role of idiosyncratic background risk deriving from labor income or entrepreneurship (Constantinides and Duffie, 1996; Jagannathan and Kocherlakota, 1996; Elmendorf and Kimball, 2000; Heaton and Lucas, 2000; Viceira, 2001). Other researchers have explored the consequences of variation in financial parameters (Campbell and Viceira, 1999, 2001), while still others have modeled preferences and decision-making under uncertainty in new, potentially more accurate ways (Kahneman and Tversky, 1979; Samuelson, 1989; Abel, 1990; Constantinides, 1990; Benartzi and Thaler, 1995; Rabin and Thaler, 2001; Lax, 2001). Some of these contributions imply rationales for why younger investors might hold more risky portfolios. But why older investors continue to reduce their holdings of risky assets after retirement remains an open question. Regardless of their age, few retirees realistically have the option to work again at steady hours or wages approaching pre-retirement levels. This suggests that the loss of labor supply as a hedge should have its most pronounced effect on portfolio choice in the years preceding rather than following retirement. As for other theories, non-random financial returns and market volatility may indeed impact elderly retirees disproportionately, and investor psychology may be tied to aging in key ways. Both of these are promising directions for future research.

This paper investigates the relationship between aging and portfolio choice using a different perspective. It argues that older investors view risky health as an undiversifiable background risk, one that increases with age on average, and that they try to hedge their risky health by investing their financial
wealth more safely. The notion that there is an important empirical relationship between health and portfolio choice is not new. Guiso, Jappelli and Terlizzese (1996) report that health status is significantly related to empirical patterns of portfolio choice among Italian households; sicker investors hold fewer risky assets. Rosen and Wu (2003) show how health status exerts a similar and robust effect on risky portfolio shares among households in the Health and Retirement Study (HRS). This paper's unique contribution is a theoretical discussion and justification for why the prospects for future health status, or health risk, should matter for portfolio choice. If health shocks are undiversifiable and permanent, then they should affect portfolio behavior, according to logic expressed in the review by Kocherlakota (1996). The model proposed in this paper shows that if adverse health shocks increase the marginal utility of consumption, the expectation of idiosyncratic health risk prompts the investor to shift more assets toward safety. It is an empirical question whether health shocks increase, decrease, or do not affect the marginal utility of consumption, and the empirical evidence on this topic is mixed. Data on health expectations and portfolio choice from the Study of Assets and Health Dynamics Among the Oldest Old (AHEAD) confirm the proposed theory, that more health risk is associated with less financial risk. This implicitly suggests that the marginal utility of consumption rises with health shocks, at least among the elderly, which is consistent with the findings of Lillard and Weiss (1997).

In the sections that follow, a multi-period, partial equilibrium model of portfolio choice and health risk is developed and tested. Section 2 motivates the focus on health risk and provides background on related issues of risk and time horizons. Section 3 derives an approximate analytical solution to a theoretical model based on the infinite-horizon, log-linearized frameworks of Campbell (1993) and Viceira (2001). In the model, healthy individuals face an undiversifiable risk of becoming permanently unhealthy, which requires them to make health expenditures. If the marginal utility of consumption rises when health falls, individuals who face health risk reduce their financial risk because they become effectively more risk averse. The model's empirical implications are assessed in Section 4, which presents a simple calibration exercise using macroeconomic data and individual-level econometric evidence in support of the basic assertion: more health risk is associated with less financial risk-taking. Section 5 concludes.
2 Background and theoretical motivation

2.1 Health shocks in theory and practice

A fundamental question is whether individuals can adequately insure against risky health. If they can, then there should be no effect of health risk on anything but the purchase of health insurance. There are several reasons why health risks may essentially be undiversifiable. First, health itself is a nonpecuniary good that cannot physically be traded between agents. Second, to the extent that health can be improved through medical treatment and behavioral changes, there are clear limitations. Many genetically inherited health characteristics realistically cannot be altered. The depreciation of health over time weakens the ability of individuals to dynamically insure against future health shocks by “saving” health from previous periods. Third, while markets for health insurance exist, they are far from perfect. American seniors face notoriously large gaps in Medicare coverage, most prominently for prescription drugs, and high costs of private supplemental plans. Thus there are several reasons why individuals may view their health as uninsurable, although it is ultimately an empirical question whether they do or not.

Several empirical studies have identified uninsurable risky health as an important determinant of precautionary saving. Hubbard, Skinner and Zeldes (1994) calibrate a numerical microsimulation and quantify the effects on saving of several kinds of idiosyncratic risk, including uninsured medical expenditures. Palumbo (1999) reports evidence of large out-of-pocket medical expenses for the elderly. He then models saving by elderly households using a state-dependent utility function based on that of Viscusi and Evans (1990), who find that the marginal utility of income declines in bad health among chemical workers. Palumbo’s model attributes a significant portion of total precautionary saving to health uncertainty, but he finds no firm evidence that health changes the marginal utility of income. This is consistent with the modeling techniques of Hubbard, Skinner and Zeldes, who assume health shocks simply absorb income rather than affect its marginal utility. Lillard and Weiss (1997) find that expectations of health shocks trigger significant precautionary saving among elderly households. They also estimate that adverse health shocks raise the marginal utility of consumption, defined as income less out-of-pocket medical expenditures.

Altogether, there is considerable evidence that uninsured health risk exists and is empirically relevant for saving behavior. Whether and how it is
relevant for portfolio behavior will turn out to depend critically on the effect of health on the marginal utility of consumption. As will be seen in Section 3, if a health shock raises the marginal utility of consumption, as found by Lillard and Weiss (1997), then health risk depresses financial risk-taking.

### 2.2 Time horizons and risk

A focal point in the study of portfolio choice, aging, and health is the role of time horizons. All things equal, increasing age leaves less time remaining before death, or a shortening investment horizon. A key question is whether this dynamic alone implies a declining risky portfolio share through age.

There are two key ingredients in modeling portfolio choice: the behavior of asset returns and the structure of investor preferences. Time-separable power utility is a standard because it encapsulates constant relative risk aversion (CRRA), a strong but useful assumption. The nature of asset returns is a much more opaque topic. The benchmark assumption, that returns are independently and identically distributed (IID), grew out of the theory of no financial arbitrage and a long track record of poor predictions. Empirical departures from IID returns can sometimes be identified (Siegel, 1994; Campbell, Lo and MacKinlay, 1997; Campbell and Viceira, 2002), but their causes remain unclear, and their implications are the topic of continuing research. For baseline models of portfolio choice, the assumption of IID returns appears to be a reasonable simplification. Power utility and IID stock returns together imply the classic result of Merton (1969) and Samuelson (1969), that long-term investors ought to behave “myopically.” That is, the optimal risky share should remain constant through time.

1 As discussed by Campbell and Viceira (2002), there are several reasons why CRRA preferences are an appealing framework. Chiefly, CRRA implies that risk-taking is invariant to wealth or income, which is consistent with macroeconomic patterns of fairly steady average asset returns during periods of growth in per capita well-being. Some microeconomists have assailed CRRA preferences and expected utility theory more generally for implying unacceptable behavior equivalences (Rabin and Thaler, 2001, e.g.), but power utility remains the primary tool for modeling intertemporal choice. As discussed by Campbell and Viceira (2002), and others, Epstein-Zin (1989) preferences are a useful generalization of power utility that allows the elasticity of intertemporal substitution to differ from the coefficient of risk aversion. But Epstein-Zin preferences require a level of complexity that typically precludes analytical solution techniques.

2 An often-heard claim is that time decreases risk because the standard deviation of cumulative IID returns increases with only the square root of the time elapsed. This
As its name suggests, myopic portfolio choice in an infinite horizon model is actually the same as portfolio choice in a finite horizon model. The number of time periods does not matter in either scenario. Campbell and Viceira (2002) demonstrate this by comparing numerical models with finite-lived investors to analytical models with infinitely-lived investors. Thus the infinite horizon is a convenient analytical tool that helps produce closed-form solutions. The effects on portfolio choice of a bequest motive are less clear. Little theoretical work has been done in this area beyond the original insights of Merton (1969), who wrote that some classes of bequest functions did not affect portfolio choice, while others might. Hurd (2002) found little evidence that bequest motives are important in describing portfolio choice among elderly Americans in the AHEAD.

For these reasons, the theoretical model proposed in the next section assumes power utility, IID stock returns, infinite horizons, and no explicit bequest motive. Since health and length of life are closely related, the empirical framework explored in Section 4 considers the explanatory power of age and remaining years of life in addition to health risk.

3 A multi-period model of portfolio choice in the presence of health risk

This section develops a theoretical model of portfolio choice closely based on that of Viceira (2001). Suppose there are two types of infinitely-lived investors with nonseparable preferences over consumption and health. The types correspond to two states of nature: healthiness and unhealthiness. Type $u$ is permanently unhealthy and must purchase health. Type $h$ is healthy and endowed with health but perceives a risk of permanently falling into ill health, becoming type $u$.

\textit{is “the fallacy of large numbers,” coined by Samuelson (1963). If returns are IID, then additional time is merely additional flips of a fair coin; each flip does not hedge any other. If an individual is unwilling to bet on one flip of a coin, he or she should be unwilling to bet on $N$ flips.}
3.1 Preferences

Let infinitely-lived individuals’ preferences be time-separable with a constant rate of time preference, \( \delta \):

\[
U = \sum_{s=0}^{\infty} \delta^s \cdot U_s(C_s, H_s).
\]

(1)

Following Picone, Uribe and Wilson (1998), suppose investors care about their stocks of health, \( H_t \), and their consumption levels, \( C_t \), in the following way:

\[
U_t(C_t, H_t) = \frac{(C_t^\psi H_t^{1-\psi})^{1-\gamma}}{1-\gamma},
\]

(2)

where \( \psi \in (0, 1) \) and \( \gamma > 0 \). Restricting the exponents on \( C_t \) and \( H_t \) to sum to unity fixes a unique \( \gamma \), which could otherwise simply be rescaled. Setting \( \gamma > 0 \) and \( \psi \in (0,1) \) ensures that marginal utilities of both goods are positive and decreasing in their arguments. When \( \psi = 1 \), (2) reduces to the standard power-utility framework with constant relative risk aversion parameter \( \gamma \). The case of \( \psi = 0 \), when the individual does not care about consumption, is neither realistic nor interesting.

