A parametric representation of mortality differentials over age and time

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Abstract

The paper presents a new method for describing and analyzing mortality differentials over age and time. The model is non-linear and includes parameters for age, time, and any number of other covariates. All variables are in a discrete format. Required data consist of detailed tabulations of deaths and exposures-to-risk, contained in a pair of multi-dimensional arrays. Estimated parameters provide a rich description of mortality differentials over age and time. One part of the description shows the level and age pattern of such differentials. Another piece is the time trend of mortality change. A third part is the age pattern of mortality change, which may vary as a function of the covariates. The model provides a means of assessing the relative magnitude of mortality variation along various dimensions. It can also be used to study interactions between covariates. The paper includes an explanation of the model and several illustrations using Finnish data from 1971-1995.

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Introduction

Differences in the level and pattern of mortality across social groups are important indicators of social stratification. Nevertheless, the measurement and analysis of such differences pose considerable difficulties. The first problem is a general shortage of adequate data to permit accurate measurement of mortality differentials in most large populations. The second problem is a lack of powerful analytic tools to represent those differences in situations where the necessary data are available. In this paper, we address the latter issue and propose a parametric model for describing mortality differentials, illustrated using high-quality Finnish data.

Data

The data consist of detailed tabulations of deaths and exposures-to-risk in Finland during calendar years from 1971 to 1995. The tabulations were prepared by Statistics Finland using individual-level records in which census data had been linked to subsequent deaths (see Valkonen et al., 1990 and 1993). During this time period, censuses were taken every 5 years (in 1970, 1975, …, 1995). Thus, covariates for both deaths and exposures during each intercensal period are derived from the most recent census (unless data for a particular variable were unavailable in the most recent census, in which case data from an earlier census was used).

Only five covariates were available in the tabulations used for this analysis: sex (2), occupation (5), education (3), marital status (4), and region of residence (5).1 For each covariate, the number in parentheses describes the number of available categories. In addition to these five covariates, deaths and exposures were classified by age group (18) and calendar year (25),2 yielding two 7-dimensional arrays with 270,000 cells each. Of course, some of these cells were empty, but 204,335 cells contained non-zero exposures. These two arrays form the raw data available for this analysis. They contain a total of

1 Sex was classified as “male” or “female.” Occupational categories were “upper white-collar,” “lower white-collar,” “workers,” “farmers,” and “self-employed persons/others.” Education was grouped as “higher,” “secondary,” and “basic/unknown.” Marital status was given as “married,” “single,” “divorced/separated,” “widowed.” Regions of residence were “Eastern and Northern Finland, city,” “Eastern and Northern Finland, countryside,” “Western Finland, city,” “Western Finland, countryside,” and “Helsinki Metropolitan area.”

2 Age groups included in the tabulations were 15-19, …, 95-99, 100+. Calendar years were 1971, …, 1995.
96,540,871 person-years of exposure and 1,133,799 observed deaths. However, the analysis conducted here was limited (somewhat arbitrarily) to age groups between 30-34 and 85-89, both to avoid small sample sizes at older ages and because the meaning of educational and/or occupational categories is less clear at younger and older ages. In this restricted age range, there were nevertheless 68,037,201 person-years of exposure and 1,052,702 deaths over the full time period.

**Methods**

A detailed description of the method employed for this analysis is given in the Appendix. Readers who prefer a mathematical treatment may prefer to read the Appendix before the main text. Here, we give only a verbal description of the method and move directly to some illustrative results. The method consists of fitting a flexible parametric model to the arrays of deaths and exposures described above, using the method of maximum likelihood. Fitted parameters of the model can be used to describe three kinds of patterns in the data:

1) the level and age pattern of mortality by social category,

2) the time pattern of mortality change (which is held constant across social categories), and

3) the speed and age pattern of mortality change by social category.

In general, the first and third categories of these results are the most interesting. The second category describes the time pattern of mortality change, but it is constrained to be equal across all social categories. Although this choice may seem questionable, it has the effect of isolating differences in the speed and age pattern of mortality change in the third category of results. These comments will be illustrated shortly.

