

An expert-based stochastic population forecast for Alaska, using autoregressive models with random coefficients (working paper*)

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Abstract

This paper presents a stochastic population forecast for Alaska by age and sex, 2005-2035. The forecast uses expert knowledge to select distributions of autoregressive model coefficients for the components of population change. The chance of large-scale disturbances to net migration is added. An Internet link to the R code used to create the forecast is given in Section 6.2.

1 Introduction and overview

Stochastic population forecasts most often make use of the cohort-component method (described in Keyfitz 1977) and the range and pattern of potential future paths for each of the components of population change (fertility, mortality and migration), to estimate uncertainty about future population. The more realistic the range and pattern of potential future paths for the components, the better the forecast. Time series modeling is a direct approach for creating realistic potential futures. (Booth 2006, Lee 1999.)

The first order autoregressive model (AR1) is a broadly useful time series model for forecasting each of the components of change. It is described with the following formula:

$$X_t = \varphi X_{t-1} + c + \varepsilon$$

Where X_t is an index value (such as the total fertility rate for fertility) at time period t ; φ is the coefficient that describes the correlation between two consecutive values

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of the index (serial correlation); c , along with φ , specifies the long-term trend and mean of the index; and ε describes random noise, which is the change between two consecutive index values that is not known from the model.

The coefficients explained above are often estimated based on historical time series data and statistical methods for finding a best fit and error. A barrier that has kept stochastic forecasting from broader use is the frequent absence of historical data to construct reasonable time series models from. It is also noted that, where areas have a long and reliable historical series of data, information from history may be inadequate for predicting the coefficients for the forecast period. (Booth 2006.)

The approach used here is to construct a range of plausible AR1 models for each component based on expert judgment, and forecast population from those. The steps in constructing these AR1 models are (1) Specify a range for φ based on knowledge of other forecasts for the component index, and/or elimination of unreasonable values, (2) Specify c or a collection or range of c 's, based on expert judgment of the long-term trend for the component index, and (3) Specify ε based on judgment of short-term error for the index. To include the chance of rare, major disturbances to migration, a probability and net level of such events is selected, and resultant simulated events are added to simulated time series forecasts for net migration. This strategy for inclusion of additional error to migration follows the one used by Lee and Tuljapurkar (1998) for mortality.

This paper proceeds with a description of each component forecast for Alaska, in five-year steps for years 2005-2035. Brief description of the application of the component forecasts with Monte Carlo-style simulation is then provided. Forecast population data, a conclusion, and sources including a link to the R statistical computing code used in this work, are given in respective order at the end.

Throughout this work, the period 2005-2035 is referred to as the “forecast period.” The five-year forecast steps, with data representing five-year averages for each respective step, are referred to as the “forecast steps.” All subjective judgments are the author’s.

2 Component forecasts

2.1 Fertility

Fertility is forecast for the 2005-10 through 2030-35 forecast steps using the sum of age specific fertility rates, known as the total fertility rate (TFR), as the index value. As shown in Figure 1, five-year averages of Alaska’s TFR between 1970 and 2005 have been above 2.0, with range from 2.16 in 1980-85, to 2.58 in 1970-75.

The first step for constructing expert-based AR1 models for the TFR is the specification of a range for φ . Subsequent to time series analysis by Lee (1974 and 1993), single-year TFR’s, or the logarithms of these, have been formulated with AR1 processes in several works, thus there are several existing models to take knowledge

from (Hartmann and Strandell 2006, and Hunsinger and Williams 2007 were reviewed for this work). A high φ that has been considered for forecasting TFR in single-year increments is 1 (a “random walk”) (Lee 1974, Hartmann and Strandell 2006), and a low φ is .88 (Hunsinger and Williams 2007). Based on estimation by simulation of the AR1 model from Hunsinger and Williams (low bound), and a random walk (high bound), five-year average TFR’s are judged to have a positive serial correlation, with φ between approximately .5 and 1. A simple uniform distribution is used to model the selected range of φ .

The second step for AR1 model construction is the specification of the level term, c . Two scenarios for the long-term level of Alaska’s TFR are considered likely enough to include: (1) That the overall level of the TFR will remain little changed from recent years, in line with the TFR projection of 2.3 by the Alaska Department of Labor and Workforce Development (Hunsinger and Williams 2007), and (2) That fertility will fall to a long-term mean of 2.1, in line with TFR projections for the United States (Ortman and Guarneri 2009). The probability of the second scenario is considered 1/3, so it will be used to make the level term for 1/3 of the forecast simulations. It may be argued that using a range for c would be more probabilistically reasonable than use of distinct scenarios with attached probabilities, and perhaps a range could be used in this case by interpolating between a low that has half the frequency of the high, but use of a range here would probably not change the results much at all, and in other cases it could discourage flexibility in setting a long-term level for the time series that directly reflects expert knowledge.

The third step is specification of short-term error, ε . Using the author’s own expert-based forecast of uncertainty for five years forward, which assumes no knowledge of that period, the AR1 residual error (ε) for Alaska’s TFR is predicted to be normally distributed, with mean 0 and standard error .243. Accordingly, there is no more than a 10 percent chance that the TFR will diverge by more than .4 from the forecast level between any two forecast steps, due to random error. This carries the assumption that residual error on the TFR is well-chosen by the expert forecaster, and that it will remain unchanged from one forecast step to the next. If it were decided that uncertainty about ε for five-year intervals is not well described by a fixed value (perhaps due to multiple experts), a range of ε , or multiple ε ’s, could be determined to sample from, and readily used in simulation. Simply using a range or collection of ε that would be fixed over the forecast period is distinct from allowing ε to vary over time, which is an option that has been carried out by Lee (1993), and Keilman and Pham (2004), and could also be used here.

The following formulas describe the forecast for Alaska’s TFR (X):

$$\begin{aligned}
 X_{i,t} &= \varphi_i X_{i,t-1} + c_i(1 - \varphi_i) + N(0, .243)_{i,t} \\
 \varphi_i &= U(.5, 1)_i \\
 c_i &= S(2.1, 2.3)_i \\
 P(c_i = 2.1) &= 1/3; P(c_i = 2.3) = 2/3
 \end{aligned}$$

Figure 1 here

2.2 Migration

For the 2005-10 through 2030-35 forecast steps, migration is forecast using the ratio of five-year net migration to the total, initial population for each forecast step as the index value. This ratio is referred to here as the “net migration ratio” (NM). Alaska’s NM varied greatly between 1970 and 1990 (see Figures 2 and 3), due in large part to the discovery of oil in Prudhoe Bay Alaska in 1968, subsequent construction and completion of the Trans-Alaska Pipeline System in the 1970’s, and a boom and bust in oil-related revenue during the 1980’s. Since 1990, this level of variation has been somewhat lower. (Hunsinger and Williams 2007.)

Though an event as large as the Prudhoe Bay discovery is considered unlikely during the forecast period, there is no doubt that major events can occur without much warning, and it’s useful to include such events in a survey of uncertainty. To manage this, a simple model of the chance and level of large-scale events is added after creation of random AR1 forecasts, which don’t include such large-scale shocks.