3.2 The budget constraint and technology

For ease of exposition, suppose health cannot be saved between periods; it is either endowed or must be purchased, and it is immediately consumed. Healthy type \( h \) individuals are endowed with health \( H^h_t \) that grows at an exogenous rate \( g \): \( H^h_{t+1} = H^h_t e^g \), and they are prohibited from buying or selling health. Each period, unhealthy type \( u \) individuals must purchase their health. Those of type \( h \) face a probability \( \pi_h \in [0,1] \) each period of permanently becoming type \( u \).

Although preferences are uniform across states, the budget constraint is not. Healthy investors of type \( h \) face

\[
W_{t+1} = (W_t - C^h_t) R_{p,t+1},
\]

(3)

As first explained by Grossman (1972), a much richer view of life-cycle behavior is afforded by models of health capital. If individuals hedge against health shocks by investing in their health, then omitting health investment from this model will exaggerate financial responses to health risk. But results in Section 4 suggest that actual responses are large.
while unhealthy investors of type u face

\[ W_{t+1} = (W_t - C_t - P_{h,t} H_t) R_{p,t+1}, \quad (4) \]

where health must be purchased at a price of \( P_{h,t} > 0 \) consumption units. All investors earn a total return on their financial portfolios equal to \( R_{p,t+1} > 0 \).

Individuals can distribute their wealth between two financial assets. One asset is risky, with total return given by \( R_{1,t+1} \equiv e^{r_{1,t+1}} \), where \( r_{1,s} \) is IID. The other asset generates a certain return \( R_f \equiv e^{r_f} \), where \( r_f \) is a constant parameter. The return on the financial portfolio, \( R_{p,t+1} \), is therefore

\[ R_{p,t+1} = \alpha_t R_{1,t+1} + (1 - \alpha_t) R_f, \quad (5) \]

where \( \alpha_t \) is the share of wealth held in the risky asset at time \( t \). The expected excess log return, \( E_t r_{1,t+1} - r_f \), is constant, and the unexpected excess log return is conditionally homoscedastic, serially uncorrelated, and normally distributed with mean zero and variance \( \sigma^2_{r_t} \).

For expository purposes, it is convenient to assume that the relative price of health is lognormal:

\[ P_{h,t} = \Pi_{s=t}^t R_{h,s}, \quad (6) \]

where the \( R_{h,s} \equiv e^{r_{h,s}} \) are lognormal IID health-inflation rates that are independent of asset returns: \( \text{Cov}_t [r_{h,t+1}, r_{1,t+1}] = 0 \). The relative price of health is an elusive empirical concept because it depends on relative rates of technological change and on changing prevalence patterns of different types of health risks. Specifying lognormality is merely a general way to capture the effect of uncertain health prices on behavior.

### 3.3 Solving the model

The individual’s problem is to

\[ \max_{c_t, h_t, \alpha_t} U, \quad \forall t, \quad (7) \]

subject to the expected budget constraint, (3) or (4). Following Campbell (1993), Campbell and Viceira (1999), Viceira (2001), and Campbell and Viceira (2002), the model’s approximate log-linear solution will be found.

The budget constraints, (3) and (4), can be log-linearized by taking first-order Taylor approximations around the mean log ratios of both consumption
and wealth, and health spending and wealth. As shown in Appendix A, if these means are stable, then the log-linear budget constraints are

\[ w_{t+1} - w_t = k^h - \rho^c(c_t^h - w_t) + r_{p,t+1}, \]  

(8)

for healthy investors, and

\[ w_{t+1} - w_t = k - \rho^c(c_t - w_t) - \rho^h(h_t + \sum_{s=0}^{T} r_{h,s} - w_t) + r_{p,t+1}, \]  

(9)

for unhealthy investors, where lowercase variables represent logs. The \( k \)'s and \( \rho \)'s are constants, all the \( \rho \)'s are positive, and \( r_{p,t+1} \) is the approximate log return on the financial portfolio, derived by Campbell and Viceira (1999):

\[ r_{p,t+1} \approx \alpha_t r_{1,t+1} + (1 - \alpha_t) r_f + \frac{1}{2} \alpha_t (1 - \alpha_t) \sigma^2_c. \]  

(10)

Proceeding requires the usual assumption of joint lognormality in consumption and asset returns (Hansen and Singleton, 1983). The solution method then involves finding log-linearized Euler equations and combining them with the log-linear budget constraint and guesses about optimal consumption rules. Paralleling Viceira (2001), the model must be solved backward, by determining the optimal choices of unhealthy type \( h \) investors first. This is because type \( h \) investors must plan contingent on their optimal behavior in the absorbing state, as type \( u \) investors.

3.3.1 Optimal choices of unhealthy investors

When unhealthy, individuals must purchase their health each period, solving (7) subject to (4). With two goods in the utility function, there are two Euler conditions that must be satisfied:

\[ 1 = E_t \left[ \delta \left( \frac{C_{t+1}}{C_t} \right)^{(1-\gamma)} \left( \frac{H_{t+1}}{H_t} \right)^{(1-\gamma)} R_{i,t+1} \right], \]  

(11)

and

\[ 1 = E_t \left[ \frac{1}{R_{h,t+1}} \left( \frac{H_{t+1}}{H_t} \right)^{(1-\psi)(1-\gamma)} \left( \frac{C_{t+1}}{C_t} \right)^{(1-\psi)} R_{i,t+1} \right], \]  

(12)

for \( i = 1, f, p \) and where the price of health follows the process described by (6). Both Euler equations must hold for each asset \( i = 1, f \) that is held
by the investor, and for the portfolio, $p$. The first Euler equation equates expected marginal rates of substitution of consumption across periods. The second does the same for health, accounting for the changing relative price of health. Between them it is implied that marginal rates of substitution between consumption and health are smoothed.

When all of the variables inside the expectations operators of the Euler equations are lognormal — that is, when consumption, health, returns, and the inflation shock are individually lognormal — then the Euler equations can be log-linearized exactly.

Appendix B derives the log-linear Euler equations for unhealthy investors:

\[
\log \delta + E_t[r_{i,t+1}] + \beta_1 E_t[c_{t+1} - c_t] + \beta_2 E_t[h_{t+1} - h_t] \\
+ \frac{1}{2} Var_t[r_{i,t+1} + \beta_1 (c_{t+1} - c_t) + \beta_2 (h_{t+1} - h_t)] = 0, \tag{13}
\]

and

\[
\log \delta + E_t[r_{i,t+1}] - E_t[r_{h,t+1}] + \beta_3 E_t[h_{t+1} - h_t] + \beta_4 E_t[c_{t+1} - c_t] \\
+ \frac{1}{2} Var_t[r_{i,t+1} - r_{h,t+1} + \beta_3 (h_{t+1} - h_t) + \beta_4 (c_{t+1} - c_t)] = 0, \tag{14}
\]

where $\beta_1 = \psi(1 - \gamma) - 1$, $\beta_2 = (1 - \psi)(1 - \gamma)$, $\beta_3 = (1 - \psi)(1 - \gamma) - 1$, and $\beta_4 = \psi(1 - \gamma)$.

Solving the model requires that guesses be made as to the form of the optimal consumption rules. Cobb-Douglas preferences in (2) imply that the individual will choose to split his or her resources between consumption and the consumption costs of health. It follows that the single consumption rule stipulating a target consumption-wealth ratio in the one-good, power utility models considered by Campbell (1993) and others generalizes to two similar and separate rules in this case:

\[
c_t = b_{c,0}^\mu + b_{c,1}^\mu w_t, \tag{15}
\]

and

\[
h_t + \sum_{s=0}^t r_{h,s} = b_{h,0}^\mu + b_{h,1}^\mu w_t. \tag{16}
\]

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4This is due to the following properties of lognormal variables: if $X \sim LN(\mu, \sigma^2)$, then $\log X \sim N(\mu, \sigma^2)$ and $E[X] = e^{\mu + \sigma^2/2}$, so $\log E[X] = \mu + \sigma^2/2$. 

10
These rules allow consumption and health costs to have separate wealth elasticities $b_s^t$ for $s = c, h$, but as will be seen, the standard result of unitary elasticities will obtain. Intuitively, a shock to wealth of a percentage point translates into a percentage point decline in both consumption and in health costs, since preferences are Cobb-Douglas. Combining the log-linearized Euler conditions, the log-linearized budget constraint, and the optimal rules produces the following result:

**Proposition 1.** Unhealthy individuals invest a share $\alpha_t^u$ of their wealth in the risky asset that is given by

$$\alpha_t^u = \frac{E_t[r_{1,t+1}] - r_f + \frac{1}{2}\sigma_r^2}{\gamma \sigma_r^2}.$$ (17)

The optimal rules are

$$c_t = b_{c,0}^u + w_t,$$ (18)

and

$$h_t + \sum_{s=0}^t r_{h,s} = b_{h,0}^u + w_t,$$ (19)

where $b_{c,0}^u$ is the target consumption-wealth ratio, and $b_{h,0}^u$ is the target health-wealth ratio.