Another result contained in the fitted parameters is information about the relative contribution of different covariates to the variability of mortality in the population being studied. Such results may be interesting in general, as they illustrate the relative importance of different dimensions of mortality variation. They may also be used to study the interaction between covariates. For example, does one covariate become more or less important when another covariate is added to the model? These ideas are illustrated in the next section using the Finnish data described earlier.
Results

Figure 1 shows results of the method when applied to Finnish data for ages 30-34 through 85-89 using all available covariates (i.e., the “saturated” model). The first five graphs (1a through 1e) show the level and age pattern of mortality by social category. Each graph shows the variation in mortality across levels of a single factor. In each case, the reference year is set at 1983 and all other factors are fixed at their average values. Thus, Figure 1c shows the predicted level and age pattern of mortality in 1983 by educational category assuming an average level of all other factors (i.e., sex, occupation, marital status, and region of residence). In these five graphs, several expected relationships are confirmed: the lowest levels of mortality are found for female white-collar workers who are highly educated, married, and living in Western Finland. Visually, it appears that the largest mortality differentials occur as a function of sex or marital status, and that regional differences are relatively small.

The time pattern of mortality change is seen in Figure 1f. The graph shows the trend in an “index of mortality decline.” This index has an arbitrary level and slope, but it shows the time pattern of change. The main point that emerges here is that mortality change was relatively faster in the first half of the time period than in the second half. A period of stagnation from 1982 until 1985 is also visible. Much more noteworthy, however, are the results in Figure 1g through 1k. Here, we see the relative speed and age pattern of mortality decline by social category. Each of these graphs shows the average annual rate of mortality decline (over the full time period) by levels of a given factor when all other factors are set to their average level. The different age patterns of mortality decline for men and women are noteworthy, with mortality decline concentrated at younger ages among men than among women.

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3 We use “covariate” as a general term, although its meaning is sometimes imprecise. For clarity, we refer to a “factor” (e.g., education), which takes on different “levels” (high, secondary, and basic/other).
4 Here and throughout, “average” refers to a weighted average, where weights are based on the relative size of the social categories.
5 In all graphs shown here, the level of the time index is fixed so that it equals zero for 1983. Thus, 1983 is designated as the reference year. The slope of a line connecting the values of the index for the first and last years is fixed so that it equals negative one. The latter choice ensures that another set of parameters (those shown in Figures 1g through 1k) measure the average annual rate of mortality decline over the full time period.
When combined with Figures 1a through 1e, graphs 1g through 1k can be read for evidence of convergence or divergence in mortality levels by social category. In the case of educational attainment and marital status, a divergence of mortality levels is especially clear. In both cases, the most advantaged group (highly educated and married persons) also enjoyed the fastest rates of mortality decline over the time period. Mortality differentials by sex tended to converge at younger ages but to diverge at older ages. Among occupational categories, white-collar workers as a whole saw their mortality decline more rapidly than other groups (thus, a widening of mortality differentials), although the position of upper- and lower-white-collar workers in relation to each other stayed about the same overall. By region, there was a general tendency toward convergence of mortality levels between the eastern and western portions of the country. However, the Helsinki metropolitan level is making slower progress against mortality at older ages and thus diverging from the rest of the country.

The model considered here assumes, implicitly, that the effect of changing the level of one factor is the same across all levels of the other factors. In many cases, this assumption may be reasonable, but it is questionable especially in the case of sex differentials in mortality. As seen in Figure 1g, the age pattern of mortality change is quite different between men and women. This result, combined with prior knowledge about the different histories of mortality decline among men and women, motivates the analysis presented in Figure 2. Here, we show the results of applying the model with all available covariates (i.e., occupation, education, marital status, and region of residence) to men and women separately.

Although typical age patterns of mortality differentials by occupation, education, marital status, and residence are broadly similar for men and women, Figures 2a through 2h illustrate a few notable differences. For men, mortality is lowest among upper white-collar workers across the age range, whereas for women, lower white-collar workers claim this distinction in some age groups. For both sexes, married individuals enjoy a considerable mortality advantage. Although widowed persons are consistently in second position, the gap between married and widowed is noticeably larger for men than for women. The ordering of the single and divorced/separated categories is reversed for men and women,
although the difference between them is so small that this reversal seems insignificant. As for regional differences, the (slight) advantage of the western region appears to be stronger for men than for women. In Figure 2g, women in their 40s in the Helsinki metropolitan area stand out for having unusually high mortality levels.