The first part of modeling uncertain NM is the selection and creation of the set of AR1 models that don’t include large-scale events, as is generally expected. Though AR1 modeling of migration has been carried out elsewhere (including Miller 2002, and Hartmann and Strandell 2006), rates and patterns of migration are highly area-specific, and it is important to account for local conditions as much as possible. φ , the correlation between two consecutive five-year values of NM, is not known for the forecast period, and no attempt is made to estimate it with historical data. Instead, a range for φ is selected by elimination of perceived-unreasonable values for it. Starting with a broadest consideration for φ , that is -1 to 1 (uniformly distributed), the chance of φ between -1 and -.5 is eliminated. This paring of uncertainty about φ is based on the judgment that, though there is chance for NM to be either positively or negatively correlated across the forecast period, any negative correlation over five-year intervals is (1) less likely, and (2) generally of a lower level.

In keeping with the Alaska Department of Labor and Workforce Development projection for migration (Hunsinger and Williams 2007), Alaska’s NM is forecast here to converge to an average of 0, so c is constrained to 0. This single forecast for c is deemed the most reasonable.

The AR1 residual error (ε) is set by judgment for five years forward to be normally distributed, with mean 0 and standard error .0243, so that there is no more than a 10 percent chance that NM will diverge by more than .04 (4 percent of the population) from the forecast mean (0) between any two forecast steps.

The following formulas describe the AR1 forecast for Alaska’s NM (X) (because the predicted mean is 0, the second term is just as well dropped, but it is left in here):

$$X_{i,t} = \varphi_i X_{i,t-1} + 0(1 - \varphi_i) + N(0, .0243)_{i,t}$$
$$\varphi_i = U(-.5, 1)_i$$

The second part of modeling NM is to add the possibility that a major event will occur, and disrupt the future of Alaska migration to a higher degree than reflected in

the related AR1 models. Following the technique used by Lee and Tuljapurkar (1998) for mortality (theirs was an extrapolation of history though, and not expert-based), to the AR1 simulations there is an added, expert-based, 20 percent chance that such an event will occur at some step in the forecast period and add .1 to NM, and an added 5 percent chance that such an event will occur during the forecast period and subtract .1 from NM. These major events are added separately to simulated AR1 forecasts for NM, and they are not autocorrelated. Multiple major events are allowed in a single forecast simulation.

Figure 2 here

Figure 3 here

2.3 Mortality

Mortality is projected for the 2005-10 through 2030-35 forecast steps using half the linear level term of the logit of remaining survivors by age in a period life table, also known as “Brass Alpha” (BA) (Keyfitz 1977), as the index value. While high-quality stochastic mortality forecasts most often use the “Lee-Carter method” (described in Lee and Carter 1992), mortality forecasts based on the extrapolation of BA have also been shown to be viable at least in the short- to medium-term (Hartmann and Strandell 2006).

An AR1 model with a fixed φ of 1, and a non-zero c , is used to define the predicted level, trend and uncertainty for BA. This type of AR1 model is usually referred to as a “random walk with drift.” While a range of values for φ on a BA time series model is an option to recognize, mortality generally has very high serial correlation (Lee and Carter 1992), and it’s believed that a random walk is singularly the best forecast model for it. To guide the forecast of BA, a short-term, knowledge-based forecast of Alaska’s life expectancy at birth (e_0) is made for the first forecast step, and resultant change in BA is calculated, and extrapolated for the remaining forecast steps.

Five-year averages of Alaska’s life expectancy at birth have increased from an approximated 66.8 in 1970-75 to 75.2 in 2000-2005 for males, and 74.6 in 1970-75 79.9 in 2000-2005 for females (see Figure 4). Assuming no knowledge of the period, the 2005-2010 e_0 ’s for males and females are predicted to be about .5 years higher than those for 2000-2005, with residual error having mean 0 and standard deviation .304 (90 percent within .5). This short-term prediction for the trend of e_0 roughly follows the total-population e_0 forecast by Miller (2002) for California, but the level of error (for five-year averages here) is fully based on expert judgment of short-term error.

Calibration of e_0 with BA is used to create base-year Alaska life tables by sex from life tables for the United States, and then to create life tables that match the author’s uncertain forecasts of five years forward for e_0 . Based on this, Alaska’s BA is predicted to increase by the approximate value of .03 for each five-year forecast step, with residual error having mean 0 and standard deviation .0182 (90 percent within .03).

The following formulas describe the forecast for Alaska's BA (X) (because the predicted φ is 1 in all cases, it can be removed from the formula, but it is left in here):

$$X_{i,t} = \varphi_i X_{i,t-1} + .03(1 - \varphi_i) + N(0, .0182)_{i,t}$$
$$\varphi_i = U(1, 1)_i$$

Figure 4 here

3 Application of the component forecasts

Using the expert-based time series models for each component of population change, 10,000 paths for TFR, NM and BA are applied in a cohort-component population projection model to create a database of 10,000 population forecasts by sex and five-year age group for Alaska. From this database, probabilistic statements on Alaska's future population can be made.

To make use of the forecast TFR's in the cohort-component method, they are multiplied by a proportional age specific profile for fertility from the National Center for Health Statistics to create age specific fertility rates, which are applied in population projection matrices to calculate the number of births for each five-year forecast step (Keyfitz 1977). This simple method for creation of age specific fertility rates from a TFR follows that of Lee (1974). While more careful methods (including Lee 1993) can account for predicted changes in the age profile for fertility, these should only have a second-order affect the forecast of births (Lee 1999). (See Figure 5.)

Figure 5 here

Forecast net migration by age and sex is calculated through the following steps: (1) Using data from Census 2000, a fixed crude out-migration rate is multiplied by the respective forecast step's initial population, and a proportional age profile for out-migration, to get a base amount of out-migration by age for both sexes. (2) Remaining in- and resultant net migration are determined by adding the calculated number of out-migrants to the forecast NM that is multiplied by the forecast step's initial population, and applying this number to a Census 2000-based proportional age profile for in-migration (see Figure 6). The forecast in- and out-migrants by age and sex are then added and subtracted, respectively, at each forecast step.

Figure 6 here

Forecast BA's are applied to a United States life table for each sex (from the National Center for Health Statistics) to calculate age- and sex specific survival rates at each forecast step, which are used in the construction of population projection matrices (Keyfitz 1977). (See Figure 7.)

Figure 7 here

4 Forecast population data

Based on 10,000 forecast simulations from the above-described models, probabilistic information about Alaska’s future population is made. Alaska’s total population will increase from 664,334 in 2005, to a middle value of 847,386 in 2035, with 90 percent range between 683,377 and 1,058,403 (see Figure 8 and Table 1). Forecast uncertainty increases greatly with each forecast step. Alaska’s growth rate for the forecast period has a median of 0.81 percent, with 90 percent range between 0.09 percent and 1.55 percent (see Figure 9 and Table 2).

Figure 8 here

Table 1: Selected quantiles for Alaska’s total population forecast

Year	.5%	5%	10%	25%	50%	75%	90%	95%	99.5%
2005:	664,334	664,334	664,334	664,334	664,334	664,334	664,334	664,334	664,334
2015:	635,894	675,269	688,339	708,967	730,411	752,800	776,975	796,642	851,023
2025:	608,085	687,436	713,464	752,444	794,505	840,271	891,656	926,462	1,037,470
2035:	547,158	683,377	725,030	784,819	847,386	917,065	1,000,175	1,058,403	1,276,267

Figure 9 here

Table 2: Selected quantiles for Alaska’s total population growth rate

Period	.5%	5%	10%	25%	50%	75%	90%	95%	99.5%
2005 to 2035:	-0.65%	0.09%	0.29%	0.56%	0.81%	1.07%	1.36%	1.55%	2.18%

One important purpose for population forecasts by age is planning for future fiscal balances. “Dependency ratios” are measures that indicate the ratio of a population age group that is generally subtracting from fiscal balances, to the population age group that is generally adding to fiscal balances. The “young-age dependency ratio,” which is the population aged 0-19 divided by the population aged 20-64 (the “working-age population”), is forecast to decrease to a median value of 52 percent in 2035, with 90 percent range between 41 percent and 65 percent (see Figure 10 and Table 3). There is an estimated 50 percent chance that the young-age dependency ratio will be no higher in 2035 than in 2005.