**Proof.** See Appendix C. □

### 3.3.2 Optimal choices of healthy investors facing health risk

If healthy individuals face a constant probability $\pi_h \in (0,1)$ each period of becoming permanently unhealthy, then they must choose consumption and portfolio shares according to the probability-weighted tradeoffs. Healthy investors therefore behave according to a single Euler condition in consumption that smooths expected marginal utilities across time and states:

$$1 = E_t \left[ (1 - \pi_h)\delta \left( \frac{C_{t+1}^h}{C_t^h} \right)^{\psi(1-\gamma)-1} \left( \frac{H_{t+1}^h}{H_t^h} \right)^{(1-\psi)(1-\gamma)} R_{i,t+1} \right] + E_t \left[ \pi_h \delta \left( \frac{C_{t+1}^h}{C_t^h} \right)^{\psi(1-\gamma)-1} \left( \frac{H_{t+1}^h}{H_t^h} \right)^{(1-\psi)(1-\gamma)} R_{i,t+1} \right],$$ (20)

for $i = 1, f, p$ as before, and where the expectations operator has already been distributed between the two additive parts of the Euler equation. Since
(20) is a sum of expectations of lognormal variables, simply taking logs of both sides will not work. As discussed in Appendix B, taking two Taylor expansions results in the following approximate log-linear Euler equation in the healthy state, where the \( \beta \)'s are defined as in (13) and (14):

\[
0 = \log \delta + E_t[r_{i,t+1}] + (1 - \pi_h)\beta_1 E_t[c^h_{t+1} - c^h_t]
+ (1 - \pi_h)\beta_2 E_t[h^h_{t+1} - h^h_t]
+ \frac{1 - \pi_h}{2} Var_t[r_{i,t+1} + \beta_1(c^h_{t+1} - c^h_t) + \beta_2(h^h_{t+1} - h^h_t)]
+ \pi_h\beta_1 E_t[c^h_{t+1} - c^h_t]
+ \pi_h\beta_2 E_t[h^h_{t+1} - h^h_t]
+ \frac{\pi_h}{2} Var_t[r_{i,t+1} + \beta_1(c^h_{t+1} - c^h_t) + \beta_2(h^h_{t+1} - h^h_t)].
\]

(21)

Again, optimal rules must be guessed and verified. Since healthy investors have only one state variable available to them, it follows that they behave according to a single rule, as in the one-good models of Viceira (2001) and Campbell and Viceira (2002):

\[
c^h_t = b^h_{c,0} + b^h_{c,1}w_t.
\]

(22)

The combination of this rule, the log-linear Euler approximation, the relevant budget constraints, and the optimal choices of unhealthy investors results in:

**Proposition 2.** Healthy individuals invest a share \( \alpha^h_t \) of their wealth in the risky asset that is given by

\[
\alpha^h_t = \frac{E_t[r_{1,t+1}] - r_f + \frac{1}{2}\sigma^2_r}{R(\psi, \gamma, \pi_h) \cdot \sigma^2_r},
\]

(23)

where

\[
R(\psi, \gamma, \pi_h) = 1 - (1 - \gamma)(\psi + (1 - \psi)\pi_h)
\]

(24)

is the healthy investor’s “effective” risk aversion, a function of the preference parameters and \( \pi_h \in (0, 1) \), the probability that the individual will become permanently unhealthy next period. The optimal consumption rule is

\[
c^h_t = b^h_{c,0} + w_t,
\]

(25)

where \( b^h_{c,0} \) is the target consumption-wealth ratio.

**Proof.** See Appendix D.

\[\square\]
3.4 Implications

3.4.1 Precautionary saving

Target ratios of consumption and health to wealth, the $b_{x,h}$'s in (18), (19), and (25), determine saving behavior in this model. As shown in Appendix C, unhealthy investors lower their target ratios of consumption and health to wealth whenever there is an increase in the volatility of asset returns or health price inflation. Precautionary saving is the result, which is standard when utility has a positive third derivative. Since volatility in health prices is likely to increase volatility in health expenditures, these findings are consistent with those of Lillard and Weiss (1997) and Palumbo (1999), who estimate large effects of medical expenditure uncertainty on precautionary saving.

Saving by healthy investors is by necessity quite different, since their behavior is contingent on optimal choices when unhealthy. Appendix D shows that increased financial risk may increase or decrease saving while healthy, depending on the magnitude of precautionary saving in the unhealthy state. Health price volatility actually decreases saving when healthy because it does not affect anything but saving when unhealthy, which rises. The effect of health risk, $\pi_h$, on saving while healthy is of indeterminate sign in this model, a result that parallels its analogue in Viceira (2001).

3.4.2 Portfolio choice

Portfolios are set according to $\alpha^u$ and $\alpha^h$ in (17) and (23), the optimal risky shares in the unhealthy and healthy states. They are different only insofar as the healthy investor’s effective risk aversion, $R(\psi, \gamma, \pi_h)$, differs from $\gamma$, the risk aversion of the unhealthy investor. Volatility in health prices has no effect on portfolio choice because it is uncorrelated with market risk and thus has no effect on the relative attractiveness of the risky asset.

The properties of $R(\psi, \gamma, \pi_h)$ depend critically on whether $\gamma \geq 1$. If $\gamma > 1$, simple algebra shows that since $\pi_h < 1$,

$$\psi + (1 - \psi)\pi_h < 1$$
$$1 - \gamma((\psi + (1 - \psi)\pi_h) > 1 - \gamma$$

$$R(\psi, \gamma, \pi_h) = 1 - (1 - \gamma)(\psi + (1 - \psi)\pi_h) < \gamma. \quad (26)$$

If the equity risk premium is positive, $E_t[r_{1,t+1} - r_f + \frac{1}{2}\sigma_r^2] > 0$, and (17),
(23), and (26) together imply that
\[ \alpha^h > \alpha^u. \]  
(27)

It also follows from (26) that
\[ \frac{\partial R(\psi, \gamma, \pi_h)}{\partial \pi_h} = (\gamma - 1)(1 - \psi) > 0, \]  
(28)

which implies that the derivative of (23) with respect to \( \pi_h \) is
\[ \frac{\partial \alpha^h}{\partial \pi_h} = -\frac{E_t[r_{1,t+1}] - r_f + \frac{1}{2}\sigma^2_r}{\sigma^2_r} \cdot \frac{1}{R^2} \cdot \frac{\partial R}{\partial \pi_h} < 0. \]  
(29)

In words, if \( \gamma > 1 \) and the equity risk premium is positive, (26) shows that the effective risk aversion of the healthy investor is less than the risk aversion of the unhealthy investor; (27) states that the optimal risky portfolio share of the healthy investor is greater than the optimal share of the unhealthy investor; (28) shows that the effective risk aversion of the healthy investor increases with health risk; and (29) shows that the optimal risky portfolio share of the healthy investor decreases with health risk.

But if instead \( 0 < \gamma < 1 \), the inequalities in (26)-(29) are all reversed, as are the model’s implications for relative portfolio behavior in the presence of health risk. When \( \gamma = 1 \) exactly, \( R(\psi, 1, \pi_h) = 1 = \gamma \) and health risk has no effect on portfolio choice at all. Thus the model’s predictions for portfolio choice hinge crucially on the level of \( \gamma \).

Unfortunately, the magnitude of \( \gamma \) is theoretically ambiguous, and there is disagreement among empirical studies seeking to measure it. Critically, \( \gamma \) determines the sign of the the mixed partial derivative of utility,
\[ \frac{\partial^2 U}{\partial H \partial C} = \psi(1 - \psi)(1 - \gamma)C^{\psi(1-\gamma)-1}H^{(1-\psi)(1-\gamma)-1}, \]  
(30)

which is negative if \( \gamma > 1 \), zero if \( \gamma = 1 \), and positive if \( \gamma < 1 \). A negative mixed partial means a decline in health increases the marginal utility of consumption. The marginal utility of some types of consumption, such as luxury goods and vacations, may diminish during illness. Other types, such as market services that replace home production during times of illness, may become more dear. Still others are probably not affected at all by health. As mentioned in Section 2, the empirical literature is split on the sign
of the mixed partial. Viscusi and Evans (1990) report that chemical workers expect their marginal utilities of income, a related concept, to fall after adverse health shocks.\textsuperscript{5} Hubbard, Skinner and Zeldes (1994) and Palumbo (1999) find no evidence based on saving behavior that the marginal utility of income responds to health shocks at all. In their paper on consumption and uncertainty among the elderly, arguably the most relevant study for current purposes, Lillard and Weiss (1997) estimate that the marginal utility of consumption rises after losses of health among elderly couples.

Intuitively, the sign of the mixed partial is critical for portfolio choice for the same reason it is important for optimal health insurance, as explored by Viscusi and Evans (1990) and others. If $\gamma > 1$, the cross-partial is negative, and a decline in health raises the marginal utility of consumption as well as the marginal utility of health. Risks to health compound risks to consumption, and the optimal amount of health insurance is greater than the actuarially fair amount required to treat health shocks. But if health risks are uninsurable, as is suggested by evidence on precautionary saving, the individual instead decreases his or her financial risks.

Thus the model’s qualitative predictions regarding portfolio choice are contingent on a preference relationship that has proved to be very difficult to measure consistently. Given that Lillard and Weiss (1997) recover a negative mixed partial based on the behavior of elderly households, it seems likely that $\gamma > 1$ in this model so that (26)-(29) are correct. Rather than explicitly attempting to measure the sign of the mixed partial, this paper proposes instead to infer it by testing the health risk model directly.

\section{Empirical evidence}

\subsection{Aggregate patterns and model calibration}

Figure 1 shows seven cross-sectional age profiles of risky portfolio shares between 1992 and 2000. The data are averaged within 2-year age groups, where the risky portfolio share, $\alpha$, is defined as risky financial wealth, such as stocks and stock mutual funds, divided by total financial wealth, which includes safer instruments such as bills, bonds, and bank accounts. The data are observed during five waves of the Health and Retirement Study (HRS) and two waves of its sister dataset, the Study of Assets and Health Dynamics.

\textsuperscript{5}By the chain rule, $\partial U/\partial Y = (\partial U/\partial C)(\partial C/\partial Y) + (\partial U/\partial H)(\partial H/\partial Y)$. 

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Among the Oldest Old (AHEAD). As Figure 1 depicts, there is a distinct downward trend in financial risk-taking through age. Investors of working age, or younger than 65, generally have $\alpha$'s between 0.25 and 0.35, or about 0.30 on average. Older investors are clustered between 0.10 and about 0.25, or perhaps 0.175 on average. Ameriks and Zeldes (2001) report similar levels based on the Survey of Consumer Finances and TIAA-CREF data.

Health risk is difficult to observe. The ideal measure is an individual’s self-assessed probability of incurring significant out-of-pocket medical expenses. Luckily, the HRS and AHEAD datasets each contain a question that queries expectations about health. Average responses to the HRS question, which asks individuals to report the chance of health limiting work activity, are plotted against age in Figure 2. A clear pattern of increasing health risk through age is evident. The average 50 year-old assigns a probability of 0.35 to work limitations due to health, while the average 65 year-old assigns almost 0.45, rising to near 0.50 by age 70. The older AHEAD cohort, comprised of individuals over 70, was asked to state the probability that medical expenses would use up all household savings over a five-year window. These responses averaged between 25 and 30 percent with no apparent age-specific pattern. Taken at face value, the data suggest health risk may rise some 25 percentage points on average between middle and old age.\(^6\)

If all of the observed decline in average risky portfolio shares through age were due to increases in average health risk, then it would appear that $\partial \alpha^h / \partial \pi_h \approx \Delta \alpha^h / \Delta \pi_h = (0.3 - 0.175)/0.25 = -0.5$. As a somewhat more refined check on the effect of health risk on portfolio choice, the functional form of $\alpha^h$ derived in Section 3 can be calibrated using aggregate financial data and assumptions about preferences and health risk. Calibration is complicated by the presence of three unknowns: preferences, $\gamma$ and $\psi$, are unobservable; and the average level of health risk, $\pi_h$, is not well measured for multiple age groups. In general it is not possible to recover all three unknowns using only two equations based on behavior of the average young and old investor.

It is possible to make a general statement about the preference parameters using only financial data and $\Delta \pi_h = 0.25$, however. Rearranging (23)

\(^6\)It is unclear how these HRS and AHEAD measures of health risk relate to the concept of $\pi_h$ in Section 3. Since the questions ask about extreme events, answers may understate the true $\pi_h$; but since they condition on periods of time longer than a year, they may overstate $\pi_h$, which takes the same periodicity as the financial variables.
produces
\[ R(\psi, \gamma, \pi_h) = \frac{1}{\alpha_h} \cdot \frac{E_t[r_{1,t+1}] - r_f + \frac{1}{2}\sigma_r^2}{\sigma_r^2}. \] (31)

Let there be two average types of investors: the average young investor faces health risk \( \pi_y^h \) and sets \( \alpha^y \), and the average old investor faces \( \pi_o^h > \pi_y^h \) and sets \( \alpha^o \). Subtracting (31) evaluated at \( \pi_y^h \) from (31) evaluated at \( \pi_o^h \) yields

\[ R(\psi, \gamma, \pi_o^h) - R(\psi, \gamma, \pi_y^h) = \left( \frac{1}{\alpha^o} - \frac{1}{\alpha^y} \right) \cdot \frac{E_t[r_{1,t+1}] - r_f + \frac{1}{2}\sigma_r^2}{\sigma_r^2}. \] (32)

Moments of the financial parameters and the estimated portfolio shares are shown in Table 1. Expanding the \( R(\cdot) \)'s using (24) and inserting values where appropriate transforms (32) into

\[ (\gamma - 1)(1 - \psi)(\pi_o^h - \pi_y^h) = \left( \frac{1}{0.175} - \frac{1}{0.3} \right) \cdot 1.8254 \]

\[ (\gamma - 1)(1 - \psi) = \frac{4.35}{0.25} = 17.4, \] (33)

where the last line uses \( \pi_y^h - \pi_o^h = 0.25 \). Since \( \psi \in (0, 1) \), (33) implies that \( \gamma > 1 \) in this model. As described in Section 3.4, the marginal utility of consumption rises with an adverse health shock when \( \gamma > 1 \). Moments of financial data therefore provide prima facie evidence that individuals should lower their financial risk-taking in response to idiosyncratic health risk. Intuitively, the expression on the left-hand side of (33) is actually \( \partial R(\psi, \gamma, \pi_h)/\partial \pi_h \) from (28); a positive value means that health risk increases effective risk aversion.

If one is willing to assume further that \( \pi_y^h = 0 \) and \( \pi_o^h = 0.25 \), as is suggested by the AHEAD data, it is also possible to recover separate estimates of \( \psi \) and \( \gamma \) with which to calibrate \( \partial \alpha_h/\partial \pi_h \). When \( \pi_h^o = 0 \), (31) can be simplified using (24) and the values in Table 1:

\[ R(\psi, \gamma, 0) = 1 - \psi(1 - \gamma) = \frac{1}{0.3} \cdot 1.8254 = 6.08 \]

\[ 1 - \gamma = \frac{5.08}{\psi}. \] (34)

Similarly, (31) evaluated at \( \pi_o^h \) produces

\[ 0.25 + 0.75\psi = -\frac{9.43}{(1 - \gamma)}. \] (35)

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Substituting (34) into (35) produces a single equation in one unknown, $\psi$, which can be solved and then substituted back into (34) to find $\gamma$, revealing

$$\psi = 0.23 \quad (36)$$

$$\gamma = 23.09 \quad (37)$$

For comparison, these parameter values are equivalent to a coefficient of relative risk aversion of around 6 when facing no health risk and about 10.5 when facing $\pi_h = 0.25$. High levels of risk aversion, such as 10 or more, are common findings in the empirical literature Kocherlakota (1996).

With these estimates in hand, it is possible to gauge the size of $\partial \alpha^h / \partial \pi_h$ using (29). Since effective risk aversion increases with health risk if $\gamma > 1$, the magnitude of $\partial \alpha^h / \partial \pi_h$ is not constant but rather decreasing in $\pi_h$. Simple algebra shows that using (36) and (37), (29) implies

$$\left\{ \begin{array}{ll}
-0.84 & \text{if } \pi_h = 0 \\
-0.46 & \text{if } \pi_h = 0.125 \\
-0.29 & \text{if } \pi_h = 0.25 \\
\end{array} \right. \quad (38)$$

Estimates of $\partial \alpha^h / \partial \pi_h$ using individual-level data from the AHEAD are most likely to match the calibrated value at $\pi_h = 0.25$, since that is the average health risk in the AHEAD sample. But the value at $\pi_h = 0.125$ is the more appropriate estimate of the average effect of health risk over the life cycle.

### 4.2 Micro-level tests of the health risk model

The calibration results provide an initial guess of what individual-level regression analysis using the AHEAD may uncover. The AHEAD dataset is better suited than the HRS to testing the theory because respondents are over 70 and almost all are asked the health risk question.\textsuperscript{7} To test whether $\partial \alpha^h / \partial \pi_h < 0$, data from the 1993 and 1995 AHEAD waves are pooled and analyzed using a time dummy. The high frequency of missing wealth observations in the data generally confounds analysis of time-series variation, but past observations can be useful in helping control for endogeneity. The unit of observation is the individual, and financial wealth is spread equally within

\textsuperscript{7}Tests based on HRS data were reported in an earlier version of this paper. They yielded similar results and are available from the author upon request.
multi-person households. The regression model is

\[ \alpha(i, t) = b_0 + b_h \pi_h(i, t) + \sum_k b_k x_k(i, t) + D_t + \epsilon(i, t), \]  

(39)

where \( \alpha(i, t) \) is individual \( i \)'s risky portfolio share at time \( t \), \( \pi_h(i, t) \) is health risk, the \( x_k(i, t) \)'s are covariates that may measure unobservable preferences, and \( D_t \) is a time dummy that picks up changes in aggregate financial parameters. Calibration of the model in Section 3 to macroeconomic data and findings by Lillard and Weiss (1997) suggest \( \gamma > 1 \), which implies that \( b_h < 0 \). As is typical, the measures of risky portfolio shares exhibit significant pooling at \( \alpha = 0 \) and 1. Following Guiso, Jappelli and Terlizzese (1996), Rosen and Wu (2003), and others, the Tobit model will be used to estimate (39), with limits at both truncation points: \( \alpha = 0 \) and 1.

Table 2 presents coefficients and their standard errors from two-limit Tobit estimation of (39) using pooled AHEAD data. Column 1 reports the coefficients from a portfolio choice regression without any health variables, while column 2 displays the results of inserting the measure of health risk. The regression in column 3 uses self-rated health status measures instead, which are the variables of choice in Rosen and Wu (2003). The results of including both health risk and health status are shown in column 4. Column 5 is identical to column 2 except that the sample has been limited to the 1995 wave as a robustness check.

The first two rows of Table 2 show that higher health risk and poorer health status are significantly and independently associated with lower risky portfolio shares. Worsening health status decreases portfolio shares slightly more than in the HRS sample used by Rosen and Wu (2003), but estimates are similar. The effect of health risk is about \( \partial \alpha / \partial \pi_h = -0.17 \), somewhat smaller in absolute value than the \(-0.29\) predicted in Section 4.1 but still significant and substantial. One standard deviation in health risk is roughly 0.3 in the data, as is a standard deviation in \( \alpha \). A one-\( \sigma \) increase in health risk is therefore associated with a decrease in \( \alpha \) of about 5 percentage points.

Table 2 also shows significant effects of several covariates. Education is a key variable, suggesting important roles for psychology and socioeconomic status. Since marriages or partnerships are associated with fairly large increase in financial risk-taking, being female only exerts a negative net impact for elderly singles. The number of children is negatively associated with financial risk, contrary to typical risk-spreading arguments and to the belief
that bequest motives may extend the planning horizon and encourage risk-taking. Stark racial patterns in financial risk-taking are evident, with blacks holding a full standard deviation less risk than whites and Hispanics behaving similarly. Each additional year of age decreases $\alpha$ between 0.005 and 0.01, strikingly close to the investment advisor’s rule of thumb. Expected remaining years of life, which are inferred from individual responses about survivorship expectations, do not exert a robust influence, however.

There are two reasons to employ instrumental variables here. Measurement error may attenuate the impact of the health risk variable, which exhibits extreme response pooling, and it would be useful to include wealth as a regressor, which may be endogenous. Response pooling is a common trait of subjective probability data, and some special treatment is prudent. Log wealth frequently has tremendous explanatory power in cross-sectional portfolio choice regressions, but it is clearly endogenous in this model. The change in log wealth is approximately a linear function of $\alpha$, as can be seen from (8), (9), and (10) in Section 3. Still, wealth may proxy for liquidity constraints or unobservable preferences, or utility may not be CRRA, so it is worth exploring models with wealth included as a regressor.

Table 3 displays results of instrumenting for the AHEAD health risk variable in order to address measurement error. Diagnostics are reported at the bottom of the table, and the instrument sets are described in greater detail in Table 4. All four models appear to be valid, but higher first-stage $R^2$’s favor columns 3 and 4, probably reflecting the usefulness of prior-wave responses as instruments. The size of the health risk coefficient jumps outlandishly in columns 1 and 2 relative to previous results, which is consistent with the effects of attenuation bias but suggests problems with those instrument sets. Using the same instrument set as in column 1, column 2 shows that as before, limiting the sample to 1995 does not change the results much. Columns 3 and 4 also restrict the sample to 1995 but instrument for $\pi_h$ using its observed value in 1993. Point estimates of the effect of health risk range between $-0.65$ and $-0.76$, more negative than the calibrated estimate of $-0.29$ but still consistent with values generated by the health risk model. Comparing columns 3 and 4 with results in Table 2 suggests attenuation bias may be

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8The model for a Tobit regression with IV is “ivtobit,” a Stata ado-file written by Joseph Harkness. Specification tests include “tobexog,” a Stata ado-file written by Christopher Baum, and “overid,” a Stata ado-file written by Baum and others that uses 2SLS. Although 2SLS is potentially biased and inconsistent when selection is present, the overid algorithm provides important insights and has not been adapted to the Tobit environment.
large but not outlandish.

Table 5 presents regressions that include wealth and instrument for both wealth and health risk in the 1995 AHEAD using instrument sets that include lagged values. Test statistics in the bottom two rows suggest that endogeneity would indeed confound regular Tobit estimation of at least three of these four models. Columns 1 and 2 use the log specification in wealth, while columns 3 and 4 use a linear specification. The impacts of other variables on portfolio choice are considerably weakened by the inclusion of log wealth, but linear wealth effects are far less damaging. This may reflect tighter endogeneity between portfolio shares and log wealth, as discussed above, or the naturally better fit of a lognormal wealth distribution in a linear regression. Since liquidity constraints, such as a minimum initial balance in a mutual fund, are probably linear functions of wealth, the linear specification may be more desirable from a theoretical perspective.

Overidentification tests offer little guidance as to the preferred specification in Table 5, but first-stage $F$-statistics appear to favor column 1. Outside of column 2, where health risk has no impact, coefficients on health risk range between $-0.2$ and $-0.45$, smaller than in Table 3 but still larger than in Table 2 and significant. Education and age retain their independent effects on portfolio choice across all columns. Across the board, remaining years of life are not significantly related to portfolio shares.

### 4.3 Remarks

Point estimates recovered from the AHEAD suggest that $\partial \alpha^h / \partial \pi_h \approx -0.3$, consistent with the predictions of the calibrated model evaluated at the average level of $\pi_h$ in the data. If $\partial \alpha^h / \partial \pi_h$ were fixed at $-0.3$, a rise in $\pi_h$ of about 25 percentage points on average over the life cycle could decrease $\alpha$ by 7.5 percentage points. Since risky portfolio shares fall about 12.5 percentage points after retirement, health risk would then explain 60 percent of the observed decrease through age. If $\partial \alpha^h / \partial \pi_h$ were indeed higher among younger individuals facing less health risk, as predicted by the model, then 60 percent would be a lower bound.
5 Conclusion

Evaluating the links between health risk and portfolio selection is a natural extension of two literatures: portfolio choice in the presence of background risk, and precautionary saving in response to health expenditure risk. Consistent with evidence in the precautionary saving literature, this paper finds that retired individuals perceive significant risks of large out-of-pocket health expenditures even though they are covered by Medicare, and that these perceptions alter economic behavior. This paper proposes a theoretical model of portfolio choice that describes how investors respond to idiosyncratic health risk, and it tests the model’s implications using aggregate and individual-level data from the Study of Assets and Health Dynamics Among the Oldest Old (AHEAD).

Individuals care about health and consumption. If the mixed partial derivative of utility is negative, adverse health shocks raise the marginal utility of consumption. Then health risk, \( \pi_h \), amplifies consumption risk, so investors will decrease their risky portfolio shares, \( \alpha \), when faced with idiosyncratic risks of becoming unhealthy. Mechanically, if the mixed partial is negative, health risk and unhealthiness will increase risk aversion, which decreases optimal risky portfolio shares if the equity risk premium is positive.

Previous research on the sign of the mixed partial suggests that it may be negative for elderly individuals, but other studies find it is positive among workers. Macroeconomic evidence hints that the mixed partial is indeed negative for older individuals, and direct empirical testing of the health risk model using data from the AHEAD provide implicit support that it is. Regression results indicate that \( \partial \alpha / \partial \pi_h \approx -0.3 \), which implies that the mixed partial must be negative. The AHEAD measure of \( \pi_h \), which averages 0.25 for retired individuals, may thus explain up to -7.5 percentage points of risky portfolio shares, which would account for 60 percent of the observed decline in average risk-taking through the life cycle.

These findings yield several implications. At the micro level, they attest to the significance of risky health for older investors, and they suggest that further improvements in old-age health security are desirable. At the aggregate level, undiversifiable health risk may be responsible for what would otherwise appear to be suboptimal risk-taking. Injecting more risk into Social Security through privatization in order to exploit the equity risk premium may be counterproductive if older investors are intentionally holding safer portfolios. Such reform may be more desirable if coupled with Medicare expansion.
Appendix

A The log-linear budget constraints

When health must be purchased. Dividing both sides of (4) by $W_t$, substituting for the price of health using (6), taking logs, and denoting logs in lowercase produces

$$w_{t+1} - w_t = \log \left( 1 - e^{c_t - w_t} - e^{h_t + \sum_{s=0}^t r_{h,s} - w_t} \right) + r_{p,t+1}. \quad (40)$$

The next step is to take a first-order Taylor approximation of the first term on the right-hand side around the mean values of $c_t - w_t$ and $h_t + \sum_{s=0}^t r_{h,s} - w_t$. Naming those two variables $X_t$ and $Y_t$ for shorthand, one can write the expansion as

$$\log \left( 1 - e^{X_t} - e^{Y_t} \right) \approx \log \left( 1 - e^{E[X_t]} - e^{E[Y_t]} \right) + \frac{1}{1 - e^{E[X_t]} - e^{E[Y_t]}} \times (-e^{E[X_t]}) (X_t - E[X_t]) + \frac{1}{1 - e^{E[X_t]} - e^{E[Y_t]}} \times (-e^{E[Y_t]}) (Y_t - E[Y_t]). \quad (41)$$

The log-linear budget constraint for older investors is therefore

$$w_{t+1} - w_t = k - \rho_c (c_t - w_t) - \rho_h \left( h_t + \sum_{s=0}^t r_{h,s} - w_t \right) + r_{p,t+1}, \quad (42)$$

where $k$ is a constant that can be inferred by collecting terms in (41), and $\rho_c$ and $\rho_h$ are given by

$$\rho_c = \frac{e^{E[c_t - w_t]}}{1 - e^{E[c_t - w_t]} - e^{E[h_t + \sum_{s=0}^t r_{h,s} - w_t]}}, \quad (43)$$

$$\rho_h = \frac{e^{E[h_t + \sum_{s=0}^t r_{h,s} - w_t]}}{1 - e^{E[c_t - w_t]} - e^{E[h_t + \sum_{s=0}^t r_{h,s} - w_t]}}, \quad (44)$$

The numerators in each formula for the $\rho$’s are positive by construction. Since wealth can never be less than consumption and health costs, the denominators are also positive, implying that $\rho_c > 0$ and $\rho_h > 0$. 

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**When health is an endowment.** Following the same steps as in the previous section, one can show that the log-linear budget constraint based on (3) is

\[ w_{t+1} - w_t = k^h - \rho_c^h (c_{t+1}^h - w_{t+1}) + r_{p,t+1}, \]  

(45)

where \( k^h \) is a constant and \( \rho_c^h \) is given by

\[ \rho_c^h = \frac{e^{E[c_{t+1}^h - w_{t+1}]} - 1}{e^{E[c_{t+1}^h - w_{t+1}]}}. \]  

(46)

This is identical to the log-linear budget constraint found in Campbell (1993) and Campbell and Viceira (1999).

**B Finding the log-linear Euler equations**

**When health must be purchased.** Consider the health Euler equation, (12). If \( X \sim LN(\mu, \sigma^2) \) is given by

\[ X = \delta \left( \frac{H_{t+1}}{H_t} \right)^{(1-\psi)(1-\gamma)-1} \left( \frac{C_{t+1}}{C_t} \right)^{\psi(1-\gamma)} R_{i,t+1}, \]  

(47)

then since \( \log X \sim N(\mu, \sigma^2) \) and \( E_t[X] = e^{\mu + \sigma^2/2} \),

\[
\log E_t[X] = \log \delta + E_t[r_{i,t+1}] - E_t[r_{h,t+1}] + \beta_3 E_t[h_{t+1} - h_t] + \beta_4 E_t[c_{t+1} - c_t] \\
+ \frac{1}{2} \text{Var}_t[r_{i,t+1} - r_{h,t+1} + \beta_3(h_{t+1} - h_t) + \beta_4(c_{t+1} - c_t)].
\]  

(48)

Repeating these steps for (11), the consumption Euler equation, produces the log-linearized Euler equations, (13) and (14).

**When there is a risk of purchasing health.** Following Viceira (2001) the log-linear Euler approximation is derived using several Taylor approximations. This technique is required because the right-hand side is a sum rather than a product of expectations of lognormal variables. Notational shorthand transforms (20) into

\[ 1 = (1 - \pi_h) E_t[e^{x_{t+1}}] + \pi_h E_t[e^{y_{t+1}}]. \]  

(49)
Taking second-order Taylor expansions around $\bar{x}_{t+1}$ and $\bar{y}_{t+1}$, moving the constant multiples out from behind the expectations operator, distributing the expectation, and simplifying yields

$$1 \approx (1 - \pi_h) e^{\bar{z}_{t+1}} \left( 1 + \frac{1}{2} \text{Var}_t[x_{t+1}] \right) + \pi_h e^{\bar{y}_{t+1}} \left( 1 + \frac{1}{2} \text{Var}_t[y_{t+1}] \right). \quad (50)$$

A first-order expansion of $e^z \approx 1 + z$ can now be used, implying

$$1 \approx (1 - \pi_h) \left( 1 + \bar{x}_{t+1} + \frac{1}{2} \text{Var}_t[x_{t+1}] + \bar{x}_{t+1} \frac{1}{2} \text{Var}_t[x_{t+1}] \right) + \pi_h \left( 1 + \bar{y}_{t+1} + \frac{1}{2} \text{Var}_t[y_{t+1}] + \bar{y}_{t+1} \frac{1}{2} \text{Var}_t[y_{t+1}] \right). \quad (51)$$

If $\bar{x}_{t+1}$, $\bar{y}_{t+1}$ and the variance terms are small, then their products are second-order small and can be omitted, and 1 can be subtracted from both sides, yielding

$$0 \approx (1 - \pi_h) \left( \bar{x}_{t+1} + \frac{1}{2} \text{Var}_t[x_{t+1}] \right) + \pi_h \left( \bar{y}_{t+1} + \frac{1}{2} \text{Var}_t[y_{t+1}] \right). \quad (52)$$

Substituting for $x_{t+1}$ and $y_{t+1}$ and combining terms produces (21) in the text.

## C Proof of Proposition 1

**Euler differences.** The standard approach is to examine the difference between Euler conditions for the risky and for the risk-free asset. Subtracting the log-linear Euler equation for health (14) with $i = f$ from (14) with $i = 1$ yields, after cancellations and two expansions of variance terms:

$$0 = E_t[r_{1,t+1}] - r_f + \frac{1}{2} \sigma_t^2 + Cov_t[r_{1,t+1}, \beta_3 \Delta h_{t+1} + \beta_4 \Delta c_{t+1}]. \quad (53)$$

By extension, differencing the consumption log Euler condition (13) between $i = 1, f$ results in a second Euler difference:

$$0 = E_t[r_{1,t+1}] - r_f + \frac{1}{2} \sigma_c^2 + Cov_t[r_{1,t+1}, \beta_1 \Delta c_{t+1} + \beta_2 \Delta h_{t+1}]. \quad (54)$$
Solving for $\alpha$. Comparing (53) to (54), it is clear that if both Euler conditions hold, then there must be a relationship between the covariances:

$$\text{Cov}_t[r_{1,t+1}, \beta_3 \Delta h_{t+1} + \beta_4 \Delta c_{t+1}] = \text{Cov}_t[r_{1,t+1}, \beta_1 \Delta c_{t+1} + \beta_2 \Delta h_{t+1}].$$

(55)

Expanding terms and observing that $(\beta_1 - \beta_4)/(\beta_3 - \beta_2) = 1$ implies

$$\text{Cov}_t[r_{1,t+1}, \Delta c_{t+1}] = \text{Cov}_t[r_{1,t+1}, \Delta h_{t+1}].$$

(56)

Combining (56) with (54) after expanding the covariance term produces

$$E_t[r_{1,t+1}] - r_f + \frac{1}{2} \sigma_r^2 = -(\beta_1 + \beta_2) \text{Cov}_t[r_{1,t+1}, \Delta c_{t+1}].$$

(57)

Combining (9) and (10) with first (15) and then (16) implies

$$\text{Cov}_t[r_{1,t+1}, \Delta c_{t+1}] = b_{c,1}^u \alpha \sigma_r^2,$n

(58)

$$\text{Cov}_t[r_{1,t+1}, \Delta h_{t+1}] = b_{h,1}^u \alpha \sigma_r^2.$$n

(59)

But since (56) holds, it must be true that $b_{c,1}^u = b_{h,1}^u$. Rearranging (57) then implies that the optimal portfolio split is

$$\alpha_t = \frac{E_t[r_{1,t+1}] - r_f + \frac{1}{2} \sigma_r^2}{-(\beta_1 + \beta_2) b_{c,1}^u \sigma_r^2}.$$n

(60)

Solving for the rule parameters. To solve for $b_{c,1}^u = b_{h,1}^u$, first note that the rules (15) and (16) imply that

$$E_t[c_{t+1} - c_t] = b_{c,1}^u E_t[w_{t+1} - w_t].$$

(61)

$$E_t[h_{t+1} - h_t] + E_t[r_{h,t+1}] = b_{h,1}^u E_t[w_{t+1} - w_t].$$

(62)

Together, these two equations imply a relationship between expected consumption growth and expected health growth:

$$E_t[h_{t+1} - h_t] + E_t[r_{h,t+1}] = \frac{b_{h,1}^u}{b_{c,1}^u} E_t[c_{t+1} - c_t].$$

(63)

In light of (63), the log Euler equation for health (14) with $i = p$ implies

$$\log \delta + E_t[r_{p,t+1}] - E_t[r_{h,t+1}] + \left(\beta_3 \frac{b_{h,1}^u}{b_{c,1}^u} + \beta_4\right) E_t[c_{t+1} - c_t] - \beta_3 E_t[r_{h,t+1}]$$

$$+ \frac{1}{2} \text{Var}_t[r_{p,t+1} - r_{h,t+1} + \beta_3(h_{t+1} - h_t) + \beta_4(c_{t+1} - c_t)] = 0.$$n

(64)
while the log Euler equation for consumption (13) with \( i = p \) becomes

\[
\log \delta + E_t[r_{p,t+1}] + \left( \beta_1 + \beta_2 \frac{b_{h,1}^u}{b_{c,1}^u} \right) E_t[c_{t+1} - c_t] - \beta_2 E_t[r_{h,t+1}] \\
+ \frac{1}{2} Var_t[r_{p,t+1}] + \beta_1(c_{t+1} - c_t) + \beta_2(h_{t+1} - h_t) = 0. \tag{65}
\]

Combining the log-linear budget constraint and the choice rules implies

\[
E_t[\Delta w_{t+1}] = k - \rho_c b_{c,0} - \rho_h b_{h,0}^u + (\rho_c + \rho_h - \rho_c b_{c,1}^u - \rho_h b_{h,1}^u) w_t + E_t[r_{p,t+1}]. \tag{66}
\]

Substituting for \( E_t[c_{t+1} - c_t] \) in (64) using (61) and (66) and noting that \( \beta_3 + 1 = \beta_2 \) results in a single equation in \( w_t \):

\[
\mathcal{A}(-\log \delta - E_t[r_{p,t+1}] + \beta_2 E_t[r_{h,t+1}]) \\
- \frac{1}{2} \mathcal{A} Var_t[r_{p,t+1} - r_{h,t+1} + h_{t+1} - h_t] + \beta_4(c_{t+1} - c_t) \\
= k - \rho_c b_{c,0} - \rho_h b_{h,0}^u + (\rho_c + \rho_h - \rho_c b_{c,1}^u - \rho_h b_{h,1}^u) w_t + E_t[r_{p,t+1}], \tag{67}
\]

where \( \mathcal{A} = (1/b_{c,1}^u)/(\beta_3(b_{h,1}^u/b_{c,1}^u) + \beta_4) \). Since \( w_t \) cannot be constant, it follows that its coefficient, \( \rho_c(1 - b_{c,1}^u) + \rho_h(1 - b_{h,1}^u) = 0 \), is zero. As shown previously, \( b_{c,1}^u = b_{h,1}^u \). Since the \( \rho \)'s are both positive, the only solution is that \( b_{c,1}^u = b_{h,1}^u = 1 \).

There is currently only one equation, (67), in the two unknowns, \( b_{c,0}^u \) and \( b_{h,0}^u \). A second relationship is \( b_{c,0}^u = b_{h,0}^u[\psi/(1 - \psi)] \), which follows directly from the fact that preferences are Cobb-Douglas over health and consumption. Combining this with (67), accounting for \( b_{c,1}^u = b_{h,1}^u = 1 \), using the consumption rules and the log-linear budget constraint, and simplifying yields an equation for the target ratio of health spending to wealth:

\[
b_{h,0}^u = \frac{A}{B} (\log \delta - \beta_2 E_t[r_{h,t+1}]) + \frac{A + 1}{B} E_t[r_{p,t+1}] + \frac{k}{B} \\
+ \frac{A}{2B} (1 + \beta_3 + \beta_4)^2 Var_t[r_{p,t+1}] + \frac{A}{2B} \beta_2^2 Var_t[r_{h,t+1}], \tag{68}
\]

and an equation for the target consumption-wealth ratio:

\[
b_{c,0}^u = \frac{A}{C} (\log \delta - \beta_2 E_t[r_{h,t+1}]) + \frac{A + 1}{C} E_t[r_{p,t+1}] + \frac{k}{C} \\
+ \frac{A}{2C} (1 + \beta_3 + \beta_4)^2 Var_t[r_{p,t+1}] + \frac{A}{2C} \beta_2^2 Var_t[r_{h,t+1}], \tag{69}
\]
where \( A = (\beta_3 + \beta_4)^{-1} = -1/\gamma \), \( B = \rho_c \left( \psi_{1-2} \right) + \rho_h \), and \( C = \rho_c + \rho_h \left( \frac{1-\psi}{\psi} \right) \). Since \( \gamma > 0 \), \( A < 0 \), and since \( \psi, \rho_c, \rho_h > 0 \), \( B > 0 \) and \( C > 0 \). By inspection, \( b_{c,t}^h \) and \( b_{h,t}^u \) both fall with increased variability in \( r_{p,t+1} \) or \( r_{h,t+1} \), because \( A/2B \) and \( A/2C \) are both negative. These are precautionary saving effects: increases in background variance cause the individual to save more.

### D Proof of Proposition 2

**Euler differences.** As with unhealthy investors, the strategy is to obtain a relationship for the risk premium by differencing the log Euler equation, (21), through asset types 1 and \( f \). Since there is no variability in health growth, \( h_{t+1}^h - h_t^h \) drops out of the variance terms, as does \( r_f \), and expanding the variance terms produces a separate \( \sigma_r^2 \) term and more cancellations:

\[
0 = E_t[r_{1,t+1}] - r_f + \frac{1}{2} \sigma_r^2 + (1 - \pi_h) \text{Cov}_t[r_{1,t+1}, \beta_1(c_t^h - c_t^i)] + \pi_h \text{Cov}_t[r_{1,t+1}, \beta_1(c_t^h - c_t^i) + \beta_2(h_{t+1}^h - h_t^h)].
\]

(70)

The second covariance in (70) has already been solved in Appendix C. The first covariance can be found by rewriting (22):

\[
\beta_1^h = b_{c,1}^h (w_{t+1} - w_t).
\]

(71)

Combining (71) with (8) and (10) implies

\[
\text{Cov}_t[r_{1,t+1}, \beta_1(c_t^h - c_t^i)] = \beta_1^h b_{c,1}^h \alpha \sigma_r^2,
\]

(72)

and combining (72), \( b_{c,1}^h = 1 \), and (70) yields

\[
\alpha_t^h = \frac{E_t[r_{1,t+1}] - r_f + \frac{1}{2} \sigma_r^2}{(1 - \pi_h) \beta_1^h b_{c,1}^h + \pi_h (\beta_1 + \beta_2) \sigma_r^2}.
\]

(73)

**Finding the rule parameters.** The log-linear Euler approximation (21) for \( i = p \) can be combined with the three optimal rules (one for healthy investors, two for unhealthy investors) to produce

\[
0 = \log \delta + E_t[r_{p,t+1}] + (1 - \pi_h) \beta_1^h b_{c,1}^h E_t[\Delta w_{t+1}] + (1 - \pi_h) \beta_2 g
\]

\[
+ \frac{1 - \pi_h}{2} \text{Var}_t[r_{p,t+1} + \beta_1^h b_{c,1}^h \Delta w_{t+1} + \beta_2 g]
\]

\[
+ \pi_h \beta_1^h E_t[\Delta w_{t+1}] + \pi_h \beta_2 E_t[\Delta w_{t+1}]
\]

\[
+ \frac{\pi_h}{2} \text{Var}_t[r_{p,t+1} + \beta_1^h \Delta w_{t+1} + \beta_2 \Delta w_{t+1}],
\]