Patterns of mortality change also display a mix of similarities and differences by sex. As seen in Figures 2i and 2j, men enjoyed a stable reduction in death rates over the entire time period, whereas women saw a rapid decline during the first half of the time period followed by a period of relative stagnation during the 1980s. The age patterns of mortality decline in Figures 2k through 2r reflect the general difference in the male and female age patterns of decline seen earlier in Figure 2g. However, they also illustrate differences by sex in the convergence or divergence of mortality differentials by social category. For example, the divergent mortality trends by occupational groups – in particular, the slower reductions in mortality among persons in the self-employed/other category – are more pronounced among men than among women. Figure 2n suggests an unusual result in the opposite direction, since men in the lowest education class at ages 45 and above are experiencing a mortality decline on par with those in higher categories. The divergence in mortality differentials by marital status is clearly much more pronounced for men than for women. Finally, Figures 2q and 2r demonstrate that the divergence of old-age mortality in the Helsinki metropolitan area, seen already in Figure 1k, is linked entirely to slower mortality reductions among women.

An alternative approach is to separate the analysis by time period. Thus, Figure 3 presents results derived by fitting the full model separately to 1971-1983 and 1983-1995. In fitting each of the models, 1983 was used as the reference year, and thus the predicted level and age pattern of mortality are quite similar in both cases. Likewise, the time pattern of mortality change is uninteresting, since both are constrained to have the same slope (see the Appendix for a technical explanation of this point). Therefore, we do not show these results in Figure 3. Rather, only the most significant results from this comparison, the age patterns of mortality decline by covariate for the two time periods, are shown in Figures 3a through 3j.
From these graphs, it is apparent that the major difference between the two time periods was a slower rate of mortality decline (and even mortality increase for some groups) at younger ages during the second time period. Furthermore, there are some notable changes over time in age patterns of mortality decline across some social categories. Although mortality decline was faster for men at younger ages during both time periods, the faster gains for women at older ages (seen earlier in Figure 1g) are limited mostly to the first time period. Therefore, the convergence of mortality levels by sex at younger ages has continued throughout the time period investigated here, whereas the pronounced divergence by sex at older ages seems to have come to an end.

Two notable trends affecting persons below age 45 are visible in Figures 3f and 3h. First, we observe a very large divergence by educational level at these ages in the second time period. Although the ordering of the groups is the same as before, the rates of mortality change are separated by a much wider gap due to an unusual pattern of mortality increase among those in the lowest educational category. Second, in a complete reversal of earlier trends, we observe a slight convergence in mortality differentials by marital status at younger ages in the latter time period. Single persons, who typically experienced the slowest pace of mortality reduction over the full period considered here (see Figure 1j), led the decline below age 45 during 1983-1995.

Aside from the few differences noted here, the ordering of social categories in terms of the speed of mortality decline across the age range is quite similar between the two time periods, or else we cannot discern clear differences due to the effects of random variation. Indeed, as we separate the available data into smaller and smaller groups, differences that seemed clear in the analysis of the full data set become less and less obvious. We also tried fitting the model separately for men and women in the two time periods, but the results became even more difficult to interpret and seemed to offer no additional insights beyond those noted already.

To complement these visual displays, we have also used the new method to quantify the magnitude of mortality variation along various dimensions, by computing the weighted sums of squares in the fitted model (see Appendix for a technical explanation). Thus, Table 1 shows the weighted sums of
squares for the model with all covariates. It is apparent that age is, of course, the most important
dimension of mortality variation. Therefore, it is better to compare the other factors after removing the
influence of age. The third column of Table 1 confirms our visual impression of Figure 1, in that sex and
marital status are clearly the most important covariates included in the model. The weight of regional
differences is almost negligible, while educational attainment and occupation play an intermediate role.\(^6\)
The table also demonstrates that mortality change over this 25-year time period accounts for less of the
overall variability than do all other factors considered here, except for region of residence.

The model can also be used to assess the interaction between covariates when one factor is added
to the analysis. For example, Table 2 shows the distribution of weighted sums of squares for four models
containing age, time, and up to three covariates (sex, occupation, and education). Model I contains only
sex as a covariate. When occupation is added (model II), the relative importance of sex as a covariate
decreases. Thus, a portion of the sex differential in mortality is explained by the fact that women were
more likely to be employed in white-collar jobs across all age groups. On the other hand, adding
education (model III) increases the proportion of variability explained by sex, since Finnish men had
higher educational levels for all age groups above age 35 during the time period considered here. Finally,
when occupation is added to a model that contains sex and education (i.e., models III vs. IV), the
proportion of variability explained by education is reduced to less than half its earlier value. Likewise,
adding education as the third covariate (models II vs. IV) diminishes the explanatory power of occupation
considerably.

Table 3 shows results similar to those in Table 1, although here the model has been applied to
women and men separately. Among women, about 94 percent of all mortality variation is due to age, but
for men this proportion is somewhat smaller (around 88 percent). This difference reflects the greater
variability of male mortality along other dimensions, especially marital status.

\(^6\) In comparing the graphic and tabular results, it is important to keep in mind that the tabular results also take
account of differences in the size of the various groups. For example, occupational differences in the graph appear
larger than they really are, since the extreme category (self-employed persons and all others) is a relatively small
group (less than 5 percent of the exposure-to-risk in the present study).
Conclusion

The model presented here seems to offer a powerful tool for describing and analyzing mortality differentials. It will not necessarily replace existing methods, but it can be used in conjunction with other approaches. The main advantage of this method is that it offers a detailed and comprehensive description of mortality differentials. One disadvantage is that it may prove to be difficult to interpret for non-specialists. A related issue is that all differentials – as well as changes in those differentials – are expressed in terms of the relative level of age-specific mortality rates. In its present form, the model cannot be used to analyze differences in the absolute level of mortality or in other aggregate measures, such as life expectancy at birth. In spite of these limitations, we believe that it is a promising method and deserves further consideration.

References


Appendix

The model proposed here is a generalization of the model proposed by Lee and Carter (1992) for use in mortality forecasting. It is, in effect, the Lee-Carter model with covariates. Formally, the model can be written as follows:

\[ m_{ijk\ldots k_p} = \exp \left\{ \beta + \beta_{k_1}^{(i)} + \ldots + \beta_{k_p}^{(p)} + \alpha_j + \alpha_{ik_1}^{(i)} + \ldots + \alpha_{ik_p}^{(p)} + \left( \gamma_i + \gamma_{ik_1}^{(i)} + \ldots + \gamma_{ik_p}^{(p)} \right) \delta_j \right\}. \]

In this notation, \( i \) indexes age and \( i = 1, \ldots, I \). Likewise, \( j \) indexes time and \( j = 1, \ldots, J \). There are \( p \) groups of covariates, or \( p \) factors, and the \( n \)th factor is indexed by \( k_n = 1, \ldots, K_n \) (in the discussion that follows, we drop the subscript on \( k \) when the meaning is clear). The overall level of mortality (in a logarithmic scale) is given by \( \beta \), with an adjustment of \( \beta_{k}^{(n)} \) for the \( k \)th category of the \( n \)th factor. The typical age pattern of mortality is given by \( \alpha_i \), with an adjustment of \( \alpha_{ik}^{(n)} \) for the \( k \)th category of the \( n \)th factor. Similarly, the age pattern of mortality decline is given by \( \gamma_i \), with an adjustment of \( \gamma_{ik}^{(n)} \) for the \( k \)th category of the \( n \)th factor. Finally, the time pattern of mortality change is given by \( \delta_j \).

In a logarithmic scale, the first portion of the model (the \( \alpha \)'s and \( \beta \)'s) is additive and relatively easy to understand. It implies merely that the level and age pattern of mortality may vary in an additive fashion across categories of the various factors. The second portion of the model (the \( \gamma \)'s and \( \delta \)'s) is multiplicative and has a less obvious interpretation. It implies that mortality change follows the same temporal pattern for all combinations of covariates, but that the average speed and age pattern of mortality change is allowed to differ without restriction across categories of the various factors. In other words, if mortality goes up or down in a given year for one group in the population, the model assumes that it goes up or down for all groups in the population. However, it may go up or down more or less slowly for each possible combination of covariates and age groups. It is even possible that mortality consistently goes up for one group but down for another, and vice versa, although a more common situation is that mortality trends move in the same direction but at different speeds for various combinations of age and covariates.
The model is fit by the method of maximum likelihood assuming a Poisson probability model. In other words, the death count in a given cell, \( D_{ijk\ldots} \), is assumed to follow a Poisson distribution with an expected value of \( E_{ijk\ldots} \cdot m_{ijk\ldots} \). It can be shown that the Poisson log-likelihood has the following form:

\[
\ell = \sum_{ijk\ldots} \left( D_{ijk\ldots} \ln(m_{ijk\ldots}) - E_{ijk\ldots} \cdot m_{ijk\ldots} \right) + \text{other terms},
\]

where the “other terms” do not depend on the unknown parameters (and thus can be ignored in the maximization procedure). The task is to find values of the parameters in the formula for \( m_{ijk\ldots} \) to maximize this equation, given an observed set of deaths and exposures.

In order to obtain a unique solution, the likelihood must be maximized subject to certain constraints. The simplest set of constraints, and the one used in fitting the model, is as follows:

1) \( \sum_k \beta_k^{(n)} = 0 \) for \( n = 1, \ldots, p \);

2) \( \sum_i \alpha_i = \sum_i \alpha_i^{(n)} = 0 \) for \( n = 1, \ldots, p \) and \( k = 1, \ldots, K_n \);

3) \( \sum_k \alpha_k^{(n)} = \sum_k \gamma_k^{(n)} = 0 \) for \( n = 1, \ldots, p \) and \( i = 1, \ldots, l \);

4) \( \sum_j \delta_j = 0 \); and

5) \( \sum_j \delta_j^2 = 1 \).

However, other constraints are possible and may be preferable for various reasons. For example, for presentation purposes, we use the following substitutes:

4) \( \delta_{j'} = 0 \) where \( j' \) is a designated reference year; and

5) \( \frac{\delta_j - \delta_{j'}}{J-1} = -1 \).
This version of constraint 4) ensures that predicted mortality levels and age patterns refer to a specific reference year (otherwise, they refer to an average mortality level over the time period). The alternative version of constraint 5) ensures that the $\gamma$ parameters are estimates of the average annual rate of mortality decline (otherwise, they express the relative pace of mortality decline across age and covariates, but it is difficult to state precisely the scale of these changes). In general, we fit the model using the first set of constraints and then modify the parameters so that they satisfy an alternative set of constraints, if desired.

Yet a third possibility for the constraints is to introduce weights as follows:

1) $\sum_k w_k^{(n)} \beta_k^{(n)} = 0$ for $n = 1, \ldots, p$;

2) $\sum_i w_i \alpha_i = \sum_i w_i \alpha_i^{(n)} = 0$ for $n = 1, \ldots, p$ and $k = 1, \ldots, K_n$;

3) $\sum_k w_k^{(n)} \alpha_k^{(n)} = \sum_k w_k^{(n)} \gamma_k^{(n)} = 0$ for $n = 1, \ldots, p$ and $i = 1, \ldots, I$;

4) $\sum_j w_j \delta_j = 0$; and

5) $\sum_j w_j \delta_j^2 = 1$.

The weights, $w_i$, $w_j$, and $w_k^{(n)}$, for $n = 1, \ldots, p$, are proportional to the marginal sums of exposure-to-risk along a given dimension. In addition, the weights are scaled so that the total weight for each factor (including age and time) is one. The advantage to using this set of constraints is that the associated parameter estimates can be used in computing weighted sums of squares, which are an intuitively plausible measure of the relative importance of the various factors in the model (see below).

We first consider the calculation of sums of squares without weights. Begin by defining $K^{(\cdot-n)} = K_1 \cdots K_{n-1} K_{n+1} \cdots K_p$, that is, the product of the number of categories in each factor except the $n^{th}$ one. The unweighted sums of squares are defined as follows:
It can be shown that:

\[ SSTotal = SSAge + SSTime + SSFac^{(1)} + \cdots + SSFac^{(p)} \]

These unweighted sums of squares offer one means of quantifying the relative importance of each factor (including age and time) in the model. The three terms of \( SSFac^{(n)} \) represent the contribution to the total variability of the \( n \)th set of covariates: the first term gives the main effect, while the second and third term express the contribution of the factor in interaction with age and time. This formulation cannot be considered the preferred solution, however, because it gives equal weight to each category within a factor, regardless of the size of the population involved.