Due to substantial migration of young adults to Alaska in the 1970’s and 1980’s, a very large share of Alaska’s population in 2005 was aged 40 to 60 (Hunsinger and Williams 2007) (see Figure 12), and the population aged 65 or more will increase greatly during the forecast period. Alaska’s “old-age dependency ratio,” or the population aged 65 or more divided by the population ages 20-64, is projected to likely increase by between 100 and 200 percent during the forecast period. Specifically, from 11 percent in 2005, the old-age dependency ratio will increase to a median value of 30 percent in 2035, with 10 percent chance that it will be more than 36

percent or less than 25 percent (see Figure 11 and Table 4). Figure 13 provides a full image of the median forecast for Alaska’s 2035 population, by age and sex, with associated 90 percent confidence intervals.

Figure 10 here

Table 3: Selected quantiles for Alaska’s young-age dependency ratio forecast

Year	.5%	5%	10%	25%	50%	75%	90%	95%	99.5%
2005:	53%	53%	53%	53%	53%	53%	53%	53%	53%
2015:	41%	44%	45%	46%	48%	50%	52%	53%	55%
2025:	36%	42%	44%	48%	51%	56%	59%	62%	69%
2035:	32%	41%	44%	48%	52%	57%	62%	65%	74%

Figure 11 here

Table 4: Selected quantiles for Alaska’s old-age dependency ratio forecast

Year	.5%	5%	10%	25%	50%	75%	90%	95%	99.5%
2005:	11%	11%	11%	11%	11%	11%	11%	11%	11%
2015:	15%	16%	16%	16%	17%	17%	17%	18%	19%
2025:	21%	23%	24%	26%	27%	28%	30%	31%	34%
2035:	21%	25%	26%	28%	30%	32%	34%	36%	45%

Figure 12 here

Figure 13 here

5 Conclusion and thoughts

Using expert judgment to select time series models for the components of population change, a consistent and reasonable stochastic population forecast has been created for Alaska. Uncertainty about Alaska’s population grows with each forecast step. There is a 50 percent chance that the young-age dependency ratio will not increase between 2005 and 2035, and greater than 95 percent chance that the old-age dependency ratio will more than double.

This stochastic population forecast, like all population forecasts, is subject to the interpretations, knowledge and inputs of the expert. No population forecaster can know what the parameters of component AR1 models should be for the future, but the techniques used here allow description of expert-based uncertainty through AR1 models, and creation of realistic distributions for future population.

6 References

6.1 Literature sources

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Hartmann, M., Strandell, G. (2006). "Stochastic Population Projections for Sweden." A methodology report from Statistics Sweden. Available online.

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Miller, T. (2002). "California's Uncertain Population Future." Technical appendix for Lee, R., Miller, T., Edwards, R. (2003). "The Growth and Aging of California's Population: Demographic and Fiscal Projections, Characteristics and Service Needs." Available online.

Ortman, J. and Guarneri, C. (2009). "United States Population Projections: 2000 to 2050." An analytical document from the United States Census Bureau. Available online.

6.2 Data and computing code sources

July 1, 2005 population estimates for Alaska, by five-year age groups and sex: Alaska Department of Labor and Workforce Development, Vintage 2009 Estimates. Available online.

Age specific fertility rate profile (national profile): National Center for Health Statistics, National Vital Statistics Reports, Births: Final Data for 2005. Available online.

Age specific mortality rates (national life table): National Center for Health Statistics, National Vital Statistics Reports, United States Life Tables, 2000. Available online.

Out-migration and in-migration age profiles: United States Census Bureau, 2000 Census, Migration by Sex and Age for the Population 5 Years and Over for the United States, Regions, States, and Puerto Rico: 2000 (PHC-T-23). Available online.

The R statistical software code that was developed by the author for this work is available online. The basis for that R code was developed by the author for the Alaska Department of Labor and Workforce Development's population projections program.

Figure 1:

TOTAL FERTILITY RATE: HISTORICAL AND 50 FORECASTS WITH 90% CONFIDENCE INTERVALS (BOLD BLACK)

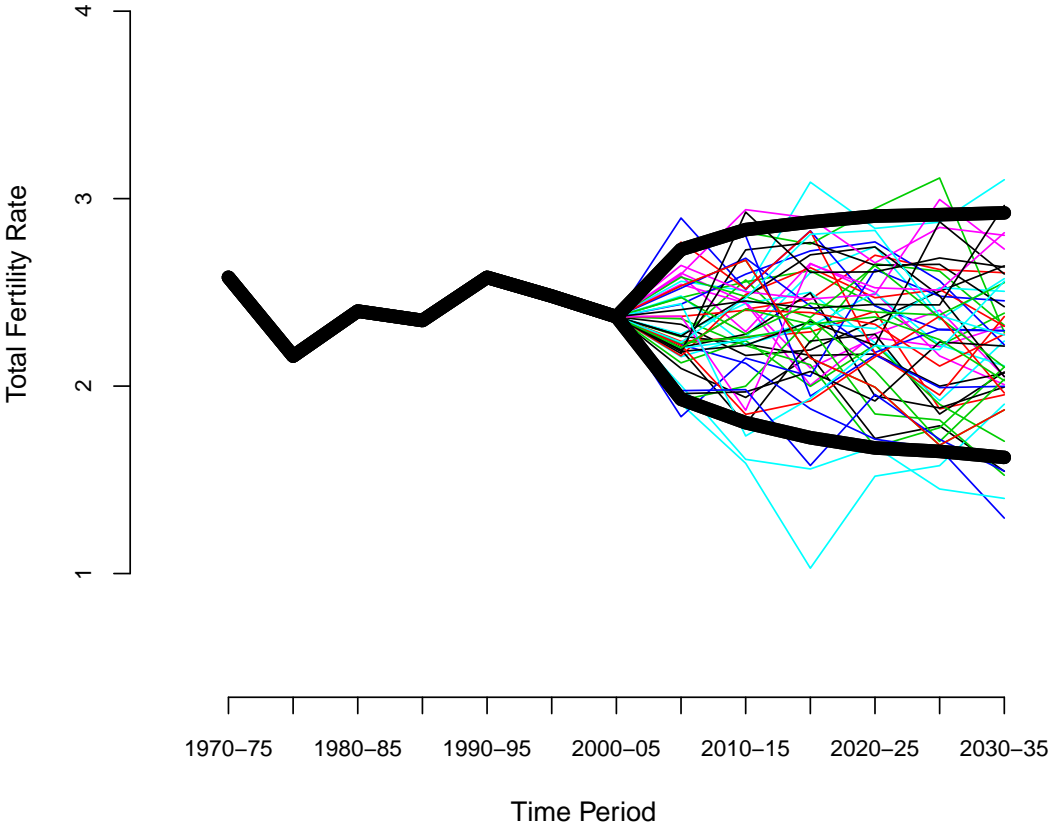


Figure 2:

NET MIGRATION RATIO: HISTORICAL AND 50 FORECASTS WITH 90% CONFIDENCE INTERVALS (BOLD BLACK)

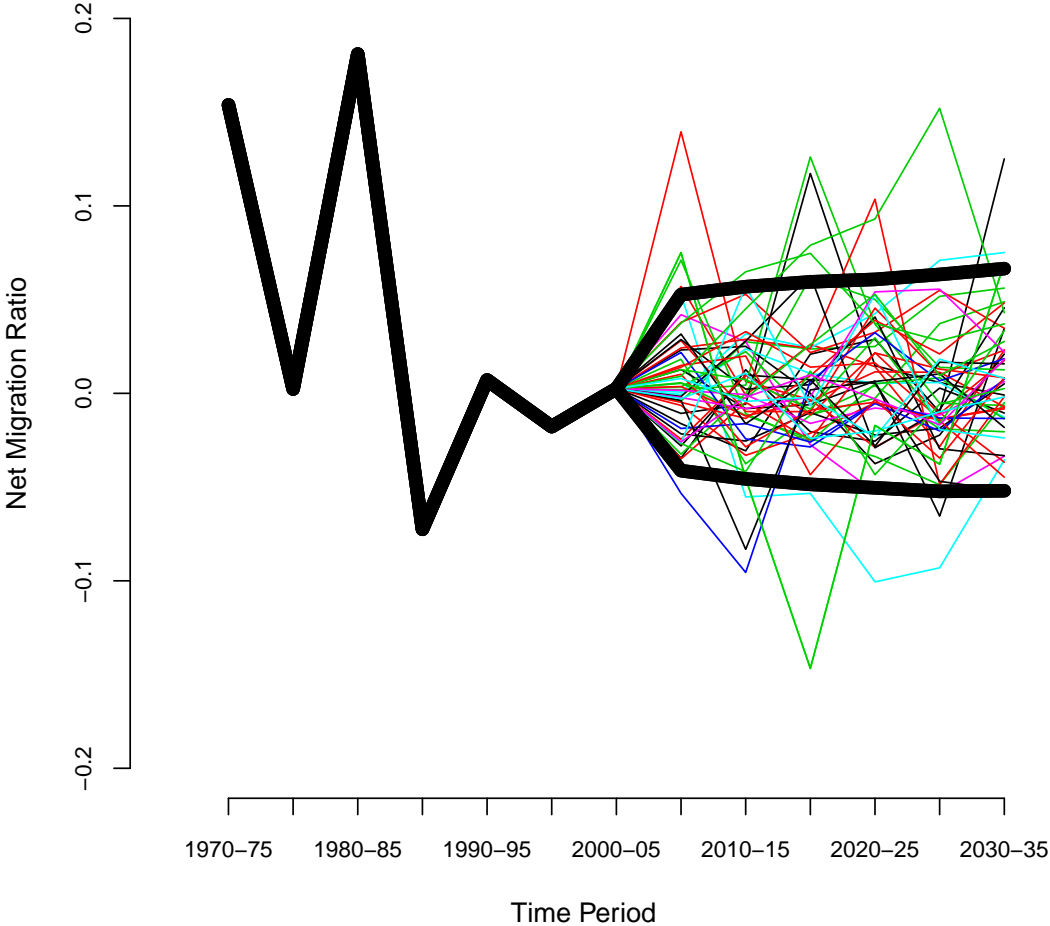


Figure 3:

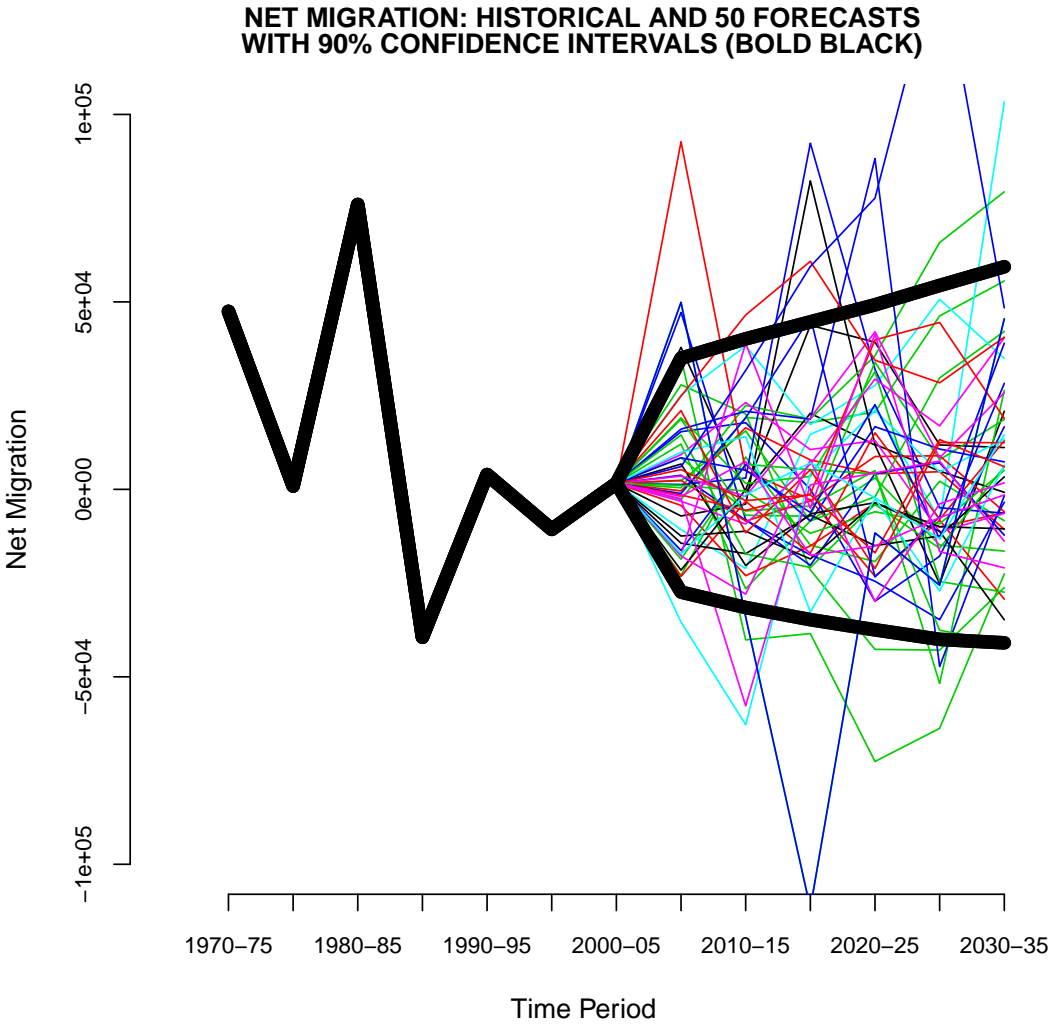


Figure 4:

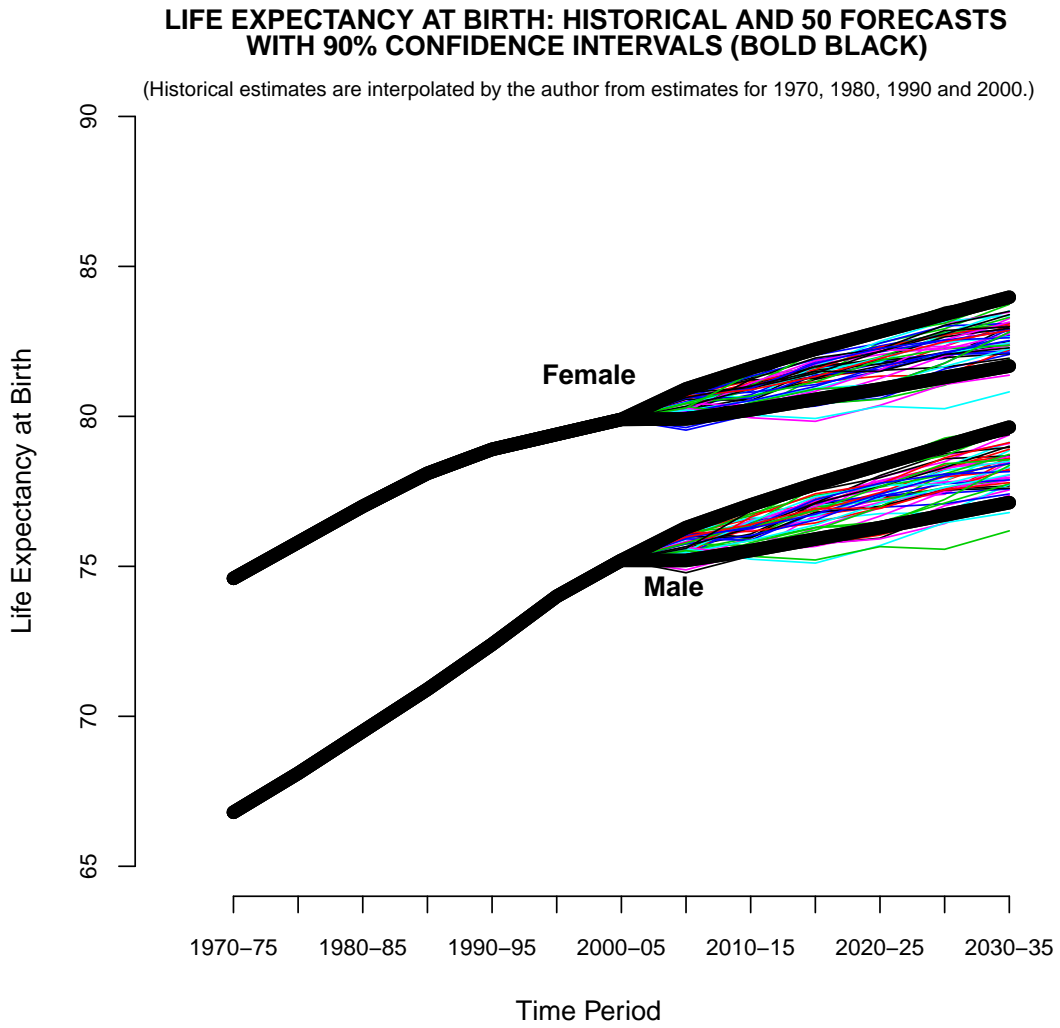


Figure 5:

**50 AGE SPECIFIC FERTILITY RATE FORECASTS FOR 2030-2035
WITH 90% CONFIDENCE INTERVALS (BOLD BLACK)**

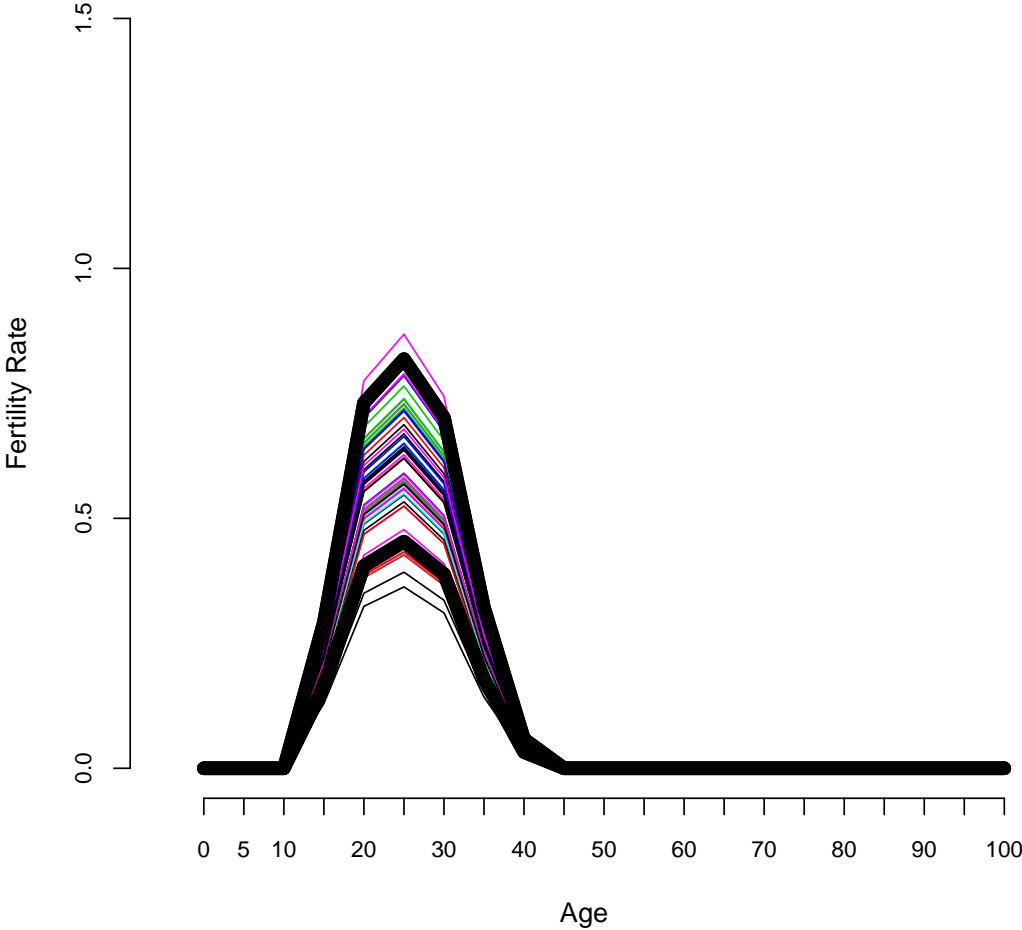


Figure 6:

**50 NET MIGRATION BY AGE FORECASTS FOR 2030–2035
WITH 90% CONFIDENCE INTERVALS (BOLD BLACK)**

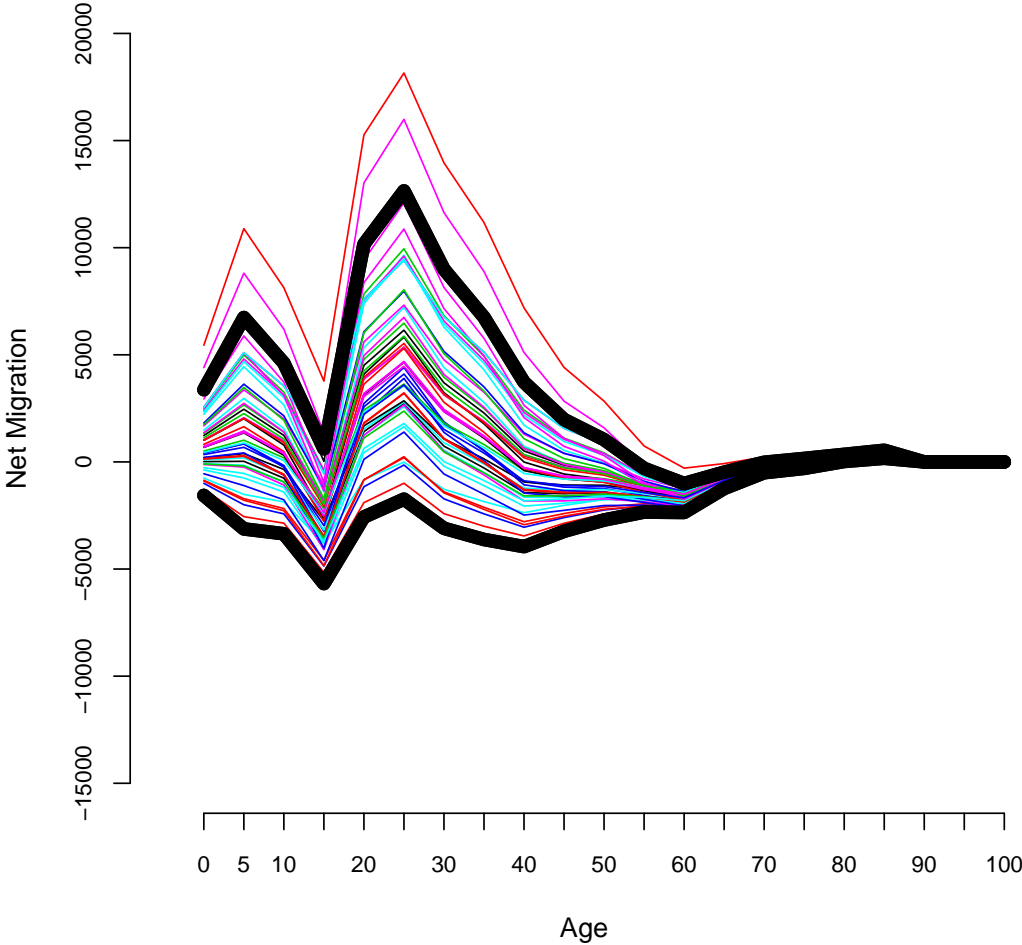


Figure 7:

**50 PERIOD LIFE TABLE SURVIVORSHIP FORECASTS FOR 2030-2035
WITH 90% CONFIDENCE INTERVALS (BOLD BLACK)**

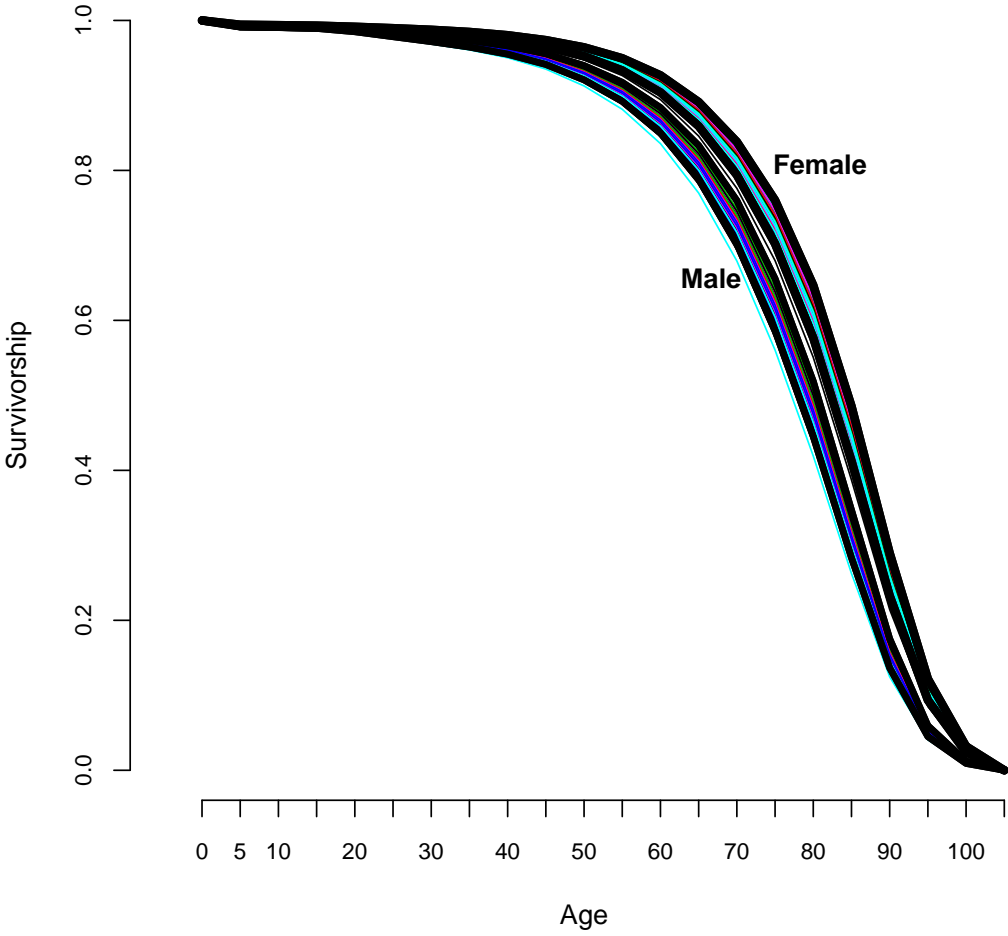


Figure 8:

TOTAL POPULATION: HISTORICAL AND 50 FORECASTS WITH 90% CONFIDENCE INTERVALS (BOLD BLACK)