(74)
where \( b_{c,1}^h = b_{h,1}^h = 1 \) has been used. In the first two lines of (74), the single-good budget constraint (8) holds, and in the second two lines, the two-good budget constraint is relevant. Substituting for \( E_t[\Delta w_{t+1}] \), health spending, and consumption using the budget constraint and the three consumption rules, distributing the expectations operator, and simplifying the variance and \( w_t \) terms yields

\[
0 = \log \delta + E_t[r_{p,t+1}] + (1 - \pi_h)\beta_1 (b_{c,1}^h k^h - \rho_c^h (b_{c,0}^h + (b_{c,1}^h - 1) w_t) + E_t[r_{p,t+1}]) \\
+ (1 - \pi_h)\beta_2 g + \frac{1 - \pi_h}{2} (1 + \beta_1 b_{c,1}^h)^2 \text{Var}_t[r_{p,t+1}] \\
+ \pi_h (\beta_1 + \beta_2) (k - \rho_c b_{c,0}^u - \rho_h b_{h,0}^u + E_t[r_{p,t+1}]) \\
+ \frac{\pi_h}{2} (1 + \beta_1 + \beta_2)^2 \text{Var}_t[r_{p,t+1}] .
\]  

(75)

By inspection, (75) is a single equation in \( w_t \) and the fixed parameters. Since \( w_t \) cannot be constant, it follows that its coefficient, \( (1 - \pi_h)\beta_1 \rho_c^h (b_{c,1}^h - 1) \), must be zero. Since \( \pi_h \neq 1, \rho_c^h > 0, \) and \( \beta_1 = \psi(1 - \gamma) - 1 \neq 0 \) as long as \( \psi \neq 1/(1 - \gamma) \), the only solution is that \( b_{c,1}^h = 1 \).

To find \( b_{c,0}^h, b_{c,1}^h = 1 \) can be substituted into (75):

\[
b_{c,0}^h = \mathcal{D} \log \delta + \mathcal{D} (1 + \beta_1 + (1 - \pi_h)\beta_2) E_t[r_{p,t+1}] + \mathcal{D} (1 - \pi_h)\beta_1 k^h \\
+ \mathcal{D} (1 - \pi_h)\beta_2 g + \mathcal{D} \pi_h (\beta_1 + \beta_2) (k - \rho_c b_{c,0}^u - \rho_h b_{h,0}^u) \\
+ \mathcal{D} \left( \frac{\pi_h}{2} (1 + \beta_1 + \beta_2)^2 + \frac{1 - \pi_h}{2} (1 + \beta_1)^2 \right) \text{Var}_t[r_{p,t+1}] ,
\]  

(76)

where \( \mathcal{D} = 1/[(1 - \pi_h)\beta_1 \rho_c^h] \) \( < 0 \) since \( \beta_1 < 0 \).

Financial risk exerts two countervailing effects on \( b_{c,0}^h \). A rise in \( \sigma_r^2 \) decreases \( b_{c,0}^h \) and raises precautionary saving while healthy. But rising variance also lowers \( b_{c,0}^u \) and \( b_{h,0}^u \), increasing precautionary saving when unhealthy and lowering it when healthy. An increase in health inflation variance also lowers \( b_{c,0}^u \) and \( b_{h,0}^u \) but has no countervailing direct impact on \( b_{c,0}^h \). An increase in health inflation variance thus increases \( b_{c,0}^h \) and lowers precautionary saving when healthy, presumably because unhealthy investors save more.

The effect of health risk on saving, \( \partial b_{c,0}^h / \partial \pi_h \), is of indeterminate sign because it depends on the signs and relative sizes of \( k, k^h, b_{c,0}^u, \) and \( b_{h,0}^u \). The results of numerically solved models of precautionary saving (Hubbard, Skinner and Zeldes, 1994; Palumbo, 1999) suggest that it is likely to be negative for reasonable parameter values.
References


Table 1: Calibration of (23) using moments of financial data

<table>
<thead>
<tr>
<th>variable</th>
<th>mean</th>
<th>standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock returns, $r_{1,t+1}$</td>
<td>0.0601</td>
<td>0.1674</td>
</tr>
<tr>
<td>Risk-free returns, $r_f$</td>
<td>0.0183</td>
<td>0.0544</td>
</tr>
<tr>
<td>$r_{1,t+1} - r_f$</td>
<td>0.0418</td>
<td>0.1774</td>
</tr>
<tr>
<td>$(E_t[r_{1,t+1}] - r_f + \frac{1}{2}\sigma_r^2)/\sigma_r^2$</td>
<td>1.8254</td>
<td></td>
</tr>
<tr>
<td>$\alpha^y$: Working-age HH’s</td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td>$\alpha^o$: Old (70+) HH’s</td>
<td>0.175</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Rows 1–3 are taken from Table 8.1 in Campbell, Lo and MacKinlay (1997). Stock returns are measured as annual log real returns on the S&P 500 index since 1926 and a comparable series prior to 1926. Risk-free returns are annual log real returns on 6-month commercial paper bought in January and rolled over in July. Assuming $\sigma_r^2 = Var_t[r_{1,t+1} - r_f]$, row 4 constructs an expression used in (17) and (23). Rows 5–7 are average risky portfolio shares across households based on data in the HRS and AHEAD datasets.
Table 2: Tobit regression results: pooled AHEAD data

<table>
<thead>
<tr>
<th>variable</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>health risk, $\pi_h$</td>
<td>$-0.1826^*$</td>
<td>$-0.1675^*$</td>
<td>$-0.1703^*$</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>(0.0219)</td>
<td>(0.0219)</td>
<td>(0.0283)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>health status</td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0061)</td>
<td>(0.0062)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>education</td>
<td>0.0623*</td>
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<td>0.0586*</td>
<td>0.0566*</td>
<td>0.0613*</td>
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<tr>
<td></td>
<td>(0.0023)</td>
<td>(0.0024)</td>
<td>(0.0024)</td>
<td>(0.0024)</td>
<td>(0.0032)</td>
</tr>
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<td>$-0.0554^*$</td>
<td>$-0.0430^*$</td>
<td>$-0.0416^*$</td>
</tr>
<tr>
<td></td>
<td>(0.0135)</td>
<td>(0.0138)</td>
<td>(0.0135)</td>
<td>(0.0137)</td>
<td>(0.0184)</td>
</tr>
<tr>
<td>married</td>
<td>0.1747*</td>
<td>0.1663*</td>
<td>0.1751*</td>
<td>0.1666*</td>
<td>0.1394*</td>
</tr>
<tr>
<td></td>
<td>(0.0145)</td>
<td>(0.0148)</td>
<td>(0.0144)</td>
<td>(0.0147)</td>
<td>(0.0194)</td>
</tr>
<tr>
<td>kids</td>
<td>$-0.0103^*$</td>
<td>$-0.0100^*$</td>
<td>$-0.0106^*$</td>
<td>$-0.0104^*$</td>
<td>$-0.0095^*$</td>
</tr>
<tr>
<td></td>
<td>(0.0034)</td>
<td>(0.0035)</td>
<td>(0.0034)</td>
<td>(0.0035)</td>
<td>(0.0048)</td>
</tr>
<tr>
<td>black</td>
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<td>$-0.3447^*$</td>
<td>$-0.3382^*$</td>
<td>$-0.3282^*$</td>
<td>$-0.3293^*$</td>
</tr>
<tr>
<td></td>
<td>(0.0300)</td>
<td>(0.0303)</td>
<td>(0.0299)</td>
<td>(0.0303)</td>
<td>(0.0400)</td>
</tr>
<tr>
<td>Hispanic</td>
<td>$-0.2170^*$</td>
<td>$-0.2529^*$</td>
<td>$-0.2127^*$</td>
<td>$-0.2484^*$</td>
<td>$-0.2697^*$</td>
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<tr>
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<td>(0.0445)</td>
<td>(0.0465)</td>
<td>(0.0444)</td>
<td>(0.0465)</td>
<td>(0.0596)</td>
</tr>
<tr>
<td>age</td>
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<td>$-0.0082^*$</td>
<td>$-0.0085^*$</td>
<td>$-0.0057^*$</td>
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<tr>
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<td>(0.0013)</td>
<td>(0.0013)</td>
<td>(0.0013)</td>
<td>(0.0013)</td>
<td>(0.0018)</td>
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<tr>
<td>remaining years</td>
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<td>0.0013</td>
<td>0.0006</td>
<td>0.0030</td>
</tr>
<tr>
<td></td>
<td>(0.0012)</td>
<td>(0.0012)</td>
<td>(0.0012)</td>
<td>(0.0012)</td>
<td>(0.0017)</td>
</tr>
<tr>
<td>$E[\alpha]$</td>
<td>0.2170</td>
<td>0.2213</td>
<td>0.2171</td>
<td>0.2213</td>
<td>0.2444</td>
</tr>
<tr>
<td>obs</td>
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<td>9,095</td>
<td>9,822</td>
<td>9,092</td>
<td>4,478</td>
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<td>$\chi^2$-stat</td>
<td>1,665</td>
<td>1,604</td>
<td>1,742</td>
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<td>833</td>
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<tr>
<td>Prob &gt; $\chi^2$</td>
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<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Notes: Standard errors are in parentheses. Asterisks denote significance at the 5% level. Data are pooled observations of individuals in the 1993 and 1995 AHEAD surveys, except for column 5, which includes only the 1995 survey. The dependent variable is $\alpha$, the risky portfolio share. All regressions are Tobits, with limits $\alpha = 0$ and $\alpha = 1$, and all include a constant term and a time dummy. Health risk is the self-assessed probability that medical expenses will use up all household savings in the next five years, expressed as a fraction, 0 to 1. Health status is reported ordinally from 1 (excellent) to 5 (poor). Education is measured in years. Married, blank, and Hispanic are indicator variables. Kids is the number of children. Remaining years are the expected number of remaining years before death as inferred from the individual’s responses to future survivorship queries.
Table 3: Instrumental-variables Tobits: AHEAD data

<table>
<thead>
<tr>
<th>variable</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>health risk, $\pi_h$</td>
<td>-3.1732*</td>
<td>-2.6904*</td>
<td>-0.7606*</td>
<td>-0.6523*</td>
</tr>
<tr>
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<tr>
<td>education</td>
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<td>0.0320*</td>
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<tr>
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<td>(0.0041)</td>
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<td>0.0728</td>
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<td>-0.0242</td>
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<td>(0.0280)</td>
</tr>
<tr>
<td>married</td>
<td>0.1062*</td>
<td>0.1209*</td>
<td>0.1304*</td>
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<tr>
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<td>(0.0322)</td>
<td>(0.0368)</td>
<td>(0.0231)</td>
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<tr>
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<td>-0.0016</td>
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</tr>
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<td>-0.2986*</td>
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<tr>
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<td>(0.0879)</td>
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<tr>
<td>Hispanic</td>
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<td>-0.3427*</td>
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<td>(0.0944)</td>
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<td>-0.0098*</td>
<td>-0.0115*</td>
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<td>(0.0037)</td>
<td>(0.0064)</td>
<td>(0.0023)</td>
<td>(0.0028)</td>
</tr>
<tr>
<td>remaining years</td>
<td>-0.0167*</td>
<td>-0.0118</td>
<td>-0.0009</td>
<td>-0.0025</td>
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<td>(0.0058)</td>
<td>(0.0066)</td>
<td>(0.0021)</td>
<td>(0.0026)</td>
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</table>

$E[\alpha]$, obs, Dataset waves, Instrument set, # of instruments, $F$-stat, $R^2$, $\chi^2$-stat, overid, Prob $> \chi^2$, overid

Notes: Standard errors are in parentheses. Asterisks denote significance at the 5% level. Italics denote an instrumented variable. Data are at the individual level. All regressions include a constant term, and column 1 includes a time dummy. The dependent variable is $\alpha$, the risky portfolio share. All Tobits are specified with two limits, $\alpha = 0$ and $\alpha = 1$. Variables are described in the notes to Table 2, while instrument sets are listed on the following page. The bottom two rows display results from overidentification tests carried out on 2SLS analogues of the IV Tobit models, where high p-values represent failures to reject correct specification and exogenous instruments.
Table 4: Instrument sets in Table 3

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>memcnt: Number of words recalled by individual during interview&lt;br&gt;bodywt: Body weight in pounds&lt;br&gt;height: Height in inches&lt;br&gt;momdieage: Age of mother when she died&lt;br&gt;daddieage: Age of father when he died&lt;br&gt;smokever: 0/1 indicator of having ever smoked&lt;br&gt;smokenow: 0/1 indicator of smoking now</td>
</tr>
<tr>
<td>2</td>
<td>same as 1 except only for 1995</td>
</tr>
<tr>
<td>3</td>
<td>same as 2 with the addition of:</td>
</tr>
<tr>
<td></td>
<td>prhlth93: $\pi_h$ measured in 1993</td>
</tr>
<tr>
<td>4</td>
<td>same as 3 with the addition of:</td>
</tr>
<tr>
<td></td>
<td>diabetes: Has been told by a doctor that he/she has diabetes&lt;br&gt;insulin: Takes insulin injections&lt;br&gt;cancer: Has been told by a doctor that he/she has cancer&lt;br&gt;hrtattk: Has been told by a doctor that he/she had a heart attack&lt;br&gt;hospital: Has spent a night in a hospital in the last year&lt;br&gt;hlpbath: Has difficulty bathing / needs help&lt;br&gt;toilet: Has difficulty using the toilet / needs help&lt;br&gt;walkblks: Has difficulty walking several blocks</td>
</tr>
</tbody>
</table>
Table 5: Instrumental-variables Tobits with wealth: AHEAD data

<table>
<thead>
<tr>
<th>variable</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>real fin. wealth</td>
<td>0.0620</td>
<td>0.0695</td>
<td></td>
<td></td>
</tr>
<tr>
<td>log real fin. wealth</td>
<td>0.1757*</td>
<td>0.1599*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>health risk, πₜ</td>
<td>-0.4524*</td>
<td>-0.2164†</td>
<td></td>
<td></td>
</tr>
<tr>
<td>education</td>
<td>0.0363*</td>
<td>0.0212*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>female</td>
<td>-0.0186</td>
<td>-0.0066</td>
<td></td>
<td></td>
</tr>
<tr>
<td>married</td>
<td>0.0976*</td>
<td>0.0370</td>
<td></td>
<td></td>
</tr>
<tr>
<td>kids</td>
<td>0.0011</td>
<td>0.0008</td>
<td></td>
<td></td>
</tr>
<tr>
<td>black</td>
<td>-0.2569*</td>
<td>-0.0415</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hispanic</td>
<td>-0.3275*</td>
<td>-0.0751</td>
<td></td>
<td></td>
</tr>
<tr>
<td>age</td>
<td>-0.0123*</td>
<td>-0.0084*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>remaining years</td>
<td>0.0024</td>
<td>0.0017</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E[α]</td>
<td>0.2738</td>
<td>0.2612</td>
<td></td>
<td></td>
</tr>
<tr>
<td>obs</td>
<td>2,997</td>
<td>2,172</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dataset waves</td>
<td>95</td>
<td>95</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Instrument set</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td># of instruments</td>
<td>11</td>
<td>19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>First-stage F-stat on wealth</td>
<td>170.46</td>
<td>92.18</td>
<td>8.77</td>
<td>5.46</td>
</tr>
<tr>
<td>First-stage R² on wealth</td>
<td>0.52</td>
<td>0.54</td>
<td>0.05</td>
<td>0.06</td>
</tr>
<tr>
<td>First-stage F-stat on πₜ</td>
<td>23.43</td>
<td>12.79</td>
<td>22.40</td>
<td>12.70</td>
</tr>
<tr>
<td>First-stage R² on πₜ</td>
<td>0.13</td>
<td>0.14</td>
<td>0.12</td>
<td>0.14</td>
</tr>
<tr>
<td>χ²-stat, overid</td>
<td>8.20</td>
<td>14.67</td>
<td>7.77</td>
<td>16.98</td>
</tr>
<tr>
<td>Prob &gt; χ², overid</td>
<td>0.5144</td>
<td>0.6196</td>
<td>0.5570</td>
<td>0.4559</td>
</tr>
<tr>
<td>Exogeneity F-stat</td>
<td>5.70</td>
<td>2.26</td>
<td>25.76</td>
<td>38.44</td>
</tr>
<tr>
<td>Prob &gt; F</td>
<td>0.00</td>
<td>0.10</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Table 6: Notes and instrument sets in Table 5

**Notes:** Standard errors are in parentheses. Asterisks denote significance at the 5% level, while † denotes significance at the 10% level. Italics denote an instrumented variable. Data are at the individual level. All regressions include a constant term. The dependent variable is $\alpha$, the risky portfolio share. All Tobits are specified with two limits, $\alpha = 0$ and $\alpha = 1$. Financial wealth levels are deflated to 1991 levels using the CPI-U-RS. Real financial wealth is measured in millions. Other variables are described in the notes to Table 2.

**Instrument sets:**

<table>
<thead>
<tr>
<th>Instrument Set</th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>prhlth93 $\pi_h$ measured in 1993</td>
</tr>
<tr>
<td></td>
<td>lfwealth93 log real financial wealth measured in 1993</td>
</tr>
<tr>
<td></td>
<td>mecnct Number of words recalled by individual during interview</td>
</tr>
<tr>
<td></td>
<td>bodywt Body weight in pounds</td>
</tr>
<tr>
<td></td>
<td>height Height in inches</td>
</tr>
<tr>
<td></td>
<td>momdieage Age of mother when she died</td>
</tr>
<tr>
<td></td>
<td>daddieage Age of father when he died</td>
</tr>
<tr>
<td></td>
<td>smokever 0/1 indicator of having ever smoked</td>
</tr>
<tr>
<td></td>
<td>smokenow 0/1 indicator of smoking now</td>
</tr>
<tr>
<td>2</td>
<td>same as 1 with the addition of:</td>
</tr>
<tr>
<td></td>
<td>diabetes Has been told by a doctor that he/she has diabetes</td>
</tr>
<tr>
<td></td>
<td>insulin Takes insulin injections</td>
</tr>
<tr>
<td></td>
<td>cancer Has been told by a doctor that he/she has cancer</td>
</tr>
<tr>
<td></td>
<td>hrtattk Has been told by a doctor that he/she had a heart attack</td>
</tr>
<tr>
<td></td>
<td>hospital Has spent a night in a hospital in the last year</td>
</tr>
<tr>
<td></td>
<td>hlpbath Has difficulty bathing / needs help</td>
</tr>
<tr>
<td></td>
<td>toilet Has difficulty using the toilet / needs help</td>
</tr>
<tr>
<td></td>
<td>walkblks Has difficulty walking several blocks</td>
</tr>
<tr>
<td>3</td>
<td>same as 1 with the exception of:</td>
</tr>
<tr>
<td></td>
<td>fwealth93 real financial wealth measured in 1993 — added</td>
</tr>
<tr>
<td></td>
<td>lfwealth93 log real financial wealth measured in 1993 — dropped</td>
</tr>
<tr>
<td>4</td>
<td>same as 3 with the addition of 2</td>
</tr>
</tbody>
</table>
Figure 1: Age profiles of $\alpha$ in 5 HRS and 2 AHEAD waves

**Notes:** Data are from the 1992, 1994, 1996, 1998, and 2000 HRS waves, and the 1993 and 1995 AHEAD waves. Each age range is labeled with its endpoint. Risky portfolio shares are constructed as the ratio of risky financial assets to total financial assets. Total financial assets include IRA/Keogh accounts; stocks and stock mutual funds; checking, saving, and money market accounts; CD’s, government savings bonds, and Treasury bills; and corporate and other government bonds and bond funds. Risky financial assets are defined as the sum of the risky portion of IRA/Keogh accounts plus stocks and stock mutual funds. The risky portion of IRA/Keogh accounts is set at half of total IRA/Keogh balances except in the 1998 and 2000 HRS, when available data indicated a 60-40 split.
Figure 2: Age profiles of health risk, 5 HRS waves

Notes: Data are from 1992, 1994, 1996, 1998, and 2000 HRS waves, and they are the averaged answers to the question, “What do you think are the chances that your health will limit your work activity during the next 10 years?” Each age range is labeled with its endpoint.