Weighted sums of squares correct this deficiency. They are defined as follows:

\[ WSSTotal = \sum_{i,j,k,...,p} w_{ijk,...,p} (y_{ijk,...,p} - \hat{\beta})^2 \]
\[ WSSAge = \sum_i w_i \hat{\alpha}_i^2 \]
\[ WSSTime = \sum_i w_i \hat{\tau}_i^2 \]
\[ WSSFac^{(n)} = \sum_k w_k^{(n)} (\hat{\beta}_k^{(n)})^2 + \sum_{ik} w_{ik}^{(n)} (\hat{\alpha}_{ik}^{(n)})^2 + \sum_{ik} w_{ik}^{(n)} (\hat{\gamma}_{ik}^{(n)})^2 \]

As before, we define weights, \( w_i \), \( w_j \), and \( w_k^{(n)} \), for \( n = 1, \ldots, p \), to be proportional to the marginal sums of exposure-to-risk along a given dimension, scales so that the total weight for each factor (including age and time) is one. If, in addition, we require that \( w_{ijk,...,p} = w_i w_j w_k^{(1)} \cdots w_p^{(p)} \) and \( w_{ik}^{(n)} = w_i w_k^{(n)} \), and if all parameters satisfy the constraints based on weights (as defined earlier), then it can be shown that:

\[ WSSTotal = WSSAge + WSSTime + WSSFac^{(1)} + \cdots + WSSFac^{(p)} \]
Thus, the weighted sums of squares partition the total variability into components attributable to age, time, and covariates. The first two terms of each covariate contribution are related to the variability in the level and age pattern of mortality, while the third term is related to the variability in the age pattern of mortality change.

Suppose we wish to quantify the magnitude of mortality variation across different categories of the \( n \)th factor. The predicted level of mortality at age \( i \) for category \( k \) is as follows:

\[
A_{ik}^{(n)} = \beta + \beta_{k}^{(n)} + \alpha_i + \alpha_{ik}^{(n)}.
\]

At age \( i \) the weighted average value of \( A_{ik}^{(n)} \) across categories is \( \overline{A}_i^{(n)} = \beta + \alpha_i \). Thus, the weighted mean square error equals at age \( i \) equals:

\[
\sum_{k} w_k (A_{ik}^{(n)} - \overline{A}_i^{(n)})^2 = \sum_{k} w_k (\beta_{k}^{(n)})^2 + 2 \sum_{k} w_k \beta_{k}^{(n)} \alpha_{ik}^{(n)} + \sum_{k} w_k (\alpha_{ik}^{(n)})^2.
\]

If we compute the weighed average of these values across age, the cross-product disappears:

\[
\sum_{i} w_i \sum_{k} w_k (A_{ik}^{(n)} - \overline{A}_i^{(n)})^2 = \sum_{k} w_k (\beta_{k}^{(n)})^2 + \sum_{k} w_k (\alpha_{ik}^{(n)})^2.
\]

This result equals the first two terms of \( WSSFac^{(n)} \). A similar argument to obtain the third term of \( WSSFac^{(n)} \) begins by defining \( B_{ik}^{(n)} = \gamma_j + \gamma_{ik}^{(n)} \). It is easy to show that

\[
\sum_{i} w_i \sum_{k} w_k (B_{ik}^{(n)} - \overline{B}_j^{(n)})^2 = \sum_{k} w_k (\gamma_{ik}^{(n)})^2.
\]

Thus, the first two terms of \( WSSFac^{(n)} \) equal the average mean square error of the mortality level across categories of the \( n \)th factor, whereas the third term equals the average mean square error of the rate of mortality change.
Table 1 – Contribution of age, time, and five covariates to total variability in saturated model, Finland 1971-1995

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<th>Weighted sums of squares</th>
<th>Percent of total</th>
<th>Percent without age</th>
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<tr>
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<tr>
<td><strong>Residence total</strong></td>
<td><strong>0.007</strong></td>
<td><strong>0.32</strong></td>
<td><strong>1.8</strong></td>
</tr>
<tr>
<td><strong>Grand total</strong></td>
<td><strong>2.168</strong></td>
<td><strong>100.00</strong></td>
<td><strong>100.0</strong></td>
</tr>
</tbody>
</table>
Table 2 – Distribution (in percent) of weighted sums of squares in four models, Finland 1971-1995

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Age</strong></td>
<td>87.86</td>
<td>86.11</td>
<td>84.56</td>
<td>85.51</td>
</tr>
<tr>
<td><strong>Time</strong></td>
<td>0.76</td>
<td>0.65</td>
<td>0.54</td>
<td>0.58</td>
</tr>
<tr>
<td>Sex Main</td>
<td>10.94</td>
<td>9.70</td>
<td>11.49</td>
<td>10.42</td>
</tr>
<tr>
<td>Sex x Age</td>
<td>0.42</td>
<td>0.31</td>
<td>0.43</td>
<td>0.34</td>
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<tr>
<td>Sex x Time</td>
<td>0.02</td>
<td>0.04</td>
<td>0.03</td>
<td>0.04</td>
</tr>
<tr>
<td><strong>Sex Total</strong></td>
<td><strong>11.39</strong></td>
<td><strong>10.04</strong></td>
<td><strong>11.95</strong></td>
<td><strong>10.80</strong></td>
</tr>
<tr>
<td>Occupation Main</td>
<td>--</td>
<td>2.60</td>
<td>--</td>
<td>1.41</td>
</tr>
<tr>
<td>Occupation x Age</td>
<td>--</td>
<td>0.55</td>
<td>--</td>
<td>0.35</td>
</tr>
<tr>
<td>Occupation x Time</td>
<td>--</td>
<td>0.05</td>
<td>--</td>
<td>0.04</td>
</tr>
<tr>
<td><strong>Occupation Total</strong></td>
<td>--</td>
<td><strong>3.20</strong></td>
<td>--</td>
<td><strong>1.79</strong></td>
</tr>
<tr>
<td>Education Main</td>
<td>--</td>
<td>--</td>
<td>2.52</td>
<td>1.07</td>
</tr>
<tr>
<td>Education x Age</td>
<td>--</td>
<td>--</td>
<td>0.40</td>
<td>0.22</td>
</tr>
<tr>
<td>Education x Time</td>
<td>--</td>
<td>--</td>
<td>0.03</td>
<td>0.04</td>
</tr>
<tr>
<td><strong>Education Total</strong></td>
<td>--</td>
<td>--</td>
<td><strong>2.95</strong></td>
<td><strong>1.32</strong></td>
</tr>
<tr>
<td><strong>Grand total</strong></td>
<td><strong>100.00</strong></td>
<td><strong>100.00</strong></td>
<td><strong>100.00</strong></td>
<td><strong>100.00</strong></td>
</tr>
</tbody>
</table>
Table 3 – Contribution of age, time, and four covariates to total variability in sex-specific models, Finland 1971-1995

<table>
<thead>
<tr>
<th></th>
<th>Weighted sums of squares</th>
<th>Women</th>
<th>Percent of total</th>
<th>Percent without age</th>
<th>Men</th>
<th>Percent of total</th>
<th>Percent without age</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>2.038</td>
<td>94.01</td>
<td>--</td>
<td>1.543</td>
<td>88.11</td>
<td>--</td>
<td>1.751</td>
</tr>
<tr>
<td>Time</td>
<td>0.012</td>
<td>0.56</td>
<td>9.4</td>
<td>0.027</td>
<td>1.52</td>
<td>12.8</td>
<td>0.025</td>
</tr>
<tr>
<td>Occupation main</td>
<td>0.021</td>
<td>0.96</td>
<td>15.9</td>
<td>0.021</td>
<td>1.18</td>
<td>9.9</td>
<td>0.020</td>
</tr>
<tr>
<td>Occupation x age</td>
<td>0.006</td>
<td>0.26</td>
<td>4.4</td>
<td>0.004</td>
<td>0.25</td>
<td>2.1</td>
<td>0.004</td>
</tr>
<tr>
<td>Occupation x time</td>
<td>0.000</td>
<td>0.01</td>
<td>0.1</td>
<td>0.001</td>
<td>0.05</td>
<td>0.4</td>
<td>0.000</td>
</tr>
<tr>
<td>Occupation total</td>
<td>0.027</td>
<td>1.23</td>
<td>20.5</td>
<td>0.026</td>
<td>1.48</td>
<td>12.5</td>
<td>0.024</td>
</tr>
<tr>
<td>Education main</td>
<td>0.021</td>
<td>0.95</td>
<td>15.8</td>
<td>0.020</td>
<td>1.12</td>
<td>9.4</td>
<td>0.020</td>
</tr>
<tr>
<td>Education x age</td>
<td>0.003</td>
<td>0.15</td>
<td>2.5</td>
<td>0.004</td>
<td>0.25</td>
<td>2.1</td>
<td>0.003</td>
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<tr>
<td>Education x time</td>
<td>0.000</td>
<td>0.01</td>
<td>0.2</td>
<td>0.000</td>
<td>0.03</td>
<td>0.2</td>
<td>0.000</td>
</tr>
<tr>
<td>Education total</td>
<td>0.024</td>
<td>1.10</td>
<td>18.4</td>
<td>0.025</td>
<td>1.40</td>
<td>11.8</td>
<td>0.024</td>
</tr>
<tr>
<td>Marital main</td>
<td>0.050</td>
<td>2.32</td>
<td>38.6</td>
<td>0.108</td>
<td>6.17</td>
<td>51.9</td>
<td>0.108</td>
</tr>
<tr>
<td>Marital x age</td>
<td>0.012</td>
<td>0.55</td>
<td>9.2</td>
<td>0.012</td>
<td>0.69</td>
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<td>0.01</td>
<td>0.2</td>
<td>0.002</td>
<td>0.11</td>
<td>0.9</td>
<td>0.002</td>
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<tr>
<td>Marital total</td>
<td>0.062</td>
<td>2.88</td>
<td>48.0</td>
<td>0.122</td>
<td>6.97</td>
<td>58.6</td>
<td>0.121</td>
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<tr>
<td>Residence main</td>
<td>0.003</td>
<td>0.15</td>
<td>2.5</td>
<td>0.008</td>
<td>0.45</td>
<td>3.7</td>
<td>0.008</td>
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<td>Residence x age</td>
<td>0.001</td>
<td>0.06</td>
<td>1.0</td>
<td>0.001</td>
<td>0.05</td>
<td>0.5</td>
<td>0.001</td>
</tr>
<tr>
<td>Residence x time</td>
<td>0.000</td>
<td>0.01</td>
<td>0.2</td>
<td>0.000</td>
<td>0.01</td>
<td>0.1</td>
<td>0.000</td>
</tr>
<tr>
<td>Residence total</td>
<td>0.005</td>
<td>0.22</td>
<td>3.7</td>
<td>0.009</td>
<td>0.51</td>
<td>4.3</td>
<td>0.005</td>
</tr>
<tr>
<td><strong>Grand total</strong></td>
<td><strong>2.168</strong></td>
<td><strong>100.00</strong></td>
<td><strong>100.0</strong></td>
<td><strong>1.751</strong></td>
<td><strong>100.00</strong></td>
<td><strong>100.0</strong></td>
<td><strong>100.00</strong></td>
</tr>
</tbody>
</table>
Figure 1 – Parameter estimates for saturated model, Finland 1971-1995, ages 30-34 to 85-99.

Note: Reference year = 1983.
Note: Reference year = 1983.
Figure 2 – Parameter estimates for saturated models by sex, Finland 1971-1995, ages 30-34 to 85-89.

Note: Reference year = 1983 for both women and men.
Figure 2 (cont.)

**g)** Death rates by age and residence, Women

**h)** Death rates by age and residence, Men

**i)** Index of mortality decline, Women

**j)** Index of mortality decline, Men

**k)** Rate of mortality decline by age and occupation, Women

**l)** Rate of mortality decline by age and occupation, Men

Note: Reference year = 1983 for both women and men.
Figure 2 (cont.)

m) Rate of mort. decline by age and education, Women

n) Rate of mortality decline by age and education, Men

o) Rate of mort. decl. by age and marital status, Women

p) Rate of mortality decline by age and marital status, Men

q) Rate of mort. decline by age and residence, Women

r) Rate of mortality decline by age and residence, Men

Note: Reference year = 1983 for both women and men.
Figure 3 – Parameter estimates for saturated model by time period, Finland, ages 30-34 to 85-89.

Note: Reference year = 1983 for both time periods.
Figure 3 (cont.)

Note: Reference year = 1983 for both time periods.