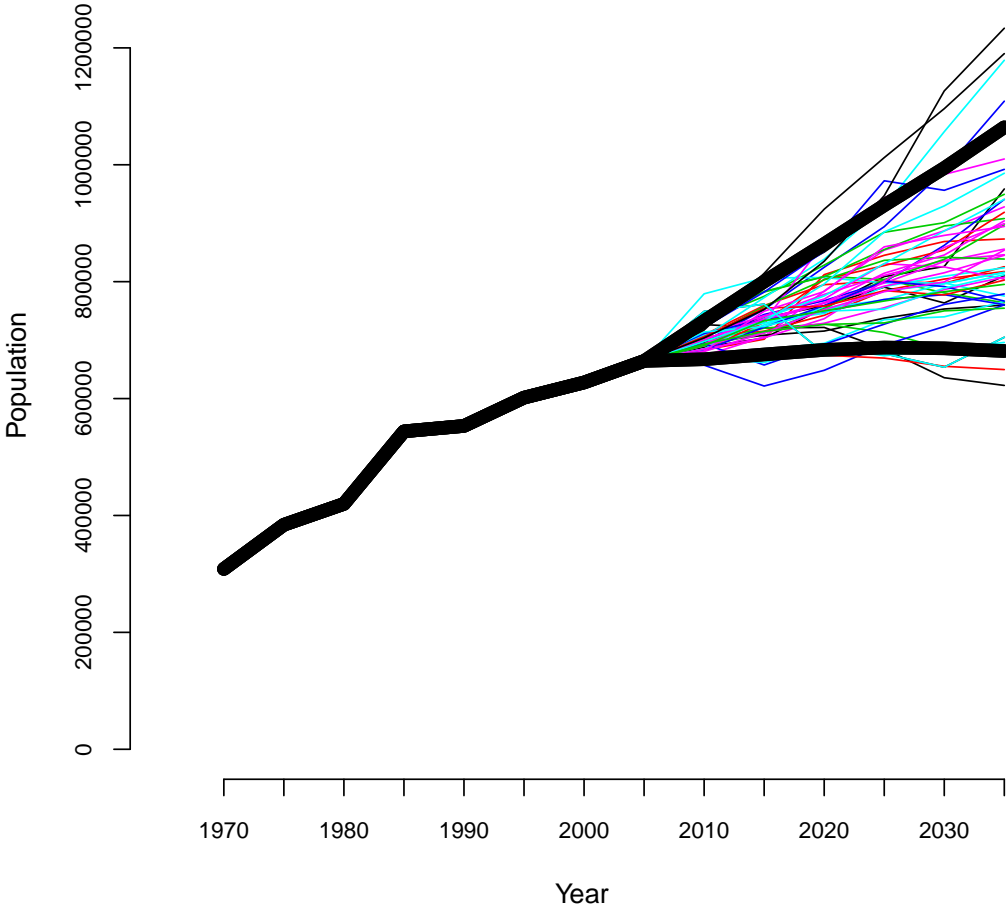


Figure 9:

**TOTAL POPULATION GROWTH RATE: HISTOGRAM
OF THE FORECASTS FOR 2005 THROUGH 2035**

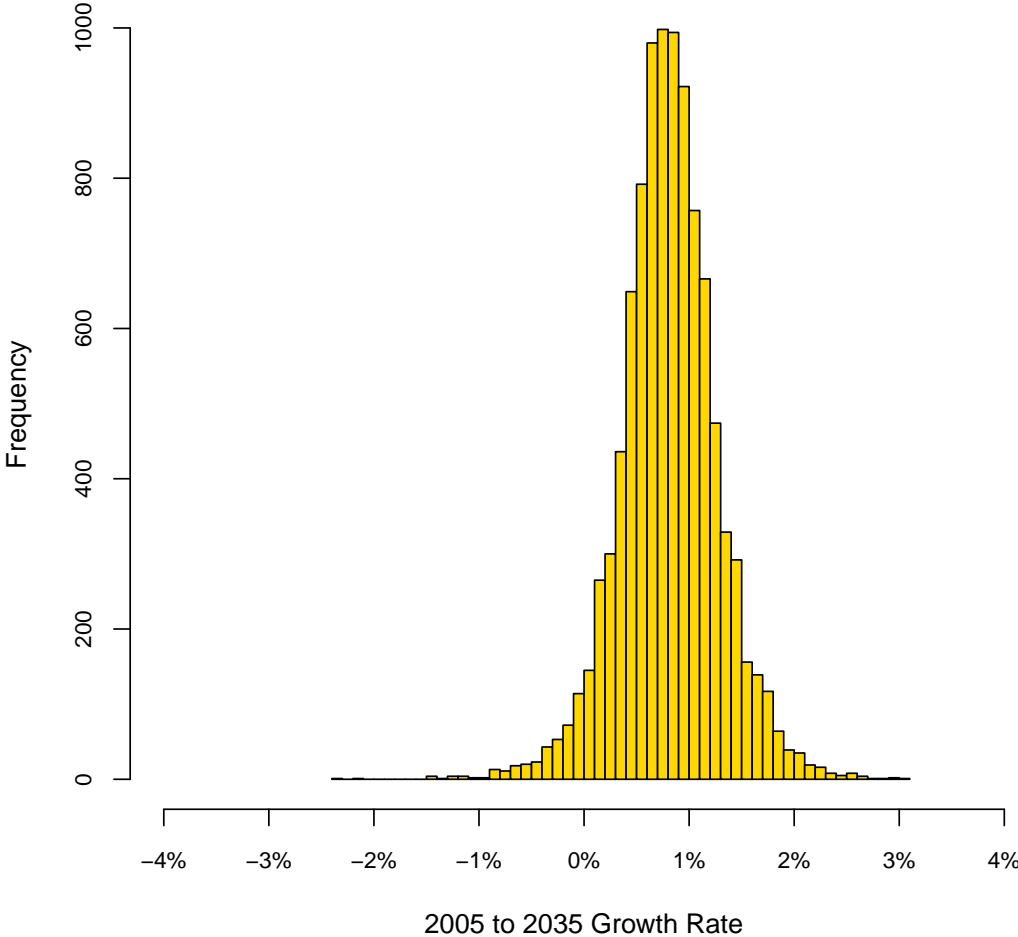


Figure 10:

YOUNG-AGE DEPENDENCY RATIO: 50 FORECASTS WITH 90% CONFIDENCE INTERVALS (BOLD BLACK)

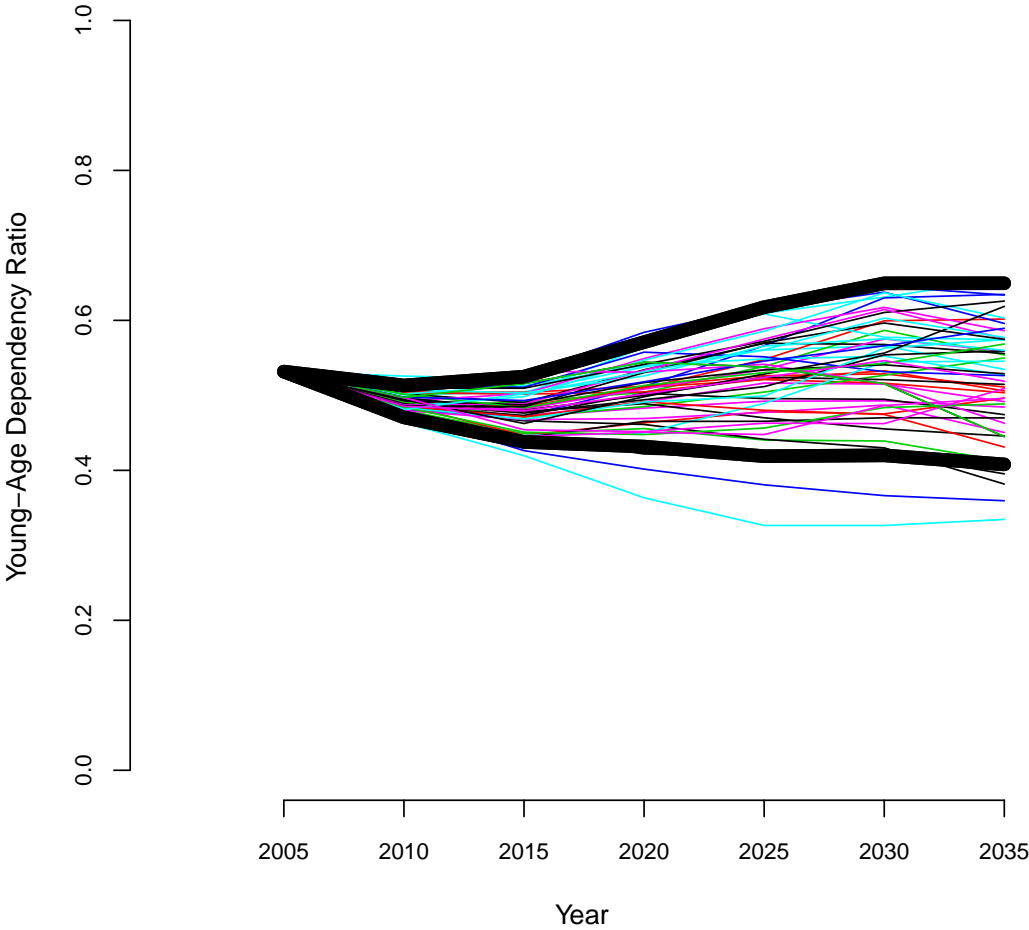


Figure 11:

**OLD-AGE DEPENDENCY RATIO: 50 FORECASTS
WITH 90% CONFIDENCE INTERVALS (BOLD BLACK)**

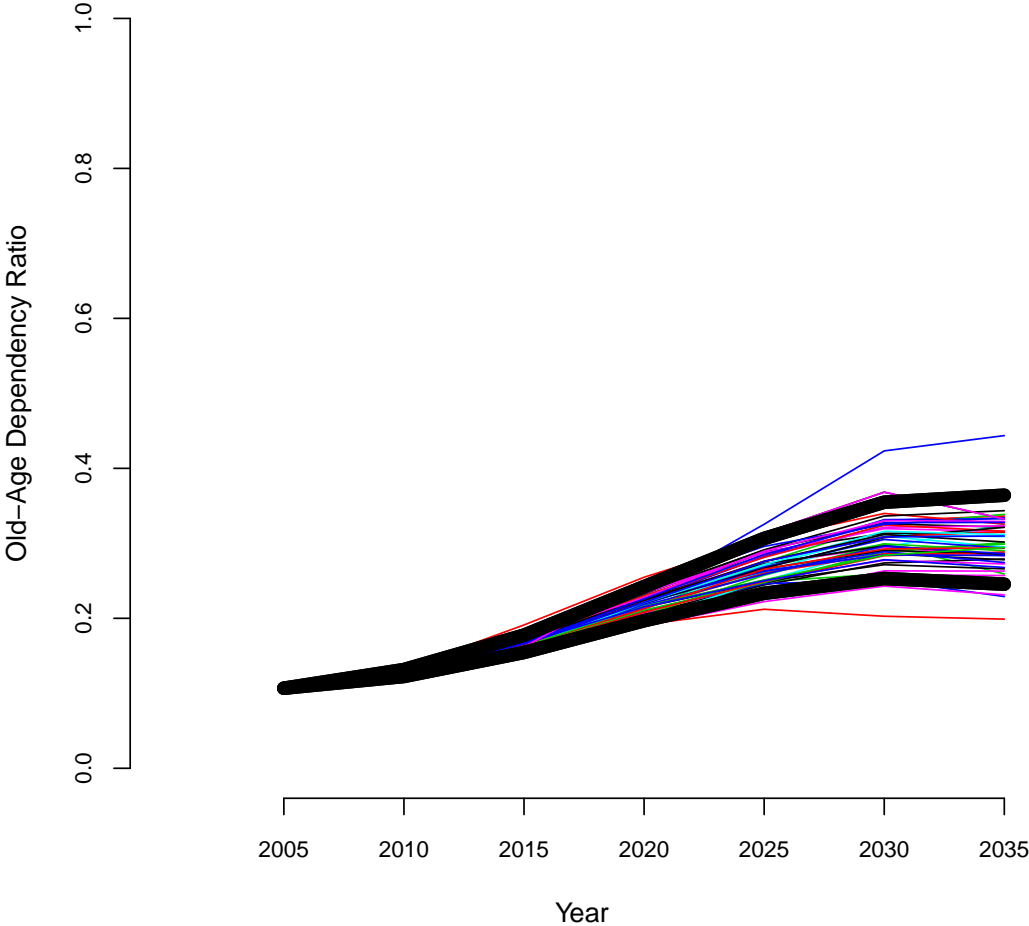


Figure 12:

ALASKA 2005 POPULATION ESTIMATES BY AGE AND SEX

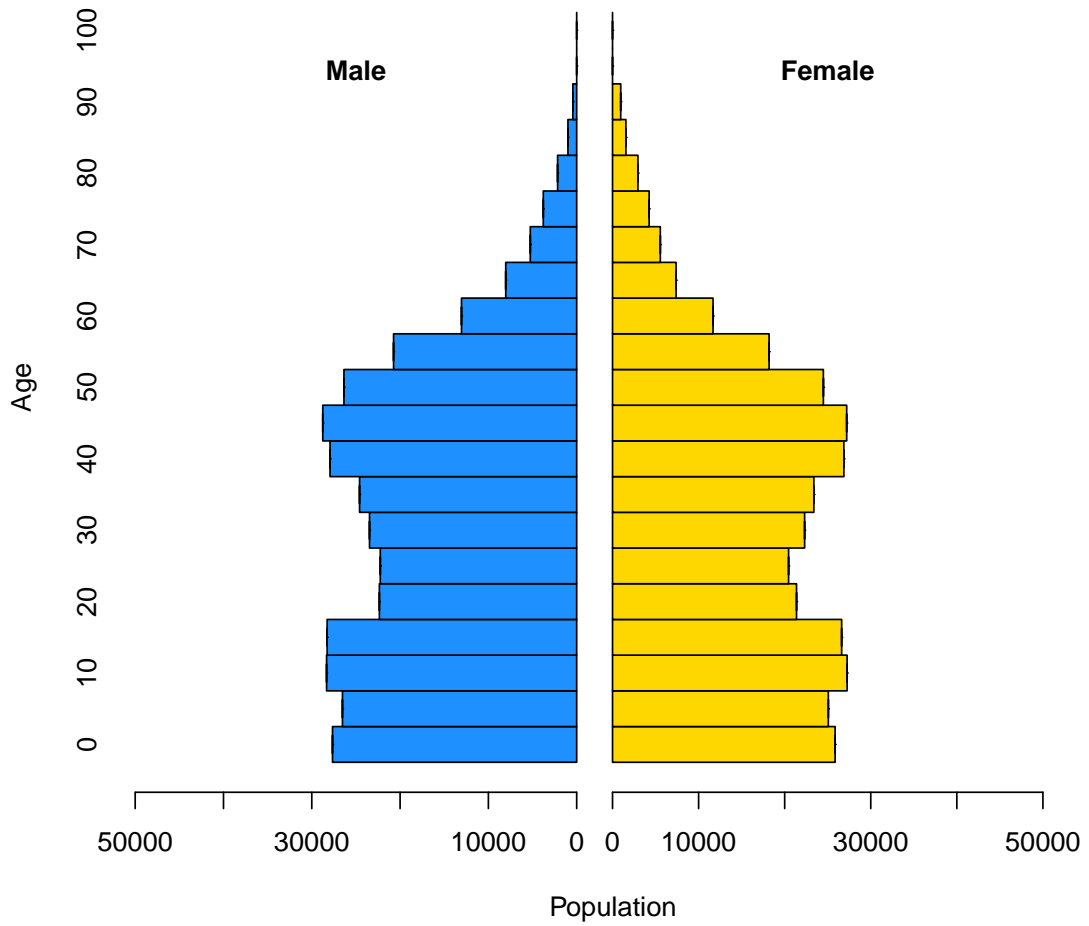


Figure 13:

